Math Card Games

Over 300 Games for Learning and Enjoying Math

fifth edition

Joan A. Cotter, Ph.D.

ABOUT THE AUTHOR

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Dr. Cotter designed the double-sided AL Abacus and wrote the pre-school to fourth grade *RightStart™ Mathematics* program and *RightStart™ Mathematics: A Hands on Geometric Approach* for middle school.

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TABLE OF CONTENTS

How to use the *Math Card Games* manual

Wow! Over three hundred games! Where do you begin? Well, here are some pointers to help you maximize the amazing learning tool you have right here in your hands.

1. This manual is broken into eight chapters: Number Sense, Addition, Clocks, Multiplication, Money, Subtraction, Division, and Fractions.

2. Each chapter begins with the easiest game on that topic then gets harder and harder as the chapter progresses.

3. Read the game's objectives. This will help you identify which game is best for a topic or need. You will find various games with the same objective, such as *Go To the Dump*, #A3, *Handshaking Game*, #A3.1, *Old Main*, #A4, and *Fish at the Dump*, #A5. All of these games focus on the facts of ten. The various games provide the students variety while they continue to work on a skill or concept.

4. Make notes as to what games were played when, by whom, and whether it was enjoyed or not. Just make general notes next to the game right in this manual, which will help you in a number of ways.

   First, fun and easy games are quickly identified and can be a review on more challenging days.

   Second, it will help identify games that the child found challenging, whether they were difficult because they weren't ready for that game, because the child was having a rough day in general, or because they just don't like that particular type of game. Of course, try the game at a later date to see if the attitude has changed.

   Third, it will provide a log of game play. Remember, 10 to 15 minutes of a game is the same as a worksheet. So when you look at the log, you can see the time spent practicing and working on math skills. A log form can be found on the next page as well as in the front of the student's workbook.

5. There is a DVD in the back of the manual. This will show you how to play 14 games. These same games are also found on YouTube.com; do a search for "RightStart Mathematics" and the games will be found under the user name of "ALabacusMathFun."

6. If you have questions on how to play a game, please call 888-272-3291 and one of our helpful customer service people will be delighted to assist you.

Enjoy this book. Keep it around for years to come and have fun with math. Have a great day and play a math card game!

Joan A. Cotter, Ph.D., Kathleen Cotter Lawler,
and the RightStart™ Mathematics team

Game Log

Date	Game Played	Players

Games help children understand, apply, and enjoy mathematics.

INTRODUCTION

Mathematics is a fundamental force in this technological and information age. The field has doubled in the last thirty years. It is critically important that every child master mathematics to become a productive member of society and an informed citizen. It is unfortunate when a person cannot pursue a desired career because of a lack of the necessary background in mathematics.

Mathematics education has alternated between teaching by rote and teaching for understanding. Rote learning is high maintenance and has led to one-third of the school year being spent reviewing. Many people with learning disabilities, about one out of seven, find rote memorization extremely difficult and cannot rote memorize 400 facts. Even though teaching for understanding takes more time initially, much less time needs to be spent in review. Students who understand math retain it longer, can apply it to new situations, and have a greater chance to succeed in advanced math courses.

This manual has games for learning all the major concepts of arithmetic with special emphasis on memorizing the facts. Counting for the facts is slow, tedious, and often inaccurate. Recent research shows that songs or rhymes learned to help memorize multiplication tables are stored in the language part of the brain, making them less useful for math. Visual strategies based on fives and tens are quicker and more permanent. These games are a fun and interesting way to learn basic math.

Mindy Holte said, “In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic.” The AL abacus, based on fives and tens, is an ideal tool for visual learning. I asked five-year-old Stan how much 11 plus 6 is. When he said 17, I asked him how he knew. He explained by saying, “I have the abacus in my mind.” This fourth edition eliminates the “dot strips” that were used in earlier editions and uses the abacus more.

WHY PLAY GAMES

When a child learns to read, she can practice those skills by reading for pleasure. In the same way, these card games combine practice with pleasure. Although learning math requires hard work, it can be enjoyable. When a person is interested in and loves his work, he can more easily tackle the distasteful segments found in any activity.

