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RightStart™ Mathematics

by Joan A. Cotter, Ph.D.

Activities For The AL Abacus

A Hands-On Approach To Arithmetic

ABOUT THE AUTHOR

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Contents

Repeat with the Cs adding 4 + 6 and the Bs adding 6 + 4. **Do you think this is always true?** Ask the children for two numbers to try.

2. Prepare worksheets (2-7 to 2-9) with problems such as 4 + 5 and 5 + 4 in pairs and let the children work independently. Observe whether or not they are using the abacus for the second sum. It takes time for children to prove to themselves that it is always true.

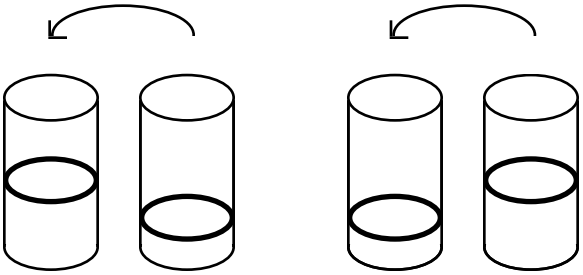
3. For children who have mastered adding 1 to a number and the commutative law, a worksheet with 1 plus a number will be a challenge. Let each child decide whether or not to use the abacus.

4	+	5	=	
5	+	4	=	
8	+	2	=	
2	+	8	=	
5	+	1	=	
1	+	5	=	
3	+	6	=	
6	+	3	=	

1	+	4	=	
1	+	6	=	
1	+	1	=	
1	+	3	=	
1	+	2	=	
1	+	7	=	
1	+	9	=	
1	+	5	=	
1	+	8	=	

Application

1. Adding water to containers provides an excellent example of the commutative law. Prepare four identical glasses, two filled to the same lower level and two filled to the same higher level as shown.



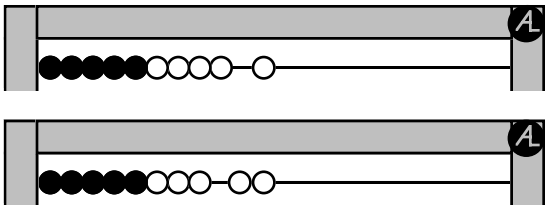
While pointing to the first arrangement, ask the children, **When I add the water from this container, how high will the water be?** Have the children venture a guess before pouring the water.

Then ask the same question while pointing to the second arrangement. [The levels will be the same.] Set up the experiment at a learning center where the children can perform it for themselves.

2. Addition problems can be worked with water using a small marked beaker, a larger marked beaker, and a pitcher of water.

THE WAYS TO MAKE 10

Ten is an important number in our number system and the facts that make 10 are also very helpful to know. Several games will be presented to help the children master these facts in a non-stressful atmosphere.



Splitting 10

1. Enter 10 on the top wire. Separate the right bead by one-half inch (1 cm) and ask a child to state the fact. [9 + 1 = 10] Move over another bead and ask another child to state that fact. [8 + 2 = 10] Continue to 1 + 9.

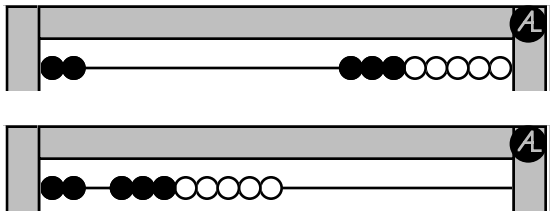
2. Give the children a blank worksheet and ask them to use their abacuses and record the combinations that make 10. After they are written, have the children find the "twins"; for example, 1 + 9 and 9 + 1. Each set of twins can be circled with a different color. Or the facts can be cut apart and the twins pasted together on another sheet of paper. One fact, 5 + 5, is called a "double" and does not have a twin.

