Teaching the Core of Number Sense

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Number sense involves understanding how we name the numerosity of quantities, how we compare and see relationships between different quantities, and how we flexibly find the resulting quantity after combining or separating quantities.

Sometimes, the term *intuitive* is included in the number sense definition, but there is very little about number sense that is intuitive. Number sense essentials must be carefully taught before numbers and relationships can seem instinctive.

This article describes a simple straightforward trajectory to help all children achieve number sense. There are several components:

- ✔ Subitizing 1 to 10
  - Perceptual subitizing from 1 to 5
  - Conceptual subitizing 6 to 10 by grouping with 5
- ✔ Part-whole circles
- ✔ Place value
  - Transparent number naming
  - Overlapping place value cards to 99
  - Overlapping place value cards to 9999
- ✔ Strategies for adding without counting

SUBITIZING 1 TO 10

Young infants can distinguish between 1, 2, and 3 objects. The instant recognition of a quantity without counting is known as *subitizing*. Three-year-olds can subitize up to five, especially after they learn that five has a middle, which distinguishes it from four. Subitizing enables the child to see the whole and its elements simultaneously while, on the other hand, counting constrains the child to focus on a single element. Most preschoolers love to reveal their age by holding up the requisite number of fingers without any counting whatsoever. In doing so, these children are demonstrating they understand the cardinality principle.

Quick identification can be extended beyond five, called *conceptual subitizing*, by grouping quantities. Although dice patterns are sometimes used for quick recognition, such patterns can be problematic since some children memorize the patterns without seeing the quantities. In addition, adding 1 to the 5-dice pattern does not produce the 6-dice pattern. A pattern that is mathematically more powerful for subitizing quantities six to ten is "five plus the remaining amount" and our fingers and tally marks provide perfect models. Note that Roman numerals, ancient abacuses, and the musical staff are also grouped in fives. Children in Japan and the Netherlands consistently use five as a subgroup.

Children can learn the sequence from 1 to 10 by raising their fingers and naming the quantities or by reading the "stairs" from top to bottom as shown in ***Figure 1***.

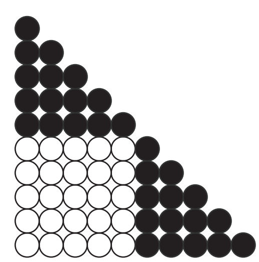

Figure 1

Researcher Butterworth (1999) states that children who can subitize perform better in mathematics long term. He also notes that children who use their fingers as representational tools (not counting) perform better in mathematics.

Only after the children know the names for the quantities from 1 to 10 should we teach them to write the corresponding numbers. A chart with tally sticks or finger representations from 1 to 10 followed by the correct numeral will help children who are confusing the numerals or writing them backwards.

PART-WHOLE CIRCLES

Part-whole circles, in which the whole is written in the large circle and the parts in the smaller circles, help children in several ways (see ***Figure 2***). With these diagrams, students can partition numbers, see the connection between adding and subtracting, solve more difficult story problems, and develop a better understanding of place value.

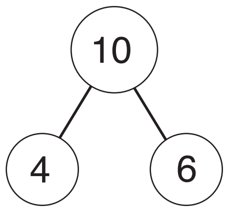

Figure 2

PLACE VALUE

Place value, the principle that the value of a digit in a written number is determined by its position in the number, is the cornerstone of modern arithmetic. Writing more than 500 years ago, the author of *Treviso Arithmetic of 1478* considered place value so important that it was listed first among the "five" operations of arithmetic. Without place value, computational algorithms make little sense. Place value organizes numbers into neat packets.

Place value has two components: a static part and a dynamic part. The static part of place value is the concept that the *value* of a digit in a number depends upon its *place*. Starting at the right, the first position is the number of ones, the second position is the number of tens, the third position is the number of hundreds, the fourth position is the number of thousands, and so forth. However, the position values are not arbitrary, but form a pattern: a ten is 10 ones, a hundred is 10 tens, a thousand is 10 hundreds, and so forth. To recognize a pattern, a minimum of three consecutive elements must be given. Therefore, to understand the place value pattern, it is imperative that children work with thousands.

