

Montessori Mathematics and RightStart™ Mathematics

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In 1967, a newspaper article about a Montessori school in the Twin Cities caught my attention. Helping children learn following Dr. Maria Montessori's methods made sense to me. As a big sister who could not resist "teaching" her eight siblings and then the mother of two young daughters, I rushed to the library to read everything I could on Dr. Montessori and her method.

My dream to become a Montessori teacher was fulfilled in 1975 when I received my diploma from Mario Montessori as a member of the first class of the Montessori Training Center of Minnesota.

Bead Frame: During training we were told division was not possible with the bead frame. I figured out how to do it and showed Mr. A. M. Joosten, the course director and contemporary of Dr. Montessori, who worked with her in India. He subsequently included it in the training. While at White Bear Montessori, I was frustrated that I was unable to explain to the children how to use the bead frame. I now realize the bead frame has several difficulties:

- Hierarchies of numbers represented sideways: They need to be in vertical columns.
- Exchanging done before second number is completely added: Addends need to be combined before exchanging.
- Answer is read going up: We read top to bottom. (Think of mile markers along freeways.)
- Inconsistent with equation order when beads are moved right: Beads need to be moved left. (Montessori herself shows beads being moved to the left, but many training centers teach moving beads to the right to accommodate counting.)
- Not visualizable: Beads need to be grouped in fives.
- Distracting: Room is visible through the frame.

Counting: Mr. Joosten introduced the Black and White Bead Stairs by pointing out they were grouped in fives so the child does not need to count. Over time, I began questioning why so many Montessori materials were devoted to counting. Number Rods, Spindle Boxes, Cards and Counters, Snake Game, Multiplication Board, Bead Frame, Dot Game, and Chains for Squares and Cubes all require extensive counting. I suspect Dr. Montessori wanted to be certain those children from the slums in Rome could answer the counting questions on an IQ-type assessment prevalent in early 20th century Italy. A Google search of the digitized editions of Montessori's earliest books for her rationale behind counting yielded only the comment that most children entering the Children's House could count to three.

Counting is not the foundation of arithmetic; indeed, only positive integers can be counted. When young children count, they focus on the immediate object being counted and lose sight of the whole, not realizing the accumulative nature of the counting words. In summary, counting:

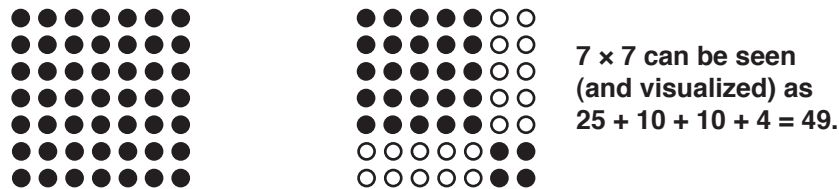
- Provides poor concept of quantity.
- Ignores place value.
- Is very error prone.
- Is tedious and time-consuming.
- Is not natural; it takes years of practice. (Children raised in primitive cultures without counting words do not spontaneously count.)
- Does not provide an efficient way to master the facts.

Dr. Montessori could not have known that 5-month-old infants can add and subtract up to 3 without counting by subitizing (Karen Wynn’s research). They actually visualize the quantities in their minds.

Visualizing: Many infants at 12 months can subitize up to four. Three-year-olds can subitize five objects if they know that five has a middle unlike four, which does not.

To appreciate visualizing, try to imagine eight identical apples in a row—very difficult. Now try to see the row of eight apples with five green and three red—most people can see them. You could say our brains are designed to match one hand, not two; that is, we can easily see five, but not ten objects. The Romans grouped their numerals in fives; for example, 8 is VIII. Piano music consists of 10 lines, but grouped in fives. The Chinese abacus, grouped in fives, was designed to make counting unnecessary. Aside from mathematics, visualization is necessary for many other activities, including reading, geography, architecture, sports, engineering, astronomy, archeology, chemistry, physics, and surgery.

Grouping in fives facilitates learning the math facts. For example, contrast 7×7 on the Multiplication Board without grouping and with grouping in fives. See the figures below.



AL Abacus: I developed Side 1 of the AL Abacus based on the Black and White Bead Stairs and used it with children with learning disabilities at an elementary Montessori school. I developed Side 2 at White Bear, where the children raced each other upon arrival to get one of the coveted four abacuses to do their math. Their math skills developed very rapidly.

Math Card Games: The more than 300 math card games were developed for my son and other children I tutored. They were designed to help children understand, apply, and enjoy mathematics. The games often require models or materials; there are games that teach strategies for memorizing the facts. They provide needed repetition without drill in an enjoyable, social context.

I also introduced some math card games to my Montessori children. The children loved them. These games are used in other Montessori environments: in Children’s Houses and upper levels.

Fractions: The linear fraction model, introduced by Dr. Catherine Stern, a Montessori trainer originally from Breslau, Germany (now Wrocław, Poland), provides a better model for fractions than circles. Dr. Montessori came to the fraction circles from geometry, not arithmetic. She incorrectly states in *Psychogeometry* that fractions cannot be equal to or greater than one. Fraction circles:

- Perpetuate the cultural myth that fractions are less than one.
- Limit understanding of fractions: they are more than “a part of a whole” or “part of a set.” (Mathematically, a fraction is a/b where b is not zero.)
- Do not give the child the “big picture.”
- Make it difficult for the child to see how fractions relate to each other.
- Are difficult to comprehend because we tend to see linearly, not circularly.