	+		=	
	+		=	
	+		=	
	+		=	
	+		=	
	+		=	
	+		=	
	+		=	
	+		=	

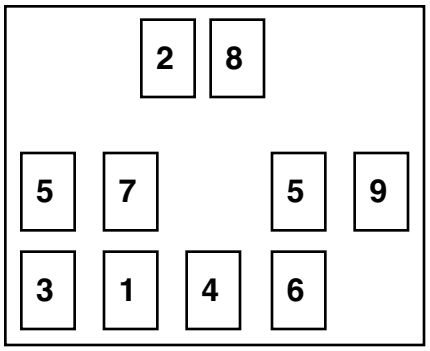
9	+	*1*	=	*10*
8	+	*2*	=	*10*
7	+	*3*	=	*10*
6	+	*4*	=	*10*
5	+	*5*	=	*10*
4	+	*6*	=	*10*
3	+	*7*	=	*10*
2	+	*8*	=	*10*
1	+	*9*	=	*10*

Games

1. HANDSHAKING GAME. Distribute among ten children one of each number 1 to 9, but two 5s. Build the stairs on the abacus; move the remaining beads near the stairs to narrow the gap, as shown. Touch the top wire and read the fact, **1 + 9**. The children having those numbers shake hands gently. Continue to 9 + 1. Then ask, **Who shook hands twice?** [All but those with the 5s] **Who shook hands only once? Why?**



2. FINDING THE PAIRS. This game for two or three players uses the same cards as the Handshaking Game. Spread the cards out face up. Without regard to turn, each player picks up a card, for example 2, and enters that number on the abacus. Next he slides over the remaining beads in the row leaving a finger's width. Ask, **How many beads are needed to make 10?** [8] Tell the child to find the 8-card and to set the pair aside.

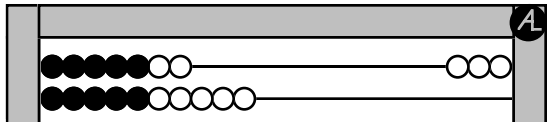


3. MATCHING MEMORY. Use the same cards to play Memory. When the first card is turned over, the player states what card is needed to make 10 before turning over the second card.

4. GO TO THE DUMP. The pairs for this game, played similar to Go Fish, are the combinations that make 10.

Missing addends

1. Write 7 + __ = 10 and say, **Seven plus what equals ten.** Put 7 on the top wire and 10 on the second wire. **How many beads do you have to add to make these equal?** [3] The children can either count the missing spaces or count the beads as they move them over. **So seven plus three equals ten.** Write the 3 in the blank. Repeat for another example: 4 + __ = 10.



Give the children a prepared worksheet (2-10 and 2-11).

7	+		=	10
4	+		=	10
9	+		=	10
2	+		=	10
5	+		=	10
8	+		=	10
1	+		=	10
6	+		=	10
3	+		=	10

2. ORAL PROBLEMS.

A. Carlos is trying to get 10 gold stars. If he already has 6, how many more must he receive? [4 stars]

B. Kay has 10 houses on her block. If she has walked by 6 houses, how many more must she walk by before she reaches the end of the block? [4 houses]

C. Tracy learned the names of 2 more of her classmates. She already knew 5 names. Now how many names does she know? [7 names]

D. What even number comes after 6? [8]

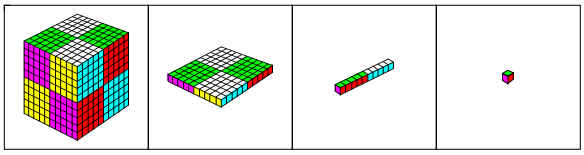
E. What odd number comes after 7? [9]

F. What even number comes before 6? [4]

E. What odd number comes before 7? [5]

HUNDREDS AND THOUSANDS

Children need to experience the quantities of 100, 200, and so forth separately before combining with tens and ones to form numbers like 582. The AL abacus with 100 beads can be used for this as well as Base 10 Picture Cards, shown simplified at the left.



The hundreds

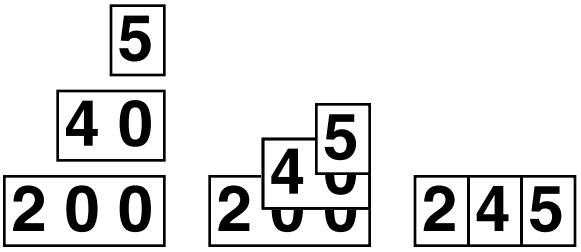
1. Ask the children, **How many beads are on an abacus?** [one hundred] Hold up two abacuses and ask, **How many beads do you think are on two abacuses?** [two hundred]

Say, **You know how we write one hundred; how do you think we write two hundred?** Show the place value card of 200 found in the appendix. Point to the 2 while saying **two**, to the middle 0 while saying **hun-**, and the last 0 while saying **-dred.** Repeat for quantities 300 and 400.