These games allow adults and children of various ages and abilities to play together. It does away with anxiety-producing flash cards, which cast the parent or teacher in the role of judge. The only person who enjoys flash cards is the person who doesn’t need them. Games create a stress-free atmosphere that allows all to learn at their own pace. Then the parent or teacher becomes a partner in the learning process.

There is another reason to make mathematics enjoyable for children. Along with the information recorded in our memories are the feelings we experienced when we learned it. These feelings are often recalled along with the fact or experience. For this reason, information stored with negative feelings tends to be forgotten. Children who associate math with feelings of failure and inadequacy will find learning difficult, and worse yet, they will ignore applications to daily life. When children recall feelings of discovery and success, they will want to continue learning and will apply that knowledge to other areas. Therefore, it is important that learning be a pleasant experience.

THE MATH GAMES

The games teach the players math while they play. The players need not know their facts before playing.

The principles that influenced the design of these games are as follows.

1. A logical and organized approach using visual strategies.
2. Concrete and mental work before paper work.
3. Minimal counting.
4. Manipulatives that are visualizable as well as visual.
5. Concept or strategy explained.
6. Interesting repetition.
7. Some method for error detection.

Rote memorizing is a low-level thinking skill. Strategies, on the other hand, give children confidence and independence. An example of a strategy is finding 9 + 6 by taking 1 from the 6 and giving it to the 9 to make 10 and 5, which is 15. Children who learn strategies have better number sense and are less likely to resort to finger counting.

Manipulatives, such as an abacus or tables, are not to be regarded as crutches. They enable the children to build a mental model, necessary for concept formation. In practice, children will refer to them less and less and finally not at all. Let each child decide when he no longer needs them. Sometimes just the security of having them nearby helps, even if they are not looked at. At the right time, a child may respond to the challenge of playing without them.

What looks like a simple step to us is often several steps for the child. That explains the variety of games. The games progressively get harder, building on previous concepts. The background section found in some of the games offers suggestions for presenting new concepts. Often a concept can be learned in more than one way, resulting in several games for the same concept.

We know how useless it is to have a corrected math paper returned a week later. To be helpful, errors must be corrected immediately. The games allow the players to discover errors themselves or with the help of the other players. Most of the solitaires cannot be won if an error is made.

PLAYING THE GAMES

This manual can be used with any mathematics program. The games provide a way to help both children needing remediation and those needing enrichment. Playing the games over the summer months keeps the players ready to learn more mathematics.

Who can teach

Anyone can play these games with children. Once the children know how to play, they can play by themselves. To help two groups simultaneously, form two circles like a figure eight and sit where the circles join.

You need not be a math expert to teach these games. Any person knowing the most basic arithmetic can do it. Algebra is not a pre-requisite for this book. If you have anxiety when faced with math you will enjoy this approach. It is my hope that after playing a few of the games you will start to enjoy math too.

What is taught

There are eight chapters: Number Sense, Addition, Clocks, Multiplication, Money, Subtraction, Division, and Fractions. Within each chapter the beginning games are easy and gradually become harder. The final games in each chapter teach more advanced concepts topics, for example, binary numbers.

It is not necessary or advisable to complete each chapter before starting another chapter. The clock games can be played while still playing the addition and the multiplication games. However, to prevent confusion, do not teach the subtraction facts until the child knows all of the addition facts. Likewise, teach the division facts only after the child knows the multiplication facts.

What age

Players of any age will enjoy these games. The beginning chapter of numeration can be use as early as three. Start with an easy game to check skills and build confidence. You can determine the difficulty level of each game by its objective. If you find a game too hard, tell the players, "Let's quit and play a different game."

How many times

Some games will be played many times, others, once or twice. Reviewing old games lets the children see their progress while reinforcing familiar concepts.

Who plays

These are good family games as well as classroom games. Play can often be with partners to even out the effect of varying abilities and to promote cooperation. Once the children understand a game, they can play by themselves. Older children can play with younger children, benefiting both. Often a game can be made more challenging for a more advanced player.

DESCRIPTION OF THE CARDS

To play these games, you need six decks of special cards, which are available from Activities for Learning. The descriptions are below.

Basic Number Cards

The basic number cards are numbered from 0 to 10. There are 12 of each number.

