| LESSON/WORKSHEET |  | CHANGE DATE | CORRECTION OR UPDATE |
| :---: | :---: | :---: | :---: |
| Lesson 23 |  | 10/05/2010 | In chart toward bottom of page the last fraction should be 1/3. |
| Lesson 27 | Worksheet 27-2 | 01/01/2012 | The second sentence should say: Also find the area for the remaining rectangles.... |
| Lesson 30 |  | 06/01/2011 | Answers: 1 digit: formula 1,8 strokes; formula 2,9 strokes; formula 3, 6 strokes. |
| Lesson 38 | Solutions for Worksheet 38 | 12/25/2010 | Problem 3: A clearer answer is: First flip 1 trapezoid vertically and put it next to other trapezoid. The 2 trapezoid make a parallelogram. The width of the parallelogram is $w 1+w 2$. The area is $(w 1+w 2) h$. So divide by 2 for the area of one trapezoid. |
| Lesson 45 |  | 12/06/2016 | Second paragraph under Solving Angle Problems, about 3/4 of the way down the page, should read: To solve the problem, keep in mind the relationships you know about angles. Since you're missing $\angle B A C$ and you know the measure of all the angles in a triangle is $180^{\circ}$, you can find the angle measurement. The answer is at the bottom of the page. |
| Lesson 48 |  | 03/01/2012 | The equations in the middle of the page should be: $a<b+c, b$ $<\mathrm{a}+\mathrm{c}$, and $\mathrm{c}<\mathrm{b}+\mathrm{a}$. |
| Lesson 55 |  | 11/21/2010 | Convex and Concave: The first word in the second sentence should be convex, not concave. |
| Lesson 57 | Solutions for Worksheet 57 | 01/01/2012 | Problem 2A: The answer should say $32=9 \mathrm{~cm}^{2}$. |
| Lesson 60 | Solutions for Worksheet 60-1 | 12/17/2015 | Problem 1: Second line of the calculations should read $c=149$ cm . |
| Lesson 60 | Solutions for Worksheet 60-2 | 02/07/2010 | Problem 6: Second line of the equation should read $c^{2}=a^{2}+b^{2}$ $=20^{2}+25^{2}=1025$, not 1050 . |


| Lesson 62 |  | 10/05/2010 | The picture is covering of the text. Should this: |  | is all the: point. <br> Points. I a point is considere you can 1 <br> Planes. sphere is point." ( $t$ plane is a |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Lesson 63 | Worksheet 63-1 and Solutions for Worksheet 63-1 | 03/26/2012 | See attached PDF. |  |  |
| Lesson 64 |  | 06/01/2012 | Second last sentence s straightedge for polygo | uld say: Use your goniome with 5 and 7 sides. | ter and |
| Lesson 66 |  | 03/01/2012 | Goal 3: To estimate the r diameter) in a circle by us circumscribed polygons. | tio of $C$ to $D$ (circumference ing the perimeter of inscrib | $d$ and |
| Lesson 69 |  | 10/05/2010 | Step 1 (at the bottom of $p$ 1 cm . Start tick marks tha The drawing at the botto 1 cm and 1 cm . <br> Step 3: Should say "Look | age): The lines being drawn are 1 cm away from lines, of the page should also be <br> for a radius of 10 cm " inste | should be not .5 cm . labled <br> ad of 5 cm . |
| Lesson 73 | Worksheet 73 and Solution for Worksheet 73 | 10/05/2010 | Midpoints were marked w | ng. See attached PDF. |  |
| Lesson 74 |  | 10/05/2010 | The picture is covering of the text. Should this: |  | WorksI has strai WorksI second ! second, $\underset{\text { warning }}{\text { wist. It is }}$ line segrThere an <br> construc minimiz |
| Lesson 76 | Solutions for Worksheet 76 | 03/08/2017 | The solution for Problem $\begin{aligned} \text { Dist. } & =1 / 4 \mathrm{C} \\ & =1 / 4 \times 12,720,000 \pi \\ & =9,985,200 \mathrm{~m} \\ & =9985 \mathrm{~km} \end{aligned}$ | 4 should read as follows: |  |
| Lesson 77 | Worksheet 77-2 and Solutions for Worksheet 77-2 | 03/26/2012 | See attached PDF. |  |  |