Place 100 identical items in a container; show it to the children and tell them that it is 100. Then show them containers with 200, 300, or other multiples of 100 and ask the children to guess the quantities.

2. Hold up the various hundred cards one at a time and ask the children to read them. Also ask them to show the quantities. If extra abacuses are unavailable, use copies of the "abacus" cards, found in the appendix, to represent one hundred. Along with the hundred cards, hold up ten cards and one cards for them to read.

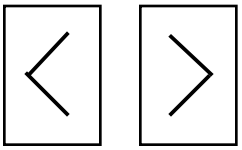
3. Show a quantity such as 245 by setting up two abacuses, each showing 100, and a third abacus showing 45. Pick up the 200, the 40, and the 5 cards; combine them by overlapping to make 245. Provide each child or group of children a set of place value cards. Give them other combinations to construct with abacuses and place value cards, such as 381 and 179.



4. Comparison of two quantities can help the children appreciate the relative magnitudes. Write on the board

321 ◯ 123

Have the children work with a partner. Give each child an abacus and 9 abacus cards; each pair also needs a "greater than or less than" card. The child on the left enters the left quantity while the child on the right enters the quantity on the right. Then they decide which quantity is greater and place the sign accordingly.



Give them two examples

452 ◯ 402 512 ◯ 517

before giving them a worksheet (7-1) to be done with a partner. After correctly placing the < or > sign, they can take turns copying it on to the worksheet.

321	◯	123
975	◯	683
526	◯	543
669	◯	907
463	◯	364
28	◯	288
943	◯	953
753	◯	750
101	◯	110

FUTURE REVIEW. After the children have worked with the < and > signs for the hundreds, ask them if they figured out the rules to do the work without the abacuses. Ask, **Which number do you look at first?** [the hundreds] Then ask, **If the hundreds are the same, what do you look at next?** [the tens] Lastly ask, **If both the hundreds and the tens are the same, what do you do?** [look at the ones]

Write a few sets on the board for them to do without abacuses. Discuss following the rules.

Adding with sums in the hundreds

1. The purpose of this activity is to introduce carrying with sums beyond one hundred. It is not necessary at this point that the children start with the ones.

Write on the board

$$\begin{array}{r} 87 \\ \underline{+\ 45} \end{array}$$

and ask the children to try it on an abacus. Then ask, **What is the problem?** [The sum will not fit on one abacus? Use two abacuses with the second placed directly below the first. Start the addition by entering 8 tens on the first abacus and 7 ones near the bottom of the second abacus. Next enter the 4 tens by filling the first abacus and continuing into the second. Finally enter the 5 ones and construct the sum 132 with the place value cards.

Let the children work in pairs to find the sums for 75 + 52, 97 + 68, and 45 + 86. Then give them worksheets (7-2).

ORAL PROBLEMS. A. A ball costs 69ç. Tou wanted two balls; what would they cost? [$1.38]

Now is a good time to explain that a DOLLAR is 100 cents, so the answer would be 1 dollar and 38 cents.