Corner Cards

The Corner cards each have four colored numbers between 1 and 10 along the sides. There are 50 Corner cards, no two alike.

Multiplication Cards

Each card in the multiplication card deck corresponds to a number in the multiplication table from 1 × 1 to 10 × 10. Thus, it has 100 cards. Some numbers, such as 1, are found only once and others, such as 6, are repeated as often as four times.

Clock Cards

There are two identical sets of the clock cards in different colors, each with 24 cards, comprised of numbers from 1 to 12 and from :00 to :55.

Money Cards

Representations of 15 pennies, 9 nickels, 14 dimes, 8 quarters, and 4 half-dollars make up the 50 money cards.

Fraction Cards

There are 75 fraction cards with 20 different fractions and 20 matching percentage cards:

1. Two each of 4/5, 7/10, 9/10.
2. Three each of 3/4, 2/5, 3/5, 5/6, 3/8, 5/8, 7/8, 3/10.
3. Four each of 2/3, 1/8, 1/10.
4. Five each of 1, 1/3, 1/4, 1/5, 1/6.
5. Eight each of 1/2.
6. Twenty percentage cards, corresponding to each fraction.

SOME PRACTICAL CONSIDERATIONS

Care of the cards

Before beginning to play, remind the players to take proper care of the cards. Show them the proper way to shuffle; never force cards between other cards–that ruins the edges. With the deck in one hand, pick up from the end about one-third of the cards. Drop some of the cards in front of the other cards. Drop more cards in front of those cards. Continue until all the cards are dropped. Repeat a few more times. Ask the winner of the last game to put the cards away.

Where to play

For the younger players, the preferred place is the floor. Children are more comfortable on the floor and the games seem more informal. A special rug used only for games makes a good playing area.

Families can play the games even while traveling. To keep the cards from slipping in a moving vehicle, place the cards on a piece of felt or terry towel.

The winner

Most of the games are competitive; that is, there is a winner (or loser). Some of the games are activities; the children seem to enjoy these just as much. Most of the competitive games are won by a combination of chance and skill. You can often help a losing player by the way you play your cards.

Occasionally, a child will need help in learning to accept defeat graciously. A few general remarks before playing, such as, “No one can win all the time,” and “We all like to win,” might help. After a child has lost a dozen games or so, losing is rarely a problem.

Incidentally, in games where the winner is decided by the most cards, do not allow the children to count the cards. It takes too much time. Simply put the stacks on a firm surface and see whose stack is highest. By pressing down on the stacks, you can determine to within one card which stack is higher.

To encourage cooperation, suggest the children play with partners or teams. Several children can play a solitaire game (despite the name); they work together trying to “beat the cards.”

The player with learning disabilities

Often, those with learning disabilities find memorizing unrelated facts very difficult and paper work tedious. These games eliminate both problems and give the child a new approach. Also, the emphasis in visualization is usually very helpful. Work in a place free from noise and visual distractions. Repeat the games many times. The best way to end a game is “Let’s play it again.”

In conclusion, it is hoped that through these games, the players will develop a lifelong interest and enjoyment in mathematics, thereby enriching their lives. I also hope some of them will become tomorrow’s mathematicians, scientists, and engineers.

NOTE FOR THE FIFTH EDITION

The first chapter has been rewritten and renamed Number Sense. In the other chapters, games have occasionally been added. Because the game numbers in these chapters are referenced in the RightStart Mathematics curriculum, the new games are numbered with a dot followed by a number. For example, the new game, C9.1, follows C9.

NUMBER SENSE

The term *number sense* refers to understanding numbers: a sense of their relative values, a sense of their structure through place value, and a sense of the changes following arithmetic operations. Some considerations to keep in mind when helping young children learn about numbers are recognizing and visualizing quantities, naming quantities, and counting.

Mere counting does not help children understand quantity. Numbers are different from other lists. In counting, seven includes quantities one to seven, but when we say the alphabet, G does not include the letters A to G.