| Lesson 79 |  | 10/05/2010 | The graphic in the $A=30=\pi r^{2}$ of the page <br> wrong <br> To find $r^{2}$, divide by $\pi$. should $r^{2}=\frac{30}{\pi} \approx 9.55$ <br> To find $r$, take the square root. $r \approx 3.09$ |
| :---: | :---: | :---: | :---: |
| Lesson 79 | Worksheet 79-2 | 09/01/2012 | Problem 7: Find the radius (not diameter) of a circle that has twice the circumference of the circle below. |
| Lesson 80 | Solutions for Worksheet 80 | 03/14/2017 | Area for the 14 " pizzas should be 153.9 in $^{2}$, not 153.8 in $^{2}$. Problem 5: answer should be $\$ 25.99$, not $\$ 23.99$. |
| Lesson 81 | Worksheet 81-1 and Solutions for Worksheet 81-1 | 03/14/2017 | Problem 1: Second sentence should read "The line below is the bottom of the tangram", not top of the tangram. Problem 13: <br> Second sentence should read "Calculate the length of the other sides of the $\operatorname{lgT}$." See attached PDF. |
| Lesson 82 | Worksheet 82-1 and Solutions for Worksheet 82-1 | 03/21/2012 | See attached PDF. |
| Lesson 82 | Solutions for Worksheet 82-3 | 10/05/2010 | See attached PDF. |
| Lesson 86 |  | 10/05/2010 | $-90^{\circ}$, wl <br> The picture is covering hand ca of the text. Should <br> Worksl this: either re the bott figure al complic |
| Lesson 87 | Solutions for Worksheet 87 | 01/01/2012 | Answer 2: A' coordinate should be: $\mathrm{A}^{\prime}(7,11)$ |
| Lesson 88 | Worksheet 88 | 04/11/2017 | The worksheet instructions have been significantly changed. See pdf. Solutions remain the same. |
| Lesson 91 | Worksheet 91 | 04/20/2017 | The worksheet instructions have been changed slightly to increase understanding. See pdf. Solutions remain the same. |
| Lesson 92 |  | 04/21/2017 | Under the heading Analysis, the paragraph should start with "Compare your second image with the original object", not last image. Also, at the end of the that section, it should read "Then find the angle of rotation from the original to the second image and record it in the table." |
| Lesson 92 | Worksheet 92-1 and 92-2 | 04/21/2017 | Worksheets: Instructions have been been changed slightly. See pdf. Solutions remain the same. |
| Lesson 93 |  | 06/01/2012 | The website address at the bottom of the page should be RightStartMath.com/geometry |


| Lesson 94 |  | 06/01/2012 | The top of the page should say to do problems 1-19 and then at the bottom of the page it should say problem 20 instead of problem 21. See attached PDF. |
| :---: | :---: | :---: | :---: |
| Lesson 95 |  | 05/02/2017 | At the end of the second paragraph, add: The number of times it can be rotates is the order of rotational symmetry. <br> And at the end of the thir paragraph, add: The order of rotational symmetry is five. |
| Lesson 102 | Solutions for Worksheet 102-1 | 06/01/2012 | Problem 4: The code should read 3.3.3.4.4/3.4.6.4. (There was an extra 4 in the second half, 3.4.6.4.4) |
| Lesson 103 |  | 10/02/2017 | Answer at the bottom of the page should say second. |
| Lesson 109 | Worksheet 109-1 and Solutions for Worksheet 109-1 | 10/14/2010 | The list of words on the lower right should be >1, 3, 3, 8, not 1 , $>1,3,8$. <br> Solutions on the right side: "Number of different polygons with $<6$ sides" should be 3 , not 1 . |
| Lesson 109 | Solutions for Worksheet 109-2 | 03/25/2011 | Problem 2: Should be c, not a, b, c. |
| Lesson 116 |  | 10/05/2010 |  |
| Lesson 118 |  | 09/10/2010 | At the bottom of the page, the infinity sign should be $\Phi$, the phi symbol. |
| Lesson 121 |  | 12/14/2017 | Problem 2: The second and third paragraphs should read as follows. In the third column, you are comparing how much the ratios differ from the golden ratio. A quick way to find these differences is to use memory on your calculator. First put 1.61803 into memory using the $\mathrm{M}+$ key. Then to subtract 1 from 1.61803, do the following: Press 1 then $-M R=$. The answer -0.61803 appears. (Ignore the negative sign when recording the answer.) <br> For the next answer: Press 2 then $-M R=$. The answer 0.38197 appears. <br> Note that some of the newer calculators have replaced the MR button with the MRC button. It will still work the same. |
| Lesson 122 | Solutions for Worksheet 122 | 02/07/2010 | Problem 1, Legal size paper: Should be 14/8.5, not 14/11, ratio is 1.65 , and golden ratio is yes. |
| Lesson 125 | Solutions for Worksheet 125-2 | 01/13/2014 | Problem 20 has two more solutions: BDGA and BCDG. |