B. Ser had 300 pennies. How many dollars is this? [3]

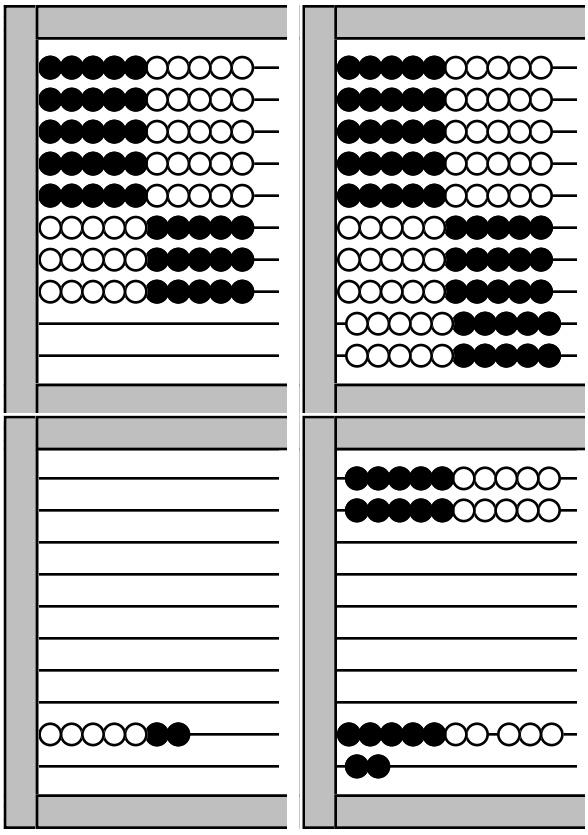
C. Mee bought a scarf at a sale for 49ç. She also bought some buttons for 75ç. What did the two items cost? [$1.24]

D. Amanda bought two gifts, one costing 86ç and the other 55ç. What was the total cost? [$1. 41]

E. Vang had 23 days of vacation from school. Eighteen days have gone by. How many days does he have before school starts again? [5 days]

F. Jonathan had saved $13 for a special gift that cost $20. How much more did he have to save? [$7]

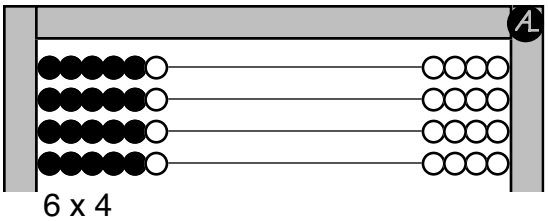
G. David walked for 28 minutes on Monday, 35 minutes on Wednesday, and 37 minutes on Friday. How many minutes did he walk that week? [100 minutes]



87 + 45	63 + 89	26 + 74
37 + 69	58 + 94	47 + 63
52 + 76	73 + 28	68 + 39

(The term "times" will be introduced later.) Ask, **How much is it?** [15] Ask a child to write the answer.

Write on the board 6 × 4 =__. Ask the children to read it [6 taken 4 times], then to enter it on their abacuses, and to find the answer. Then ask to enter 7 × 8. Encourage them to see the tens groupings as shown.

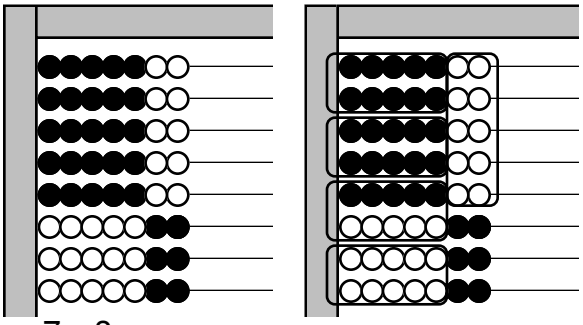


6 x 4

Next, ask them to find 9 × 5. Since they know 10 × 5, subtracting 5 from 50 will give the product of 45.This also works for multiplying by 8s.

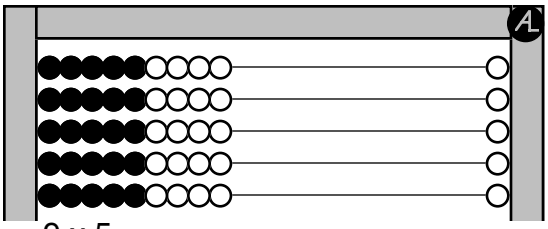
Give them a page (8-6, 8-7) of problems, letting them find the answers in any manner they choose.

FUTURE REVIEW. To help the children see these facts visually, enter an array and show it to the children for 2-3 seconds. Then ask them to state the multiplication fact and the product. (For the classroom, an overhead abacus is available from the publisher.)


7 x 8

Multiplying with 0s and 1s

1. The object of this lesson is that the children discover and learn the rules associated with multiplying with zeroes and ones. Work with them only long enough to be sure they understand how to do it. Ask them to look for patterns.



