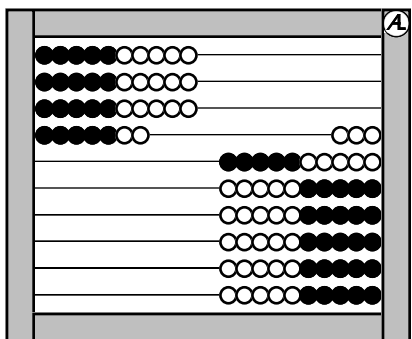
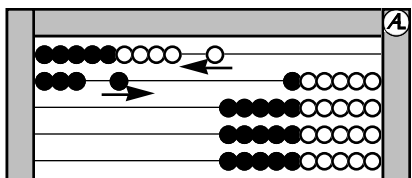


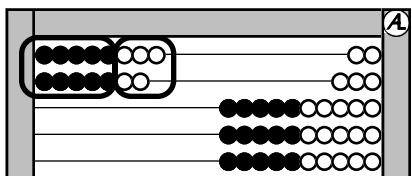
Place value cards.



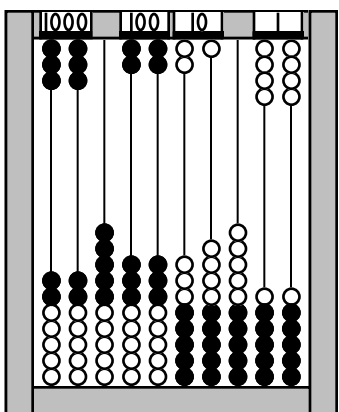
AL abacus (side 1) with 37 entered.



Transforming $9 + 4$ into $10 + 3$.




Seeing the sum of 8 and 7 as 10 (2 fives, the black beads) and 5, the number of white beads.



AL abacus (side 2) with 6438 entered.

- Young children don't realize counting represents quantity.
- Very error prone. Children under 6 are not good counters.
- No motivation to learn facts.
- Inefficient and time-consuming.

Visualizing quantities.

- Babies, at 5 months, can add and subtract up to 3.
- Group by 5s. Impossible to imagine 8 objects without grouping. 

Place value is the most important concept of arithmetic.

- Teach *math way* of counting: after 10, say ten 1 (11), ten 2 (12), ten 3 (13), . . . , 2-ten (20), 2-ten 1 (21), . . . , 9-ten 9 (99).
- All Asian children learn math with math way of naming numbers they understand place value in first grade. Average U.S. child understands it at the end of fourth grade.
- Essential for understanding algorithms.
- Place-value cards: encourage reading in normal order; starting with ones column and then tens columns is backwards.
- Essential to use 4-digit numbers to understand trading (carrying).

What makes a good manipulative (according to Japanese).

- Easily visualized.
- Representative of the structure of mathematics.
- Easily managed.

The AL Abacus.

- Grouped in fives and tens.
- Used for operations, strategies, money.
- Evens and odds; also needed for side 2 of abacus.

Some addition strategies.

- What makes 10: seen on abacus, Go to the Dump game.
- Adding 9: complete the 10.
- Two 5s: two fives = 10; then add "leftovers." For $8 + 7$, the leftovers are $3 + 2$; so the sum is 15. See figure at left.

Learning the facts.

- Strategies first: It takes time for new strategy to become automatic.
- Games far superior to flash cards.
- Timed tests and graphs.

Importance of mental computation.

- Understanding more important than procedures.
- Develops number sense (common sense with numbers).
- Necessary for estimating.
- Easier to start at the left: e.g. $34 + 48 = 34 + 40 [74] + 8 [82]$.

Adding 4-digit numbers on the abacus.

- Important for understand trading: that 10 ones = 1 ten, 10 tens = 1 hundred, 10 hundreds = 1 thousand.
- Children need to write down on paper what happens after number is added on the abacus.