The dynamic part of place value refers to transferring between positions, as often occurs when performing arithmetic operations. For example, when adding 25 + 18, part of the 13 ones (5 + 8) must be transferred to the tens position to restore the sum to its proper form because no position can have more than nine.

Transparent Number Naming

Children speaking Asian languages are aware of place value at an early age because of the words they use for naming quantities. Following 10, their next numbers are: ten 1, ten 2, ten 3, ten 4, . . . , 2-ten, 2-ten 1, 2-ten 2, 2-ten 3, . . . , 9-ten 9. These children need only 11 words to count to 100; on the other hand, English speakers need 28 words. Making things even more difficult for speakers of English, the number names for 11 and 12 seem arbitrary and 13 to 19 are spoken in reverse order. Spanish number words have similar inconsistencies. You could say that we give our children numbers in bulk while the Asians give their children numbers prepackaged.

Teachers encounter a similar obstacle when teaching reading. Generally, the name of a letter is not the same as the sound it represents. The solution is using phonics to teach the sound equivalent for each letter. The analogous solution for approaching early number sense is teaching the transparent number name, the name of the quantity.

I have asked children at the end of first grade, "What is ten plus three?" Children taught traditionally bob their heads three times before answering; they are counting on from ten. Children knowing transparent number naming will answer immediately; they think ten 3, which they know is thirteen.

Overlapping Place Value Cards to 99

While oral transparent number naming gives children an introduction to place value, written numbers provide a deeper understanding. ***Figure 3*** shows place-value cards for 2-ten and for 5. When reading the 20-card, point to the 2 while saying 2 and point to the 0 while saying "ten." It is the 0 that gives the 2 its value of 2-ten. The cards are then overlapped to show 2-ten 5. The teens present no irregularities using transparent naming with place-value cards; for example, see 1-ten 3 (13), shown in ***Figure 4***.

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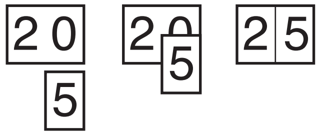

Figure 3

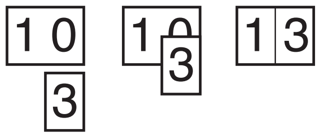

Figure 4

After a few months using transparent naming, the children transition to the regular way of number naming.

Overlapping Place Value Cards to 9999

Read the 600-card by pointing to the 6 while saying "six" and pointing to the two zeros in turn while saying "hun-dred." Likewise, read the 5000-card by pointing to the 5 while saying "five" and pointing to the three zeros in turn while saying "th-ou-sand." See ***Figure 5***.

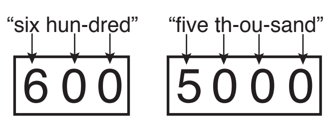

Figure 5

One advantage of this approach is that students are encouraged to read numbers in the normal direction, from left to right. Often they are taught to start at the right and assign the values of ones, tens, hundreds, and so forth, to each digit as they proceed to the left.

A second advantage is a broader understanding of the place value system. If you ask a student how many tens are in 840, a student taught to look at columns will say there are 4 tens. While it is true there is 4 in the tens place, there are actually 84 tens in 840.

A third advantage of using place-value cards is that students are prepared for scientific notation; seeing 600 as 6 followed by two zeros is similar to 6×10^2, in contrast to thinking of the 6 as being in the third column.

STRATEGIES FOR ADDING WITHOUT COUNTING

To add with the traditional counting model, students are taught to count out the quantities for each addend and then count to find the sum. Gradually, they advance to counting fingers, on number lines, or even counting words themselves.

To appreciate what it means to learn to add with the counting model, assume for a moment that we count using the alphabet. A is 1, B is 2, C is 3, and so forth. Now let us add F + E. First we count out F counters (A, B, C, D, E, F). Next we count out E counters (A, B, C, D, E). What is the sum? Either count all or count on from F to get the sum K.