Drawing Board Geometry: Because of my drafting skills (I have a degree in electrical engineering), while at White Bear Montessori I developed the Drawing Board Geometry material, consisting of a small drawing board, a 12-inch T-square, a 30-60 triangle, and a 45 triangle. I presented lessons to the extended-day children on constructing an equilateral triangle and then dividing it into halves, thirds, fourths, sixths, eighths, and ninths. The children wanted to continue, so they divided the triangle into 12ths, 18ths, 24ths, and more. Stephanie did not stop until she divided the triangle into 256 equal parts. The children also constructed and divided hexagons and squares. The children had constructed many of the figures in *Psychogeometry* although the book had not yet been published.

RightStart™ Mathematics: I left White Bear because I felt I needed to communicate to teachers and parents that children were capable of so much more than they were currently learning in schools. I pursued a Masters in Curriculum and Development with a secondary mathematics teaching license. I taught math to grades 6–8 in St. Paul. That was an eye-opener for me because those students understood very little, having memorized everything.

A few years later, I started work on my doctorate at the University of Minnesota. My dissertation become Grade 1 of the RightStart™ Mathematics program. The differences between the experimental and control classes was remarkable. Three times on a national test, first grade classes with two different teachers scored at the 99th percentile, compared to their previous 60–70 percentile ranking. The school asked me to continue writing, which I did for the next four years, completing RightStart™ Mathematics in the year 2001.

Currently, RightStart™ Mathematics is being revised to reflect changes in both important mathematics, applications, and advances in brain research and in teaching practices.

All RightStart™ Mathematics manuals are written with Montessori principles in mind:

- The child is given an overview before working with components.
- Either the concept or the word must be known before the three-period lesson.
- Physical models for the child to work with that represent abstract concepts. Additionally, they must be visualizable (grouped in fives).
- The child can be an independent learner by using strategies.
- The social nature of learning is enhanced through playing math card games. The games have control of error built in wherever possible.
- The teacher is given explanations, especially in the Second Edition.

Color coding is not used to designate the hierarchies. Dr. Montessori, not knowing that 12 percent of boys have some color deficiency, choose the worst colors, green, blue, and red to represent the ones, tens, and hundreds. Sadly, some children develop “color value” in lieu of *place* value.

RightStart™ Mathematics: A Hands-on Geometric Approach: After teaching hundreds of teachers about drawing board geometry, I was asked what comes next. Over a period of four years, I wrote *RightStart™ Mathematics: A Hands-on Geometric Approach* for students in upper elementary and middle school. It is designed to be done independently or within a small group. Topics include 90 percent of the usual curriculum for that age, but approached geometrically before algebraically.

Common Core Standards: While I certainly do not agree with all the progressions expected in the Common Core State Standards nor even the terminology, they are a good faith effort to

improve mathematics instruction in the U.S. The standards stress understanding, but do not dictate how to teach. Even more important than the Common Core lists of what is to be learned at each grade are the underlying Mathematical Practices:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

To pursue the first Mathematical Practice requires understanding of the nature of mathematical problems. A problem is not a problem if the solution is obvious. Practicing computational skills is not problem solving; such practice is an exercise, an activity done to perfect a skill.

Problem solving is more like solving a puzzle. A practice found in some Montessori schools is color coding story problems by operation; this solves the problem for the child. All the child needs to do is the computation. In essence, the child is told where to put the next puzzle piece. We must remember, “Never do for a child what he can do for himself, as every useless aid hinders the growth of a child.”

For example, the following problems require more than computation:

- Thirteen children are going on a field trip. If four children can ride in a car, how many cars are needed?
- Pauline has 13 petunias to plant. She wants exactly 4 in a row. How many rows can she plant?
- Four children have \$13 to distribute evenly. How much does each one receive?
- Four children split 13 granola bars equally. How much does each one receive?
- Jack packages 13 cookies with 4 per bag and eats the leftovers. How many does he eat?

Note that these five problems have the same numbers and the same operation, but each solution is different. They require thinking.

Montessori mathematics needs less emphasis on computation. While computing was paramount in the early 1900s, calculators and computers now perform most of this work.

Children with learning disabilities. Children with dyslexia or dyscalculia generally learn best with visual activities. Rote counting is very difficult for them. They need models to be visualizable. We have received many testimonials about children who have succeeded with RightStart™ Mathematics after repeated failure using other programs. This approach and method will be presented at the International Dyslexia Association, Upper Midwest Branch in April, 2014.

The future. We want our Montessori children to pursue technological fields. It appears a lower percentage of them go on to study in these related areas. Do they feel unprepared? Have they lost interest? I am also concerned that Montessori magnet schools and charter schools score lower on high-stake tests in mathematics. Just like science from the early 1900s was updated in the Montessori environment, Montessori mathematics needs updating today.