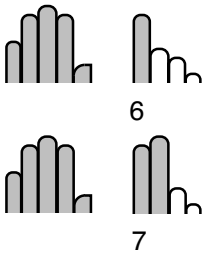
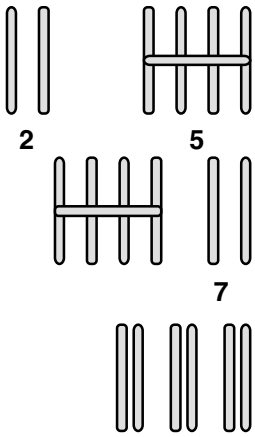
The Power of Abacus-Based Math

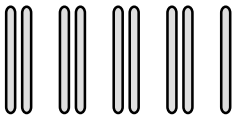


Yellow is the Sun

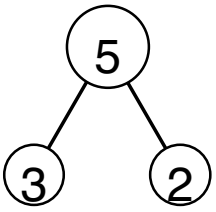
Yellow is the sun.
Six is five and one.
Why is the sky so blue?
Seven is five and two.
Salty is the sea.
Eight is five and three.
Hear the thunder roar.
Nine is five and four.
Ducks will swim and dive.
Ten is five and five.



An even number.



An odd number.



Part-part-whole. Helps children solve problems and write equations.

In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic.

–Mindy Holte

Mathematics is changing.

- Our world is becoming ever more mathematized.
- Math itself is expanding, e.g., fractals, statistics, encryption.
- Computers and calculators both change and are changed by new advances in math.
- Math itself is becoming more visual, e.g., graphing calculators, fractals, encryption. More geometry is being used.

Mathematics education is changing.

- More known about brain and learning, e.g., child under stress stops learning.
- Real learning means making connections.
- Learning styles: majority of children do not learn best by listening. A study showed teachers spend > 80% time talking.
- Visual thinkers, the gifted and many with LD, find rote memorizing difficult. They need to see whole picture, not small steps.
- Standards suggest what, when, and how math is taught.
- More than arithmetic must be taught: Geometry, algebra, probability, and statistics are all on state exams from K on.

Economics of mathematics education.

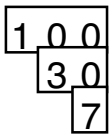
- In international studies, such as TIMSS and PISA, the U.S. scores low compared to other countries.
- In 2004 of the 1.2 million students who took the ACT test, only 40% were deemed ready to study college algebra.
- 25% of college students take remedial math, 37% in CA.
- 50% of engineers & computer scientists receiving Ph.Ds are foreign born.
- Only 51% of patents going to U.S. citizens, down from 90%.

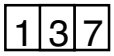
Why understanding is necessary.

- Understanding aids memory: 93 min to learn earn 200 nonsense syllables, 24 min to learn 200 words of prose, and 10 min to learn 200 words of poetry.
- Better learning.
- Less memorization and review needed.
- Essential for applying to real problems.
- Impossible to memorize advanced math.

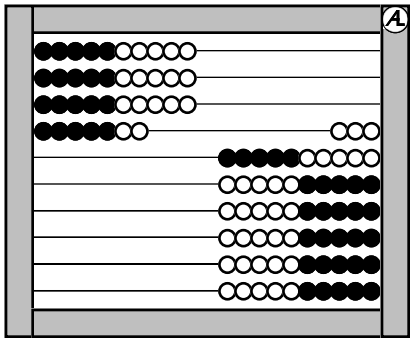
Counting, a rote activity, does not help a child master math concepts.

- The alphabet example of how we teach children simple adding shows some of the difficulties. ($F + E = K$)
- Ignores place value.

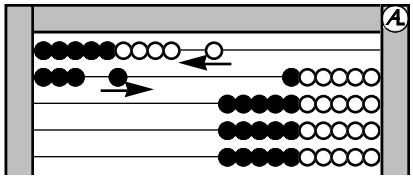




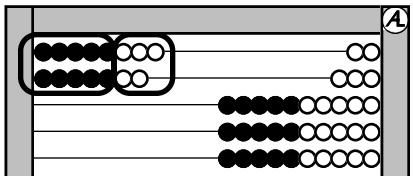
Place value cards.