Researchers have found that 5-month-old babies can distinguish between 1, 2, and 3 objects and half of 12-month-old babies can distinguish up to 4 objects. When you point to objects in counting, the young child loses the concept of the whole and assumes you are naming the objects. That is why if you ask a young child to count four objects and then ask her to give you four, she will frequently give you only the fourth object. Therefore, never count (by pointing to each object) fewer than 5 objects. Instead, refer to the collection by the number, for example, 4 apples.

To recognize quantities 6 to 10, they must be grouped. To understand the necessity of grouping, try to see mentally a group of eight apples in a line without any grouping–impossible. Next try to see five of those apples as red and three as green; most likely you can visualize them. Grouping by fives corresponds, of course, to our fingers. Such grouping is not a new idea; the Romans grouped in fives with their numerals: V for 5 and L for 50. Another example is piano music, which is written with two groups of five lines. Who could read it if the staffs were not separated?

The words a child uses for naming numbers plays an important role in understanding place value. English is inconsistent in naming numbers 11 to 99, particularly from 11 to 19.

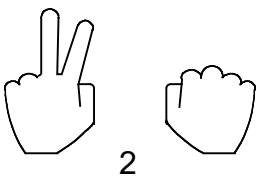
On the other hand, Asian languages follow a simple pattern. Their number words are as follows: 1 to 10, then ten 1 (11), ten 2 (12), ten 3 (13). The twenties are 2-ten, 2-ten 1, 2-ten 2, and so forth. In Asian languages, only 11 words are needed to count to 100, while 28 words are needed in English. Research shows that children learning the "math" way of number naming understand place value about three years earlier than those who do not. Therefore, teach children to count with ten 1, ten 2, . . . , 9-ten 9. Do this for several months.

Activities in this chapter focus on naming and recognizing quantities, evens and odds, and place value with tens and ones.

N1 ACTIVITIES USING FINGERS TO SHOW QUANTITIES

Most children are taught to raise a certain number of fingers when asked their age. One of the joys of having a birthday is being able to hold up one more finger.

Because we read from left to right, ask the child to use his left hand to represent the quantities 1 to 5. It does not matter which fingers on the left hand the child uses. Do not count. To teach three, do not say, "This is one, two, three"; say, "This is three." Then teach 6 to 10 with 5 on the left hand and the amount over 5 on the right hand. See the figures below.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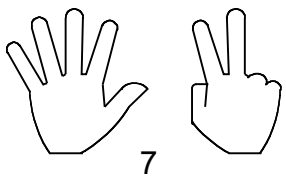
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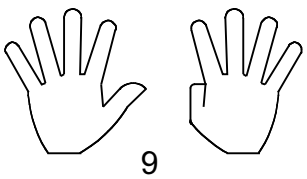
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ADDITION

Research has shown that children learn the facts easier if they use strategies. Visual strategies are more powerful than other types of strategies because our visual memories are quick and permanent. A summary of visual addition strategies using the AL abacus is given in the Appendix on page 10.

The terms needed to discuss addition are addend and sum, or total.

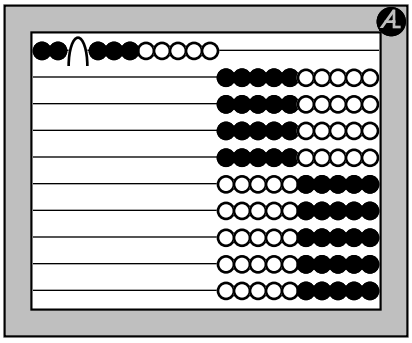
$$\begin{array}{rl} 8 & \text{addend} \\ +\,4 & \underline{\text{addend}} \\ 12 & \text{sum (total)} \end{array}$$

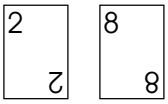
About half of the games in this chapter concentrate on small groups of facts using these strategies. The strategies used are (a) the facts that make 10, (b) adding 1 or 2, (c) the doubles and near doubles, (d) adding 9s and 8s, (e) facts that make 11, and (f) facts that make 9. The Corners games and Chain games reinforce larger groups of facts.

In some of the games, players will be collecting many pairs of cards. Ask the players to form two piles face up, one for each card of the pair. This allows everyone to verify the last pair and it prevents a difficult shuffling problem. At the end of the game combine the two piles. By pressing down on two piles, it is possible to determine which pile has the most cards to within one card.