| Lesson 125 | Solutions for Worksheet 125-2 | 12/06/2019 | Problem 11: On the left side, the third one down, $A B G$, should be POG. On the right side, the fifth one down, IPH, should be $B M G$. Problem 18. Additional answers are $B C G$ and $B M G$. |
| :---: | :---: | :---: | :---: |
| Lesson 127 |  | 12/29/2017 | In the middle of the page below the circle, the calculation for the area of the circle, second line, is missing the pi symbol. It should read: $A=\pi(2)^{2}$. |
| Lesson 128 |  | 10/05/2010 | The figure in the center of the page is using the wrong symbol in the last line, it should look like this: $=\sqrt{10} \approx 3.162$ |
|  | Solutions for Worksheet 128 | 12/06/2019 | The answer for Problem 2, the square root of 26 , should be 5.099, not 5.100. |
| Lesson 129 | Worksheet 129-2 and Solutions for Worksheet 129-2 | 01/08/2018 | Question 7 should read: Where do you start when tracing an Euler path that has only two odd points? <br> Question 8 should read: Where do you end when tracing an Euler path that has only two odd points? See attached PDF. |
| Lesson 130 |  | 10/05/2010 | The three graphics shown had the wrong symbols. $a=10 \times .60 \quad \frac{2}{4}=\frac{3}{6} \rightarrow \frac{2}{3}=\frac{4}{6} \quad 12=.75 \times b$ |
| Lesson 131 |  | 09/01/2012 | See attached PDF. |
| Lesson 131 | Worksheet 131-1 and Solutions for Worksheet 131-1 | 09/01/2012 | See attached PDF. |
| Lesson 131 | Worksheet 131-2 and Solutions for Worksheet 131-2 | 09/01/2012 | See attached PDF. |
| Lesson 133 |  | 01/23/2018 | Replace the lesson with the attached PDF. |
| Lesson 133 | Worksheet 133 | 01/23/2018 | Problem 4: Change the question to read "What is the hypotenuse of a right triangle when the legs...." |
| Lesson 135 |  | 10/05/2010 | Graphic in the middle of the page is using the wrong symbol. This is what it should look like: $h=154 \times \tan 14.5$ |
| Lesson 145 |  | 04/26/2017 | Under Problem 6 heading, second paragraph should read, "You need to use trigonometry to fin the pile radius", not pile heiaht. |
| Lesson 145 | Solutions for Worksheet 145-2 | 04/01/2013 | Problem 6: The 4816 should be 4796 in both places. |
| Lesson 146 | Solutions for <br> Worksheet 146-1 | 10/25/2010 | Problem 1: $H$ should be .707 , not .5 and $V$ should be, $.24 \mathrm{dm}^{3}$ and $236 \mathrm{~cm}^{3}$, not $.17 \mathrm{dm}^{3}$ and $167 \mathrm{~cm}^{3}$. |
| Lesson 147 | Worksheet 147-4 | 01/13/2014 | Problem 13:: Second sentence should be Cut it out and fit it into your tetrahedron. |


| Lesson 152 |  | 10/05/2010 | The symbols ar€ $V$ (cylinder) $=\pi r^{2} h$ And $h=2 r$ <br> So $V($ cylinder $)=\pi r^{2} 2 r=2 \pi r^{3}$ <br> $V($ sphere $)=\frac{2}{3} V($ cylinder $)=\frac{2}{3} \times 2 \pi r^{3}=\frac{4}{3} \pi r^{3}$ |
| :---: | :---: | :---: | :---: |
| Lesson 154 |  | 03/25/2011 | Above the second figure, it should say "Lastly, there are six axes." rather than "Lastly, there are three axes". |
| Lesson 159 |  | 10/05/2010 | The figure at the bottom of the should look like this: $H=\frac{1}{2} \sqrt{2} \times a$ |
| Lesson 159 | Worksheet 159 and Solutions for Worksheet 159 | 01/13/2014 | See attached PDF. |
| Lesson 162 | Worksheet 162-2 | 01/13/2014 | Problem 21: Formula should be $V(\mathrm{rh})=\frac{1}{3}(12+10 \sqrt{2}) a^{3}$ |
| Lesson 162 | Solutions for Worksheet 162-2 | 01/13/2014 | Problem 21: Answer should be $8.7 \mathrm{dm}^{3}$. |
| Lesson 163 | Solutions for Worksheet 163-2 | 04/01/2013 | Calculation for $A($ dod $)$ should be $12 \times 5 \times A$ (triangle $A B O)$ not ABC |
| Lesson 165 |  | 10/05/2010 | Angles in a polyhedron: In the third sentence, the degree symbol was wrong or missing. It should read: Each triangle has $180^{\circ}$ so the suface angles total $180 \times 4=720^{\circ}$. |
|  | Final test | 09/01/2012 | Heading for Questions 9-12: Angle 1 is 139.1, not 139.9. |
|  | Final test Worksheets \& Solutions | 04/15/2016 | Heading for Questions 13-15 is missing. Should read: Triangle ABC is similar to triangle XYZ. What is the perimeter of each triangle? What is the ratio of their perimeters? PDF is attached. |