9 x 5

Write on the board

$$0 \times 3 = \_\_$$

and ask, **How much do we enter each time?** [0] **How many times do we enter it?** [3] Go through the motions of entering nothing on 3 wires. **So what is the answer?** [0]

Next write on the board

$$8 \times 0 = \_\_$$

Give the children time to think how they would do it before showing them. **How much do we enter each time?** [8] **How many times do we enter it?** [0] Move 8 beads on the top wire a slight distance, stop, and say, **We cannot enter it even once. So how much is 8 × 0?** [0]

If necessary give them more examples, 0 × 4, 9 × 0. Then give them a worksheet (8-8) with 0s.

FUTURE REVIEW. After the "0" worksheet is completed, ask what happens when there is a 0 in a multiplication problem. [The answer is 0.]

2. Tell the children that now they will work with multiplying 1s. There is no confusion with problems like 1 × 7, but 7 × 1 can cause difficulties. Write on the board

$$7 \times 1 =$$

9 x 3 = ___	0 x 3 = ___	7 x 1 = ___
7 x 5 = ___	9 x 0 = ___	1 x 5 = ___
5 x 3 = ___	5 x 0 = ___	8 x 1 = ___
8 x 8 = ___	0 x 7 = ___	1 x 3 = ___
6 x 9 = ___	4 x 0 = ___	1 x 7 = ___
6 x 3 = ___	0 x 10 = ___	9 x 1 = ___
9 x 10 = ___	6 x 0 = ___	1 x 9 = ___
9 x 4 = ___	0 x 2= ___	5 x 1 = ___
5 x 9 = ___	0 x 8 = ___	4 x 1 = ___
8 x 5 = ___	7 x 0 = ___	0 x 1 = ___
9 x 9 = ___	0 x 6 = ___	1 x 2 = ___
5 x 6 = ___	0 x 0 = ___	10 x 1 = ___
9 x 5 = ___	1 x 0 = ___	1 x 1 = ___
8 x 6 = ___	0 x 9 = ___	1 x 4 = ___

Ask, **How many times do we enter the 7?** [1] Ask them to find the answer. Assign the worksheet (8-8).

FUTURE REVIEW. After the work with the "1" worksheet is completed, ask, **What happens when you multiply by 1?** [The answer is the number being multiplied by 1.]

Multiplying with take and give

Review the names given to answers. Ask, **What do we call the answer in addition?** [sum] **What do we call the answer in subtraction?** [remainder or difference] **The answer in multiplication is called the PRODUCT. In the problem of 3 taken 2 times equals 6, what is the 6 called?** [product]

Another way to find the product is called "take and give." Younger children often enjoy working with the take and give that they ask for more of the "harder" problems.

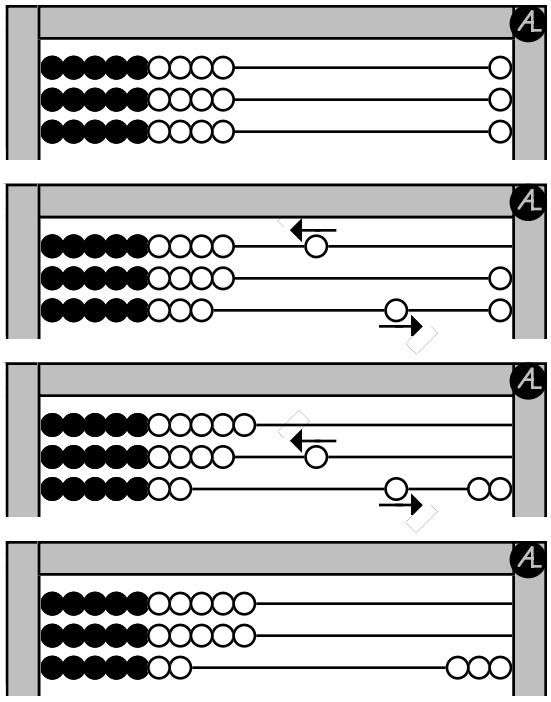
Tell the children that you are going to show them a shortcut for solving problems with higher numbers. Write on the board

$$9 \times 3 = \_\_$$

and ask them to enter the problem. Explain that the product can be read easily if the rows are tens. Therefore, to make the first row a ten, TAKE a bead away from the last row and GIVE it to the first row. Do the same thing for the second row. Then the product can be read as 27.