A1 FIND THE PAIRS

The first few games deal with learning the facts that make 10. This game for one or two prepares the children for finding pairs equaling 10.





2 + 8 = 10.

Objective: To find the pairs that total 10.

Background: To determine a pair using the AL abacus, ask the child to enter the number on the card onto the abacus, for example, 2. Next slide over the remaining beads in the row (8) leaving a finger's width. Ask, "How many beads are needed with 2 to make 10?" [8] Ask the child to find the 8-card.

Manipulatives: An AL Abacus for each player.

Number of players: One or two.

Cards: Two of each basic number card from 1 to 9.

Layout: Scatter the cards face up.

Object of the game: To pair all the cards.

Play: Without regard for turn, the players each pick up any card, determine the number needed to make 10, and search for that card. The pairs are placed on two stacks. Repeat with another card. Continue until all the cards are paired.

A2 FIND THE TENS MEMORY

This memory game collects pairs totaling 10.

Objective: To provide practice in finding pairs with sums equaling 10.

Manipulatives: The abacus. See Find the Pairs (A1) for instructions on using the abacus.

CLOCKS

Children are aware that the numbers on a clock dictate many events in our lives. In some ways the preponderance of digital clocks makes telling time more difficult. Learning to read an analog clock is still a very necessary skill. Analog clocks provide a mental model, for example, to tell how much time there is between 10:42 and 11:00.

Children can play the clock games without knowing any addition or multiplication facts. The first seven games only require reading numbers to 12. After that the child must be able to read numbers to 60.

Actually time-telling is based on a simple principle. The hour (shorter) hand uses low numbers while the minute (longer) hand uses higher numbers, which are invisible. These games provide children with the necessary practice.

The games approach telling time in the following six steps:

1. First, the children work exclusively with numbers representing the hour hand. After a while, she will know the positions even on an unmarked clock.
2. Next the children work with the minute hand. She learns the placement of the "invisible" numbers. You can prepare children with this phase in advance by teaching them to count by 5s. Play the Multiples Memory Game (P2). They can also learn it by entering 5s on the abacus and naming the quantity.
3. The next few games help the children to put the two hands together to tell time.
4. Then there is series of games that teach the children to recognize a half hour later or earlier and other common times.
5. Next the children become familiar with the minutes between the numbers.
6. Finally, the children read any number of minutes before the hour. This step is delayed until the children have thoroughly mastered formal time telling to prevent confusion with before and after the hour.

The clock cards needed for playing these games consist of two sets of 24 cards, in different colors. Each set has 12 hour cards, with numbers from 1 to 12, and 12 minute cards, with numbers :00, :05, :10, ... :55. Two clock representations are included in the appendix and may be reproduced.

Finally, use time in everyday conversations to the children. Remember to keep the games fun. Most of them should be played more than once, some many times. Soon the children will be able to read analog clocks.

C1 HOUR CARDS AROUND THE CLOCK

Although we realize the numbers around the clock are consecutive, children need to experience this.

Objective: To become aware of the order of the numbers on a clock.

Number of players: One.

Cards: Use either set of 12 hour-cards and Clock A from Appendix page 13.

Layout: The cards are scattered face up on the right side of the clock.

Object of the game: To place the cards around the clock in the proper positions.

MULTIPLICATION

The first several multiplication games can and should be played while the children are enjoying the addition and other games. Cards needed for the multiplication games are the basic number cards and the multiplication cards. Each multiplication card represents one number of the multiplication table, up to 10 × 10. The 11s and 12s are not basic facts and are not included.

Memorizing the multiplication facts is approached through visualizing the multiples (also called skip counting) patterns, as shown below, and through visualizing arrays. The children should not recite the multiples in order to find a multiplication fact.

The terms needed to discuss multiplication are: multiplicand × multiplier = product. That is, 8 × 4 is interpreted as 8 repeated 4 times, not 8 groups of 4. This is consistent with the other operations of arithmetic:

8 + 4 means start with 8 and transform it by adding 4.

8 – 4 means start with 8 and transform it by removing 4.

8 ÷ 4 means start with 8 and transform it by breaking it into either 4 groups or groups of 4.

