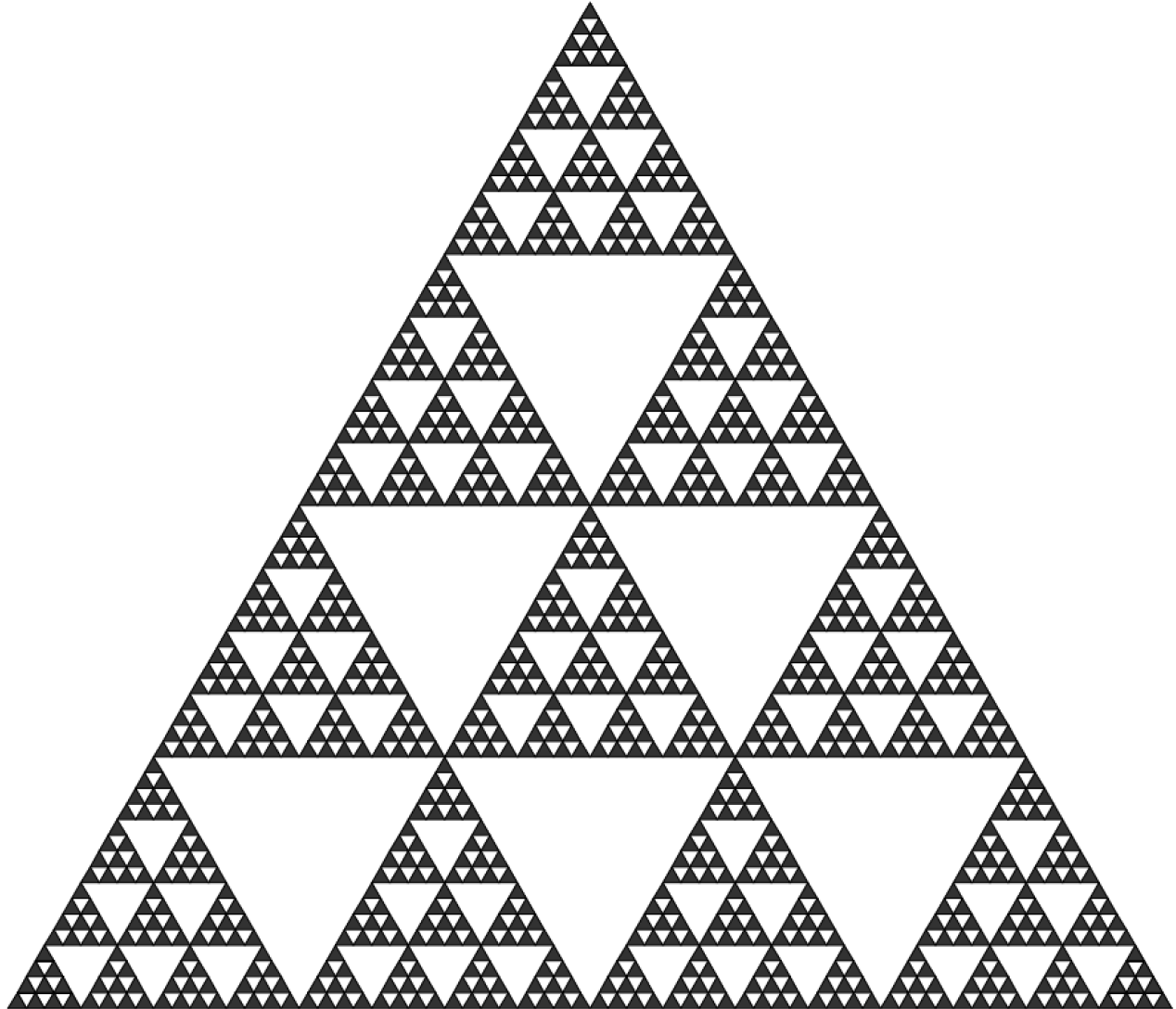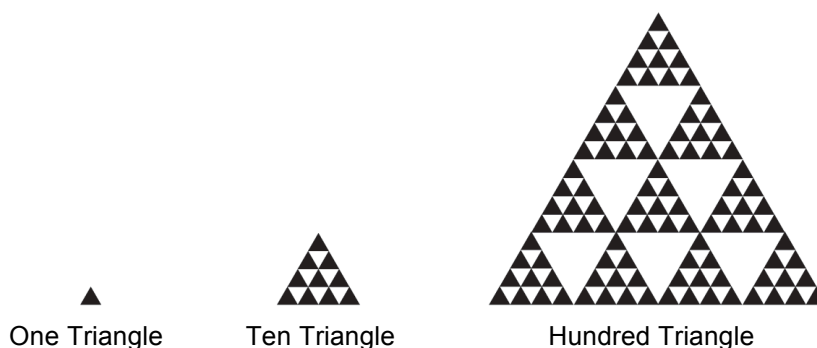# COTTER TENS FRACTAL