Ask them to work problems, 7×8 and 8×9. Any number of beads may be removed at a time as long as the same number is restored on another wire. Provide worksheets (8-9, 8-10).

Also give them worksheets (8-11) with the problems written in the vertical form; tell the children that they are worked the same way. Also tell them that there is another way to read a multiplication problem; 9×3 can be read as "9 times 3."



9 x 3 = ___
7 x 5 = ___
5 x 3 = ___
8 x 8 = ___
6 x 9 = ___
6 x 3 = ___
9 x 10 = ___
9 x 4 = ___
5 x 9 = ___
8 x 5 = ___
9 x 9 = ___
5 x 6 = ___
9 x 6 = ___
8 x 6 = ___

6	9	3
x 5	x 3	10
8	6	6
x 4	x 3	x 6
5	2	7
x 9	x 1	x7
4	7	8
x 1	x 3	x 2

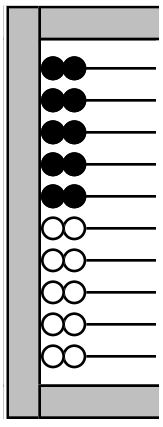
Writing multiplication tables

1. Say to the group that they will be writing the multiplication tables. Although there are no new facts, the tables organize all the multiplication facts, which they need to learn. They already know some of them. After the tables (8-12 to 16) are completed, they can be fastened together.

Write on the board a sample table, for example, the 2s as shown on the mini worksheet shown on the right.

To start, enter the first 2 and record the product. Subsequently add another 2 and write the result. Do not clear until the entire table is completed.

2 x 1 = ___
2 x 2 = ___
2 x 3 = ___
2 x 4 = ___
2 x 5 = ___
2 x 6 = ___
2 x 7 = ___
2 x 8 = ___
2 x 9 = ___
2 x 10 = ___



Unit 9
Division

Division answers two questions; for example, 24 divided by 3 tells either how many 3s are in 24 or how much is in each of 3 groups. In this unit, the children will be shown different ways of dividing numbers. During this work, they will memorize many of the facts. Later in the unit, the children will be tested to determine how many facts they know, and strategies can be given for those they still need to learn.

Division has a profusion of signs to represent it, namely, ÷,)‾, —, and /. The first of these, the division sign, primarily is used to show numeric calculations in arithmetic; it is often found on calculators. The second symbol, the "doghouse," is reserved for calculating with paper and pencil. The third symbol, the dividing line, is widely used in most branches of mathematics; it is also extensively used in divisions resulting in quotients less than 1, usually referred to as fractions. The fourth symbol, the slanted line, is used in print, including recipes, on highway signs, and in work on computers.

Division by 0 is a topic that does not need to be introduced to primary students. The older child can appreciate the impossibility. Help a child to understand by asking how many 0s are in the number: it makes no sense, so we say it is impossible.

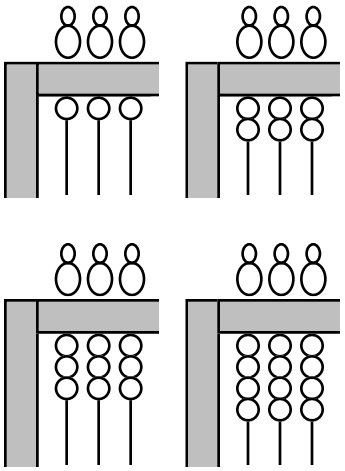
Short division is a skill that is frequently needed and should be mastered first. Long division, however, is seldom done by hand today because of the widespread use of calculators. In this unit, it is approached as an extension of short division.

INTRODUCING DIVISION

Division will be studied from several different perspectives, including counting, skip counting, and the inverse of multiplication.

Division by counting

Tell the children that now they will start division. Pose this problem, **If three friends have 12 pens that they want to share evenly, how can they share them?** Guide them to the solution of passing them out, 3 at a time, until they are gone. Repeat with other objects, such as 18 books shared evenly among 6 children.