Therefore, 8 × 4 should mean start with 8 and transform it by adding it a total of 4 times.

Multiples

Multiples are learned more quickly and easily when children see patterns (Appendix page 18). Arrange the 2s in two rows that show the ones repeating. The second row is 10 plus the first row. Notice also that all the multiples for the 2s are even.

2	4	6	8	10
12	14	16	18	20

Arrange the 3s in groups of 3. Look in each column and notice the ones decrease by one. Also note the 0-9 sequence starting at the lower left, moving up, and continuing at the bottom of the next column. In each column the tens have the pattern, 0, 1, 2. Also add the digits: in the first column they are 3 (1 + 2 & 2 + 1); in the second column, 6; and the third, 9.

3	6	9
12	15	18
21	24	27
30		

With the 4s grouped in two rows, the ones are aligned. Observe that the first row of 4s is every other number of the 2s. The second row is 20 more than the first row.

4	8	12	16	20
24	28	32	36	40

The 5s are easily learned. Children like to recite them in sing-song fashion.

5	10
15	20
25	30
35	40
45	50

The 6s can be thought of as the even 3s for the first row. In the second row the ones are the same and the tens are 3 more.

6	12	18	24	30
36	42	48	54	60

Even the 7s have a certain pattern. Each number from 1 to 9 is used once in the ones. In each column, the ones increase by 1 and the tens increase by 2. In the rows, the tens increase by 1.

7	14	21
28	35	42
49	56	63
70		

The 8s, of course, are every other 4 for the first row. Also notice that in each row, the ones are the even numbers in descending order. In each row the tens increase by 1 and the second row is 40 more than the first row.

8	16	24	32	40
48	56	64	72	80

The 9s are the most interesting of all; the sums of the digits in each case equal 9. The digits can also be reversed to obtain

9	18	27	36	45
90	81	72	63	54

MONEY

Money is one application of arithmetic that virtually all children find interesting. It really is not very complicated. It does take practice, which the following games provide. Young children in developing countries make change with ease as street-corner vendors.

The children need to know how to count by 5s and 10s before starting this chapter. Also they should know the names of the coins and be able to use them in everyday speech.

The money cards consist of 50 cards with coin representations, including 15 pennies, 9 nickels, 14 dimes, 8 quarters, and 4 half-dollars. However, in this chapter the corresponding letters P, N, D, Q, and H will identify the coins in the figures.

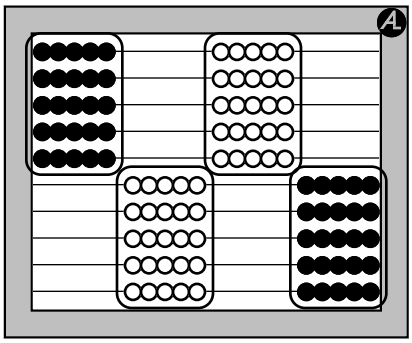
M1 SORT THE COINS

After the child can distinguish between the various coins, teach the names, *penny* and *nickel*. If possible, use real coins. Explain that the value of a penny is 1 cent. Ask the child to enter 1 on the abacus. Repeat for a nickel, entering 5 on the abacus.

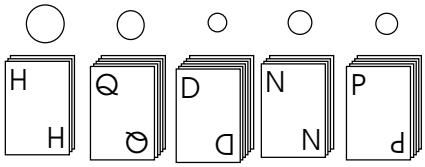
Next teach about the *dime*. Explain that a dime is smaller than a nickel because it used to be made of silver, which is quite expensive. Enter 10 on the abacus.

Continue with the dollar: that is 100 cents. Ask her to enter 100. Then teach the *half-dollar*. Ask her to enter half the beads. [50] Ask, "How many half-dollars make a dollar?" [2]

Lastly tell the child that the word *quarter* means one-fourth, which is half of a half. Show it on the abacus as shown on the right. Ask, "How many quarters make a dollar?" [4]



Showing a quarter of a dollar.



Activity: This first game asks the child to sort the money cards into five piles. Use coins to head each column.

M2 SIMPLE MONEY WAR

Wars will occur often in this game.

Objective: To provide practice comparing the relative worth of coins.