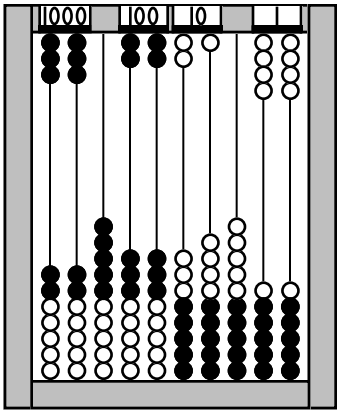
AL abacus (side 1) with 37 entered.



Transforming 9 + 4 into 10 + 3.



Seeing the sum of 8 and 7 as 10 (2 fives, the black beads) and 5, the number of white beads.



AL abacus (side 2) with 6438 entered.

- Young children don't realize counting represents quantity.
- Very error prone. Children under 6 are not good counters.
- No motivation to learn facts.
- Inefficient and time-consuming.

Visualizing quantities.

- Babies, at 5 months, can add and subtract up to 3.
- Group by 5s. Impossible to imagine 8 objects without grouping.

Place value is the most important concept of arithmetic.

- Teach *math way* of counting: after 10, say ten 1 (11), ten 2 (12), ten 3 (13), . . , 2-ten (20), 2-ten 1 (21), . . . , 9-ten 9 (99).
- All Asian children learn math with math way of naming numbers they understand place value in first grade. Average U.S. child understands it at the end of fourth grade.
- Essential for understanding algorithms.
- Place-value cards: encourage reading in normal order; starting with ones column and then tens columns is backwards.
- Essential to use 4-digit numbers to understand trading (carrying).

What makes a good manipulative (according to Japanese).

- Easily visualized.
- Representative of the structure of mathematics.
- Easily managed.

The AL Abacus.

- Grouped in fives and tens.
- Used for operations, strategies, money.
- Evens and odds; also needed for side 2 of abacus.

Some addition strategies.

- What makes 10: seen on abacus, Go to the Dump game.
- Adding 9: complete the 10.
- Two 5s: two fives = 10; then add "leftovers." For 8 + 7, the leftovers are 3 + 2; so the sum is 15. See figure at left.

Learning the facts.

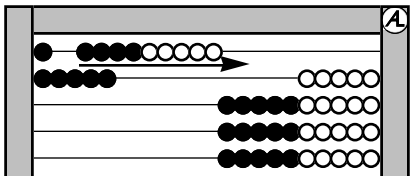
- Strategies first: It takes time for new strategy to become automatic.
- Games far superior to flash cards.
- Timed tests and graphs.

Importance of mental computation.

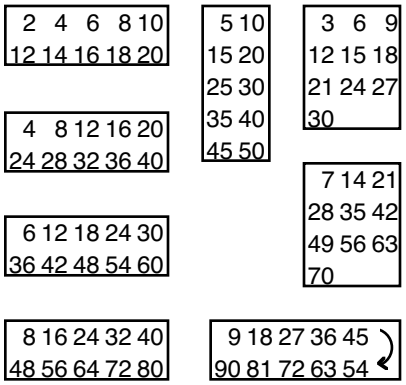
- Understanding more important than procedures.
- Develops number sense (common sense with numbers).
- Necessary for estimating.
- Easier to start at the left: e.g. 34 + 48 = 34 + 40 [74] + 8 [82].

Adding 4-digit numbers on the abacus.

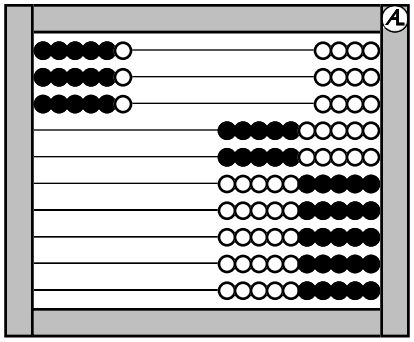
- Important for understand trading: that 10 ones = 1 ten, 10 tens = 1 hundred, 10 hundreds = 1 thousand.
- Children need to write down on paper what happens after number is added on the abacus.



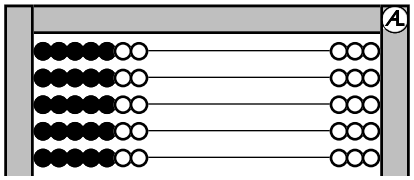
Subtracting 15 – 9 by subtracting the 9 from 10, then 1 + 5 = 6.



Skip counting patterns.



6 x 3 (6 taken 3 times).



7 x 5 (7 taken 5 times).

Why thousands so early.