Fractals are a new branch of mathematics that is only a few decades old. There are two types of fractals, random and regular. Scientists use random fractals for computer modeling in order to study some of nature's irregular patterns and structures. Regular fractals, also called geometric fractals, consist of larger structures that are identical in shape to the smaller structure. The Cotter Tens Fractal is a regular fractal.

The Cotter Tens Fractal helps the child experience our place-value system concretely by representing how our number system is constructed with tens. It starts with ten small colored equilateral triangles arranged in the shape of a larger equilateral triangle. This is the Ten Triangle. Ten of these triangles arranged in the same pattern form the Hundred Triangle. A One Triangle, a Ten Triangle, and a Hundred Triangle are shown on the next page.

COTTER TENS FRACTAL

One Triangle   Ten Triangle   Hundred Triangle

Ten Hundred Triangles in the same pattern form the Thousand Triangle, the Cotter Tens Fractal. This fractal will expand indefinitely in the same way as our number system does.

One purpose of building this fractal is to help the children visualize and experience the repeating tens structure of our number system. Another purpose is integrating mathematics and art.

There are three methods to build the Cotter Tens Fractal. Descriptions and materials needed are listed below.

Method 1.

This method will build a Ten Triangle, then use ten of the Ten Triangles to create a Hundred Triangle. Finally, Ten Hundred Triangles are needed to create the third iteration. All together, one thousand triangles will be needed.

This is best done with a group of children. Although is labor intensive and time consuming, the results are unforgettable.

Materials Needed:
- Δ Worksheet 1.1 – need 10 copies on bright or dark colored paper
- Δ Worksheet 1.2 – need 25 copies on white paper or same color as the background paper
- Δ 8½" by 11" paper – 40 sheets of white or same color as the background paper
- Δ Tape and glue
- Δ Background paper* – two sheets of paper 36" wide and 85" long. Often this can be found at an office supply shop, print shop, or newspaper press. Sheets of paper can also be taped together.
     - * Some have chosen to go without a background; they tape the Hundred Triangles directly onto a wall. See photos on pages 9 and 10.

COTTER TENS FRACTAL

Method 2.

This second method uses pre-printed Tens Triangles. This means that only one hundred of the triangles are needed, rather than the thousand little triangles. The results are easier to obtain and is almost five feet tall, just like the first method.

Materials Needed:
- Δ Worksheet 1.1 – need one copy on blue paper
- Δ Worksheet 1.2 – need one copy on white paper or same color as the background paper
- Δ Worksheet 2 – need 25 copies
- Δ 8½" by 11" paper – 40 sheets of white paper
- Δ Tape and glue
- Δ Background paper* – two sheets of paper 36" wide and 85" long. Often this can be found at an office supply shop, print shop, or newspaper press. Sheets of paper can also be taped together.
   * Some have chosen to go without a background; they tape the Hundred Triangles directly onto a wall. See photos on pages 9 and 10.

Method 3.