Cards: The 50 money cards.

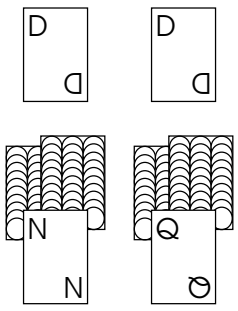
Number of players: Two.

Deal: Divide the cards evenly between the players.

Object of the game: To capture all the cards. Shorter versions can be played with the winner decided by the one having the most cards after a certain length of time or after using the initial stack of cards.

Play: Each player lays his top card on the table face up and states the value. The player whose card has the greater value takes both cards. Players take turns deciding who collects the cards. (This prevents one person from doing all the thinking.)

Player 1. **Player 2.**



The first cards played are both dimes, causing a war.

SUBTRACTION

Before starting on this chapter, be sure the players thoroughly know their addition facts. When a person is trying to memorize a set of facts based on another set of facts, less confusion results if one set is thoroughly mastered first. A driver who has thoroughly learned a complicated route in one direction will find the return trip easy. But if he attempts the return trip before mastering the original trip, he is likely to become confused.

Subtraction facts are best learned through strategies. See the Appendix page A-11 for various subtraction strategies.

The games in this chapter teach the subtraction facts and how to apply them in subtracting one number from another. The beginning games focus on a small group of facts; later games use more facts. A few games teach the properties of subtraction. Other games, such as converting to binary or base three, require application of subtraction. Negative numbers are introduced and used in several games.

Subtraction can be thought of in three ways:

1. It answers the question, how many more are needed. This is the inverse of addition, and the result is often called the *missing addend*. It is written as 7 + ___ = 15.
2. It answers the question, how much more is one quantity than another. The results of the comparison is called the *difference*. The smaller number is subtracted from the larger.
3. It answers the question, how much is left, or remains, when a portion is taken away from a larger quantity. The result is referred to as the *remainder*.

The terms used to discuss subtraction are minuend, subtrahend, remainder, and difference.

$$\begin{array}{r} 12 \\ -\ 4 \\ \hline 8 \end{array}$$

minuend
subtrahend
remainder or difference

For young children the best term for "borrowing" is *trading*. They understand when a trade is fair. They do not understand that terms like *exchanging, renaming,* and *regrouping* mean equality.

In some of the games, the 1 in the tens place is assumed. The fact 8 – 9 is to be thought of as 18 – 9. Children have no problem with this notation and readily accept it as eighteen minus nine. We do the same thing whenever the need to "borrow" arises. For example, in the problem 43 – 27, we think 13 – 7 even though the 1 is absent.

Simplified Subtraction

The subtraction algorithm, or procedure, generally taught in the United States is not the only one is general use; students in Latin America use another algorithm. In fact, there are at least seven methods.

Described below is the Simplified Subtraction method. In this simpler method, which is easy to learn and easier to use, the work proceeds from left to right like division, rather than right to left like addition. The method is explained below.

According to research, it is easier for most children to complete the work for trading, or borrowing, before performing the actual subtracting.

In the following example two trades are necessary.

DIVISION

Division has often been a difficult topic in arithmetic, but it need not be. The division facts are not a separate collection of facts to learn; they are the inverse of the multiplication. However, wait until the children have mastered the multiplication facts before starting division.

Division is the process of separating into equal groups. It answers one of two questions, either the size of the groups or the number of groups. For example, a problem that asks, how many bags are needed if you put three apples into each bag, is a number of groups question. A problem that asks, how would you divide 12 cookies equally among 3 friends, is a size of groups question.

Four terms describe the various numbers of the division process.

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \end{array} \text{ remainder} \qquad \begin{array}{r} 8 \text{ r1} \\ 4\overline{)33} \end{array}$$

The quotient (the word is Latin for how many times) tells how many times the divisor goes into the dividend. The remainder tells what is left over. The relationship to multiplication is quotient × divisor + remainder = dividend, for example, (8 × 4) + 1 = 33.