Show 12 ÷ 3 on the abacus. Use golf tees or markers from a game to represent the persons (3 for this example). Enter beads evenly until the total quantity (12) is reached. Point to the first wire, or marker and ask, **How many did this one receive?** [4] Repeat for the second and third wires. See the figures.

Show them how to write it saying, **This is one way we write it**. Write on the board

$$12 \div 3 = 4$$

while saying, **Twelve divided by 3 equals 4.**

Guide them through two other examples, such as, 14 ÷ 2 and 20 ÷ 4 before giving them worksheets (9-1).

12 ÷ 3 = ___
10 ÷ 2 = ___
12 ÷ 4 = ___
15 ÷ 5 = ___
16 ÷ 4 = ___
21 ÷ 7 = ___
14 ÷ 2 = ___
30 ÷ 10 = ___
35 ÷ 5 = ___
8 ÷ 4 = ___

FUTURE REVIEW. After they have completed the worksheets, write and ask them how they would do $0 \div 3$, **How much would each person receive?** [0]

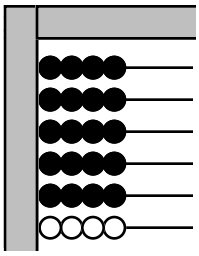
Division by skip counting

Tell the children that there is another way to write division. Write on the board

$$4\overline{)24}$$

and explain that it is read as 24 divided by 4 and means how many 4s in 24. One way to find the answer is to enter and count by 4s until 24 is reached. The number of wires tells how many 4s are in 24. Show them where to write the answer, above the ones place of the 24.

Repeat for $3\overline{)18}$ and $5\overline{)20}$. Then give them worksheets (9-2) for practice.



$$\begin{array}{r} 6 \\ 4\overline{)24} \end{array}$$

$4\overline{)24}$	$4\overline{)32}$
$5\overline{)50}$	$2\overline{)16}$
$3\overline{)12}$	$7\overline{)28}$
$4\overline{)28}$	$10\overline{)90}$
$5\overline{)20}$	$6\overline{)18}$
$8\overline{)40}$	$7\overline{)35}$
$5\overline{)15}$	$4\overline{)8}$

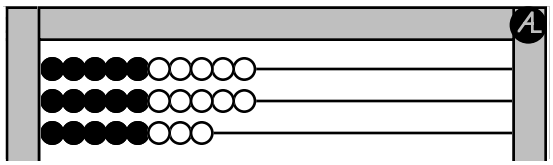
Division by take and give

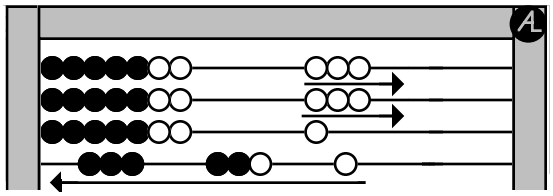
Division by counting and by skip counting are somewhat tedious, but many children enjoy division by take and give. It consists of rearranging a quantity into the desired array in one of two methods.

1. Write on the board

$$7\overline{)28}$$

and ask the children to enter 28. Tell them, **We want to find out how many 7s are in 28, so we want only 7 on each wire.** Ask the children to remove the extra beads by take and give. See the figures. **So how many 7s are in 28?** [4]





Repeat for $5\overline{)20}$ and $9\overline{)18}$. Then assign them similar worksheets (9-3).

$7\overline{)28}$	$8\overline{)56}$
$4\overline{)16}$	$3\overline{)18}$
$3\overline{)21}$	$9\overline{)72}$
$6\overline{)30}$	$2\overline{)16}$
$5\overline{)25}$	$3\overline{)24}$
$8\overline{)32}$	$7\overline{)42}$
$4\overline{)28}$	$2\overline{)14}$

2. The second method involves arranging the beads evenly on 7 wires. Say, **We want to find out how much is on each of 7 rows.** Again write on the board

$$7\overline{)28}$$

and ask the children to enter 28. Tell them that we want to distribute the 28 over 7 wires. Use take and give; start at the 7th wire with the transferring. Although 4 beads were transferred at a time in the figures below, any quantity can be transferred at a time. See the figures. The worksheet (9-4) is similar to the last one.