The third method uses pre-printed Ten Triangles to create the Hundred Triangle and pre-printed Hundred Triangles to create the Cotter Tens Fractal. Although the growth of the fractal is not quite as obvious, this method is a quicker and often appreciated when the number of people involved is low or space is limited.
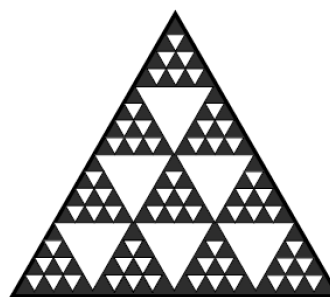
Ten Triangle   Hundred Triangle
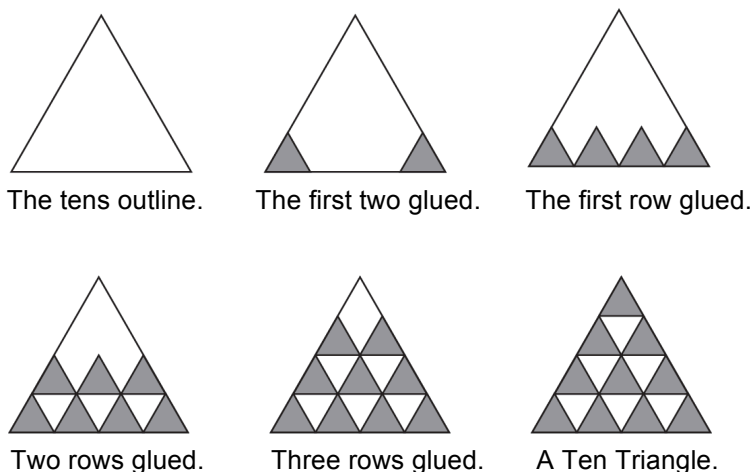
Materials Needed:
- Δ Worksheet 3.1 – need one copy for every two children
- Δ Worksheet 3.2 – need one copy for every two children
- Δ Worksheet 3.3 – need one copy for every two children
- Δ Worksheet 3.4 – need three copies for every child
- Δ Glue

Cotter Tens Fractal

The First Iteration: Making the Ten Triangle, Method 1.

Prepare by copying Worksheet 1.1 on colored paper; you will need 10 copies on the same color paper. Copy Worksheet 1.2 on a white paper; you will need 25 copies.

Give the child the strips of small equilateral triangles from Worksheet 1.1 to cut out. Remind them that they must be cut very carefully. Each sheet has 105 triangles, or 5 extra triangles. Also cut out the medium triangles from Worksheet 1.2.

Show them how to make the pattern with 10 of the triangles. Start by placing four triangles in a row along the bottom edge of the medium triangle from Worksheet 1.2. Then place three triangles above those, two above those, and the last one on top. See the figures below.

The tens outline.	The first two glued.	The first row glued.

Two rows glued.	Three rows glued.	A Ten Triangle.

Gluing starts the same way, starting from the bottom row. Apply weights overnight to keep them flat. Make 100 of these Ten Triangles.

To help the child keep the concept in mind, ask: How many little colored triangles are in each medium triangle? [10] How many medium triangles do you need to make a hundred? [10]

The First Iteration: Making the Ten Triangle, Method 2.

To build understanding of the fractal, use the same process as Method 1. Build only one triangle per child; each child will need 10 small triangles form Worksheet 1.1 and 1 medium triangle from Worksheet 1.2. Pre-printed Ten Triangles will be used for the second iteration.

Cotter Tens Fractal

The First Iteration: Making the Ten Triangle, Method 3.

Cut out ten equilateral triangles from Worksheet 3.1. Although two sets of ten are included on the worksheet, only one set of ten is needed per Ten Triangle.

Arrange the ten equilateral triangles as shown in the left figure below. Start by placing four triangles in a row along the bottom edge of the medium triangle from Worksheet 3.4. Then place three triangles above those, two above those, and the last one on top. See the figures below. Glue each triangle in place to make the Ten Triangle.

Ten small triangles arranged
to form the Ten Triangle.
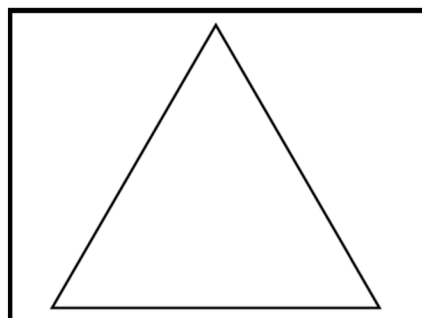
Worksheet 3.4 on which to glue
the 10 small triangles.