1. How many times do you think the diameter will fit around the circle?
2. Measure the diameters $(D)$ and circumferences $(C)$ for both circles. You choose the units for measuring. Record your answers in

$\qquad$
3. Draw points that are exactly 2 cm from the $\times$ shown below. Make at Date $\qquad$ least 25 points in all directions from the $x$.

4. How many points is it possible to draw? infinite
5. What figure do you get when you connect the points? circle
6. What is the length of the longest line segment you can draw inside your figure? $4 \mathbf{c m}$

Read Lesson 43. Draw lines from the underlined words to the matching figure. Then draw lines from each word to its definition.

$\qquad$
Date $\qquad$

1. How many times do you think the diameter will fit around the circle? $\qquad$
2. Measure the diameters $(D)$ and circumferences $(C)$ for both circles. You choose the units for measuring. Record your answers in the table below and on the graph on the next worksheet. Also measure the diameters and circumferences for five other circles.

| Object | $\underline{D}$ | $\underline{C}$ | $\frac{C}{D}$ |
| :---: | :---: | :---: | :---: |
| A | $6 \mathrm{cm*}$ | $18.8 \mathrm{~cm} *$ | 3.15 |
| B | $12 \mathrm{~cm} *$ | $37.7 \mathrm{~cm} *$ | 3.14 |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |



## Date

$\qquad$

1. Find the midpoints of each side of the triangle. Mark them with points, $M_{1}, M_{2}$, and $M_{3}$. 2. Construct the altitudes (perpendicular lines) to each side of the triangle. Label the points at which the altitudes intersect the sides as $P_{1}, P_{2}$, and $P_{3}$.
2. Label the orthocenter (the point where the perpendicular lines intersect) $O$.
3. Bisect $X O, Y O$, and $Z O$. Label the points $B_{1}, B_{2}$, and $B_{3}$.

5*. Construct the circle that circumscribes the triangle. (Hint: Problem 5 on Worksheet 72-2)
Compare its diameter to the diameter of the nine-point circle. $\qquad$


$5^{*}$. Construct the circle that circumscribes the triangle. (Hint: Problem 5 on Worksheet 72-2)
Compare its diameter to the diameter of the nine-point circle. twice as great 3. Lisect $X O, Y O$, and ZO. Label the points $B_{1}, B_{2}$, and $B_{3}$.
4. Bres intersect)
$5^{*}$. Construct the circle that circumscribes the triangle. (Hint: Problem 5 on
 1. Find the midpoints of each side of the triangle. Mark them with points, $M_{1}, M_{2}$, and $M_{3}$.

[^0]

$\qquad$
7. Use your drawing tools to divide the circle below into 12 equal sectors. Cut out the circle. 8. On the back side of the circle, draw a radius. (It needn't be exact.) Roll the circle along the top edge of the parallelogram below as shown. The radius of the circle is $r$, what is the width of the parallelogram. $\qquad$


9. Cut apart the 12 sectors of your circle. Arrange them on the parallelogram above as shown. Glue or tape, if desired.
Imagine dividing the circle into a million parts, instead of 12 , and arranging the sectors the same way. Then the sides of the parallelogram would look straight.
10 . The height of the parallelogram is $r$. Using Question 8, what is the area of the parallelogram? $\qquad$ What is the area of a circle? $\qquad$


$5^{*}$. Construct the circle that circumscribes the triangle. (Hint: Problem 5 on Worksheet 72-2)
Compare its diameter to the diameter of the nine-point circle. twice as great 3. Lisect $X O, Y O$, and ZO. Label the points $B_{1}, B_{2}$, and $B_{3}$.
4. Bres intersect)
$5^{*}$. Construct the circle that circumscribes the triangle. (Hint: Problem 5 on
 1. Find the midpoints of each side of the triangle. Mark them with points, $M_{1}, M_{2}$, and $M_{3}$.