There are four ways to write division problems as shown below. The first three are introduced in the games. The slanted form (/) is confusing for children and is rarely used in computations.

$$\begin{array}{r} 3 \\ 5\overline{)15} \end{array} \qquad 15 \div 5 = 3 \qquad \frac{15}{5} = 3 \qquad {}^{15}\!/_{5} = 3$$

How important is long division? Long division is unnecessary for single-digit divisors. It does not help in understanding the process, is very inefficient, interferes with learning a more efficient method, and is not a necessary preparation for double-digit division. Short division, where nothing is written below the dividend, is the preferred operation. Unfortunately, about 25 percent of adults in the U.S. did not learn short division in school. However, it is quite simple to master and is explained on page 112.

After mastering short division, most children should learn long division. It is necessary, for example, to observe repeating decimals. On the other hand, children with learning problems should not bother to learn it. With calculators so readily available, they will never use it. Their time is better spent in learning other mathematics.

Some division concepts are included in the Fraction chapter.

D1 FIND THE QUOTIENTS

This game asks the player or players to match the dividends with the corresponding quotients for a particular divisor.

Objective: To see the relationship between multiplication and division.

Background: Ask a child to enter 12 on an abacus. Then ask, “How many 3s are in 12?” Since only 3 beads can remain on each wire, remove 3 extra beads from the first row (see the next page) and move them to the third wire. This is done in one smooth operation, with the right hand removing 3 beads and the left hand entering 3 beads. Remove another 3 beads and move them to the fourth wire. Finally, move the last extra bead to the second wire. Repeat the question. [4] “There are four 3s in 12 and 3 × 4 is 12.” Repeat with other examples.

FRACTIONS

Many people become uncomfortable when fractions are mentioned. I know an excellent cabinetmaker who can add fractions only with the help of a tape measure. A mathematics teacher tells this story: During sixth grade while attending a parochial school, she was introduced to dividing fractions. She followed the invert and multiply rule but believed that the children in the public school had a different rule to use. She was not bothered that the answers would be different. To her it was only a paper and pencil exercise; she was unconcerned about striving for a consistent or useful solution.

Introduce children to fractions early—in kindergarten or first grade. They are part of everyday life. When children do not learn fractions until the fourth or fifth grades, they are so convinced that 5 is always more than 4 that they have a problem with $\frac{1}{5}$ being less than $\frac{1}{4}$. In working with fractions, children should only use the horizontal form, not the slanted line.

Most work with fractions should be done with fraction charts, a copy of which is in the Appendix on page 21. Sturdier charts, wood, plastic, and magnetic, are available from Activities for Learning, Inc. at RightStartMath.com. Although circles are often used to represent fractions, comparing fractions with them is difficult. Are we comparing areas, angles, or arcs? Showing fractions greater than one is also more difficult with circles.

To be successful with more advanced fractions, children must be thoroughly familiar with multiplication and division of whole numbers. This background is needed to simplify a fraction to lowest terms and to find a lowest common denominator.

Some might feel that fractions are becoming obsolete. The metric system does not need them and calculators use decimals. However, fractions are the culmination of arithmetic and are essential for understanding algebra and other advanced topics. In essence, fractions are division.

Generally, the games should be played in the order given. Do not be in a hurry to play the next game. Frequently go back to games already learned; the children will often play them from a new perspective.

Although the fraction cards do have the fractions printed on both top and bottom of the cards, for clarity the figures in this chapter show the fraction only at the top.

F1 BEGINNING FRACTION ACTIVITIES

These initial activities introduce fractions concretely. They have been used with children as young as five and six and with adults.

Cut one fraction chart into its individual pieces. The children will also need an uncut fraction chart.

Activity 1: Show the children the fraction chart and ask them to use their pieces to build it. They should not build it on top of the model.

Activity 2: Tell the children that what the word *fraction* means is to break, or divide, into pieces; it is derived from the Latin "frangere." Until this place in arithmetic, 1 was considered the smallest unit. Now one will be fractured, although always into equal pieces.

Give the children strips of paper the same length as the 1 on the fraction chart. Demonstrate and then have them fold a strip in half and tear or cut it apart at the fold. Write the fraction and explain that it means 1 divided by 2. The line between the numbers means *divided by*. Repeat for $\frac{1}{3}$ and $\frac{1}{4}$ with other strips.

INDEX