To help the child keep the concept in mind, ask: How many little colored triangles are in each medium triangle? [10] How many medium triangles do you need to make a hundred? [10]

The Second Iteration: Building the Hundred Triangle, Method 1.

To create the Hundred Triangle, a base is needed. Tape four sheets of 8½" by 11" paper together to create a 17" by 22" rectangle.

Tell the child that now they can make the Hundred Triangles. Take 10 Ten Triangles and lay them out on the base the same way as they did in the previous iteration: four on the bottom, three on the next row, then two and one Ten Triangle at the top. See the figure on the next page.

Tell them to glue the Ten Triangles in place. When the glue is dry, trim the base paper to the shape of the triangle.

COTTER TENS FRACTAL
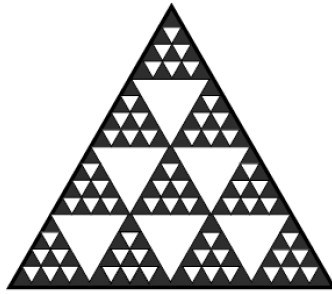
Hundred Triangle

Ask the child: How many Ten Triangles do you see? [10] How many small colored triangles do you see? [10-ten, or 1 hundred] To help them understand, count the ten-triangles as 1-ten, 2-ten, 3-ten, and so forth. At 10-ten ask: What is another name for 10-ten? [1 hundred]

Make a total of 10 Hundred Triangles. When these are completed, ask the child to guess what is next. [building the Thousand Triangle]

The Second Iteration: Building the Hundred Triangle, Method 2.

To create the Hundred Triangle, a base is needed. Tape four sheets of 8½" by 11" paper together to create a 17" by 22" rectangle.

Prepare by copying Worksheet 2; you will need 25 copies. Have the child cut out the four Ten Triangles (not the tiny triangles) on each sheet for a total of 100 Ten Triangles. Remind them that they must be cut very carefully. The Ten Triangle made in the first iteration may be used in place of a pre-printed Ten Triangle.

Tell the child that now they can make the Hundred Triangles. Take 10 Ten Triangles and lay them out on the base the same way as they did in the previous iteration: four on the bottom, three on the next row, then two and one Ten Triangle at the top. See the figure above.

Tell them to glue the Ten Triangles in place. When the glue is dry, trim the base paper to the shape of the triangle.

Ask the child: How many Ten Triangles do you see? [10] How many small colored triangles do you see? [10-ten, or 1 hundred] To help them understand, count the ten-triangles as 1-ten, 2-ten, 3-ten, and so forth. At 10-ten ask: What is another name for 10-ten? [1 hundred]

Make a total of 10 Hundred Triangles. When these are completed, ask the child to guess what is next. [building the Thousand Triangle]

COTTER TENS FRACTAL

The Second Iteration: Building the Hundred Triangle, Method 3.

Tell the child to cut out the Ten Triangles (not the tiny triangles) from Worksheet 3.2. Although two sets of ten Ten Triangles are included on the worksheet, only one set of ten is needed per Hundred Triangle.

After the triangles are cut out, ask: Can you show me 2-ten? [two tens triangles] Can you show me 4-ten? [4 triangles] Can you show me ten? [1 ten triangle] These questions will help the child understand the relationships and representations of the triangles.

Arrange the 10 Ten Triangles as shown in the figures below. Start by placing four triangles in a row along the bottom edge of the medium triangle from Worksheet 3.4. Then place three triangles above those, two above those, and the last one on top.

Tell the child to glue each triangle in place to make the Hundred Triangle.
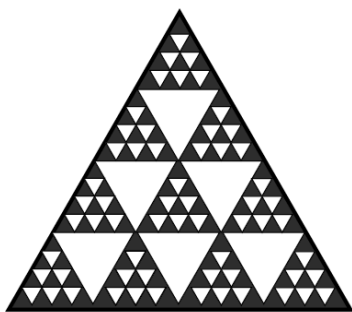
Ten Ten Triangles arranged
to form the Hundred Triangle.
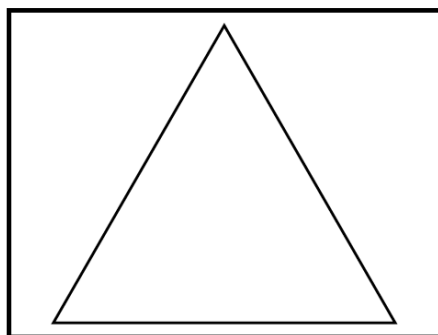
Worksheet 4 on which to glue
the 10 Ten Triangles.