- To appreciate a pattern, we need at least three samples.
- To understand trading, the child must trade 10 ones for 1 ten, 10 tens for 1 hundred, and 10 hundreds for 1 thousand.

Introducing subtraction.

- Only after addition is mastered. It is psychologically negative.
- Going up is easier for some facts; e.g. 11 – 9. Used for making change. Also, avoid "take away."
- 15 – 9: subtract 9 from 10.

Skip counting (multiples).

- Needed for multiplication facts, fractions, and algebra.
- Start as soon as 1-100 is understood; use patterns.

Skip counting pattern explanations.

- Twos. The second row is 10 plus the first row. They are the even numbers.
- Threes. Consider the ones: they increase starting at the lower left with 0 (30) and continue up the first column and over to bottom of the second column and to the third column. Next consider the tens: 0, 1, 2 in each column. Sum of the digits: 3 in the first column (1 + 2, 2 + 1, 3 + 0), 6 in the second column, and 9 in the third.
- Fours. The second row is 20 more than the first row, every other even number.
- Fives. They have an obvious singsong pattern.
- Sixes. The first row is the even 3s. Second row is 30 more than the first row.
- Sevens. Within each row the tens increase by 1. The ones increase by 1 starting at the upper right (21) and continuing down the column and over to the next column.
- Eights. In each row the ones are the decreasing even numbers. The second row is 40 more than the first row, also every other 4.
- Nines. The sum of the digits in all cases is 9. The ones decrease while the tens increase. The second row has the digits of the first row reversed, as shown by the arrow.
- Skip Counting Memory game.

Multiplication seen visually on abacus. (See figures at left.)

Goal of math—not to turn students into $7 calculators— but thinking persons who can apply math to new situations.

Galileo: "The great book of nature can be read only by those who know the language in which it was written. And this language is mathematics.

Ref: Cotter, Joan. Using Language and Visualization to Teach Place Value. (for NCTM members: http://my.nctm.org/eresourcesview_article.asp?article_id=2261)

A downloadable PowerPoint presentation on this topic is available at http://www.alabacus.com.

GO TO THE DUMP

(From *Math Card Games: 300 Games for Learning and Enjoying Math. Fourth edition* by Joan A. Cotter (2005); published by Activities for Learning: Hutchinson, MN.)

Objective To learn the combinations that total 10

Number of player 2 to 4

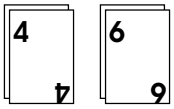
Cards 4 or 6 of each basic number card 1 to 9

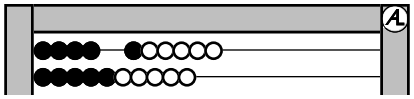
Deal Each player takes 5 cards; the remaining cards face down form the dump, or stack.

Object of the game To collect the most pairs that equal 10

Preparation Before starting, the players check over their hands for pairs that total 10. To do this, they look at each card in turn, determine what is needed to make 10 and look for that number among their other cards. (Some children may need to spread the cards out on the playing surface.) Encourage the children to use their abacuses.

Store paired cards face up on *two* piles. (This allows verification and keeps the cards shuffled for the next game.)





6 is needed with 4 to make 10.

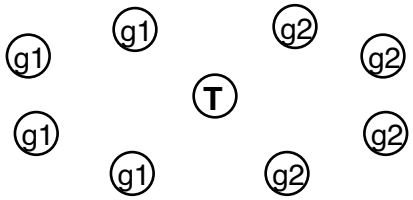
Play When all are ready, the first player asks the player on the left for a number *needed* to complete a pair. If the second player has it, it must given to the first player, whereupon the first player receives another turn. If the player asked does not have it, the player says, "Go to the Dump," which is also the signal to begin a turn. The second player takes a turn by asking the player on the left and so forth.

Meanwhile, the first player concludes the turn by picking up the top card from the dump. Additional turns are not given even if a needed card is picked up.

A player running out of cards takes five more cards, but the turn is ended. When the dump is exhausted, players may ask any player (not only the players on their left) for a card.

At the end of the game, players combine their two stacks and compare the heights. (Counting the cards is too time consuming.) No shuffling is necessary for subsequent games.

Note One way to monitor two games simultaneously is to sit in the middle of a figure 8. See the figure below.



Monitoring two games: g1 is one game, g2 is the other game, and T is the teacher.

(The PowerPoint presentation and 4-page handout available at www.ALabacus.com)