If you did not have counters, how would you count from F? You might use your fingers: raise one finger for A, while saying G. Then raise another finger for B, while saying H. Continue until you raise E fingers and say K. If fingers are forbidden, a tedious mental dialogue ensues: A is G, B is H, C is I, D is J, and E is K.

Now that you know how to add, memorize the facts. What is C + D? E + E? H + G? So we resort to flash cards, which do not work for one out of seven children, especially those with learning disabilities. Even for the remaining six out of seven, the results are often short-lived, necessitating frequent review.

Adding and subtracting within 20 can be accomplished without the need for counting. Some strategies are described below. Incidentally, children in Japan do not use the counting-on strategy.

Facts Equal to Ten

The first set of facts children should learn are those whose sums equal ten: 1 + 9, 2 + 8, 3 + 7, 4 + 6, and 5 + 5. They can easily be subitized with beads or tiles that are grouped in fives. The Go to the Dump game, similar to Go Fish, provides plenty of practice. Instead of asking for cards that match to make a pair, children ask for cards that make a pair that add up to 10. Children can also play a variation of the game to learn the facts equaling 11 or 9.

Adding 9 or 8

To add 9 + 6, children should think of taking 1 from the 6 and giving it to the 9, resulting in

10 + 5. Place value work assures that the child knows this as 1-ten 5, or 15. Likewise, adding 8 + 4 can be done by taking 2 from the 4 and giving it to the 8, resulting in 1-ten 2, or 12.

Adding with Two-Fives

To add 6 + 8, children could see the 6 as 5 + 1 and the 8 as 5 + 3. The two fives equal 10; the "leftovers" equal 1 + 3, resulting in the sum of 1-ten 4, or 14. As another example, add 7 + 6. First, children see 7 as 5 + 2 and 6 as 5 + 1; the two fives are 10 and the leftovers are 2 + 1, giving the sum of 1-ten 3, or 13.

COUNTING

Rote counting to 100 has become the entrance through which we drag our young children into the world of mathematics. This rote memorization task can be very difficult for some children, including those who have learning disabilities, who have poor memories, who have specific language impairment (SLI), who have Downs syndrome, or who have other challenges. The corollary of expecting a student to "count forward beginning from a given number," rather than from one, is even more daunting. Try this: Without starting from the beginning, what word follows "hill" in "Jack and Jill"?

A recent study using this non-counting approach in Finland found that "The children with SLI began kindergarten with significantly weaker early numeracy skills compared to their peers. Immediately after the instruction phase, there was no significant difference between the groups in counting skills" (Mononen, Aunio, Koponen, 2014). In other words, children develop the ability to count without asking them to memorize that long list of words by heart. Building on their natural ability to subitize, children understand that number words are names for quantities. It is well-known that learning occurs when we connect to what we already know: subitizing connects to visual images, counting connects to abstract words. Preschool teachers who learn about subitizing can eliminate many unnecessary hours they spend teaching one-to-one correspondence.

Researchers have extensively studied the complexity of counting and delineated the stages of children's progression. Karen Fuson (1988) listed them as: (a) String Level, when the words are forward connected, an undifferentiated whole; (b) Unbreakable List, when the child must start from the beginning each time; (c) Breakable Chain, when the child is able to start at various places; (d) Numerable Chain, when the child understands the words in a numerical sense; and (e) Bidirectional Chain, when the child can count either forwards or backwards. Often students are asked to compare two numbers. Sadly, some students count until they come upon one of the numbers. That number will be the lesser of the two.

We know that children's later mathematics success depends upon their early number sense. Counting is a by-product of early number sense, not the goal. We need to reassess the counting model where we spend a year teaching students to count and two years telling them not to. Let us eliminate the unintended consequence of exacerbating any achievement gap when the poor counters start school by teaching right from the beginning that counting words are names of quantities, not abstract words. We can help more students reach the goal of the Common Core State Standards to better prepare American students from Kindergarten through 12th grade for college and career.

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