Ask the child to compare this Hundred Triangle to the Tens Triangle done previously. Ask: Where do you see ten dark triangles? [any of the triangles grouped in tens] How many little dark triangles are in the whole fractal? [1 hundred] For a higher level thinking question, ask: If you had ten of these triangles, how many little triangles would you have altogether? [one thousand]

COTTER TENS FRACTAL

The Third Iteration: Building the Thousand Triangle, Method 1.

To create the Thousand Triangle, background paper is needed. Tape together two sheets of paper 36" wide and 85" long creating a 72" by 85" rectangle. Sheets of paper can also be taped to create the necessary background. Some have chosen to go without a background; they simply tape the Hundred Triangles directly onto a wall.

Lay out the 10 Hundred Triangles, using the same pattern as before to build the Thousand Triangle. Then tell them to glue the Hundred Triangles in place on the background paper, then fasten the fractal to a wall or tape the Hundred Triangles to a wall using the same layout.

In conclusion, ask: How many little colored triangles are in the Cotter Tens Fractal? [1000]

The Third Iteration: Building the Thousand Triangle, Method 2.

To create the Thousand Triangle, background paper is needed. Tape together two sheets of paper 36" wide and 85" long creating a 72" by 85" rectangle. Sheets of paper can also be taped to create the necessary background.

Lay out the 10 Hundred Triangles, using the same pattern as before to build the Thousand Triangle. Then tell them to glue the Hundred Triangles in place on the background paper, then fasten the fractal to a wall or tape the Hundred Triangles to a wall using the same layout.

In conclusion, ask: How many little colored triangles are in the Cotter Tens Fractal? [1000]

The Third Iteration: Building the Thousand Triangle, Method 3.

Ask the child to cut out the ten Hundred Triangles (not the tiny triangles) from Worksheet 3.3. After they are cut out, say: Show me 2 hundred. [two Hundred Triangles] Show me 4 hundred. [4 triangles] Show me 1 hundred. [1 triangle]

Ask the child to arrange the 10 triangles as done in the previous iterations. Start by placing four triangles in a row along the bottom edge of the medium triangle from Worksheet 3.4. Then place three triangles above those, two above those, and the last one on top. See figures on the next page. Tell them to glue each triangle in place to make the Thousand Triangle, the Cotter Tens Fractal.

COTTER TENS FRACTAL
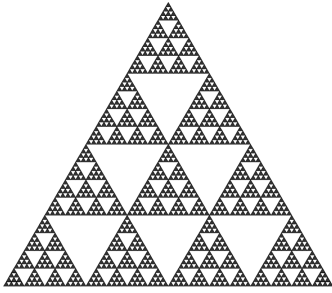
Ten Hundred Triangles form
the Thousand Triangle.
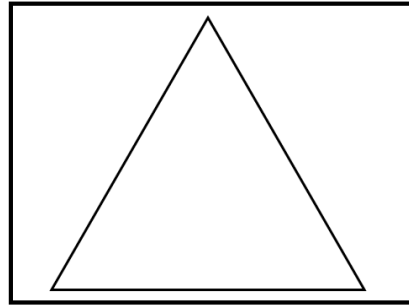
Worksheet 4 on which to glue
the 10 Hundred Triangles.

Ask the child to compare this Thousand Triangle to the Ten Triangle they did with the first iteration and the Hundred Triangle from the second iteration. In conclusion, ask: How many little dark triangles are in the Cotter Tens Fractal? [1000]

Show us your work.

We would love to see your work! If you'd like us to post your Cotter Tens Fractal on our website for others to admire, please send your photos to info@RightStartMath.com.

Cotter Tens Fractals using Method 1 without a background.

Cotter Tens Fractal

Michael's family helped with his Cotter Tens Fractal.

A group of children working on the Cotter Tens Fractal from start …..

…..to finish!!

Cotter Tens Fractal using a dramatic background color.