[^1]


2. How many pieces are in a tangram?
3. How many different shapes?
4. What fraction of the pieces are triangles?
2. How many pieces are in a tangram?
3. How many different shapes?
4. What fraction of the pieces are triangles?
2. How many pieces are in a tangram?
3. How many different shapes?
4. What fraction of the pieces are triangles?
5. Which applies to all the triangles in the tangram?
Right Isosceles Equilateral Congruent


|  | G6・モ6\＄ | と6・モ0工\＄ | łSOつ［PłOL |
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 5．If the Montreal store sold a 16＂pizza，what should be the price？



1．Find the area to the nearest tenth of a square in．for each size of pizza in both charts． same day in December． the charts below．These were the prices at the same pizza chain on the Prices for cheese pizza in the United States and in Canada are listed in Worksheet 80，Pizza Problems Worksheet 80，Pizza Problems Name
$\begin{array}{ll} & \text { Name } \\ \begin{array}{l}\text { Draw the three figures aligned } \\ \text { to the top, right, bottom, and } \\ \text { left lines as marked. }\end{array} & \text { Align Top }\end{array}$

Shapes are shown aligned to two sides. Draw them in the


Draw the three figures aligned to the top, right, bottom, and left lines as marked.


Date $\qquad$
-


Draw the four shapes aligned to the top, right, bottom, and left lines as marked.

(FIGURES WILL VARY ACCORDING TO HOW THEY'RE STACKED.)

$\qquad$

Shapes are shown aligned to two sides. Draw them in the center before they were aligned.

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Worksheet 83-1, Reflecting
Reflect each figure about the line of reflection, shown as a center line. Use your drawing tools. Check your work with a reflecting
tool, if available.


Name $\qquad$

3.

4.

7.

5.


6.


## $----$


9.

$\qquad$
$\qquad$


5．Find the center of rotation for $J K L M$ and its image．
4．Using the same center and angle of rotation，draw four new
images，continuing the rotation around $P$ ．Label each new image．
1．Connect each point of the triangle with its corresponding image． Use a light line．Then construct the perpendicular bisector of each line．Where they intersect，label the point $P$
2．Draw circles with center $P$ and radii $\overline{P C}, \overline{P A}$ ，and $\overline{P B}$ ．
What is the angle of rotation between $\triangle A B C$ and $\Delta A^{\prime} B^{\prime} C^{\prime} ?$
Make sure the rotation angle gives a whole number
（no fractions）of images around the point of rotation！
4．Using the same center and angle of rotation，draw four new
images，continuing the rotation around $P$ ．Label each new image．

$\qquad$


Problem c．

$\qquad$
$\qquad$
5. What do you observe about the angles between lines $m$ and $n$ and the angle of rotation?
6. In an object reflected twice, where is the center of rotation? If
necessary, extend lines in the figures to see the relationship.
7. Do these results about the lines of reflection work with the objects and images in Worksheet 89-2 and 89-3? Explain.
工_ـ_



You may find it helpful to label the points on your figure.

## More Double Reflections

GOALS 1. To explore double reflections with different angles
2. To discover the location of the center of rotation

MATERIALS Worksheets 92-1, 92-2
Drawing board, T-square, 45 triangle, 30-60 triangle
Ruler
A set of tangrams (optional)
Goniometer
ACTIVITIES Other double reflections. In Lesson 89, you discovered what happens when an object is reflected vertically and horizontally. The results give the same image as if the object had been rotated 180. This lesson explores reflecting through other angles.
Problem 1. On the worksheet, the first problem shows a square formed with tangram pieces. First reflect it around line $m$. Before you can use your drawing tools, you will need to measure to find a starting point. Repeat the reflection over line $n$.
Do this before reading further.
Analysis. Compare your second image with the original object. Is it a reflection or a rotation? The answer is at the bottom of the page.
Measure the angle between the two lines of reflection, $m$ and $n$. Consider line $m$ rotating into line $n$ to find whether the angle is positive or negative. Remember that if the rotation is counterclockwise, the rotation is positive. Write the angle in the table on the second worksheet.

Next find the center of rotation. Then find the angle of rotation from the original to the second image and record it in the table.
Worksheet. Complete the worksheets. (Answer: rotation)


A building in Virginia Beach, Virginia.

## Lines of Symmetry

GOALS 1. To review line of symmetry
2. To compare a line of symmetry to a line of reflection
3. To find the lines of symmetry in a figure
4. To learn the terms maximum and minimum
5. To learn about the infinity symbol, " $\infty$ "
mATERIALS Worksheets 94-1, 94-2
Drawing board, T-square, 30-60 triangle, 45 triangle
A set of tangrams
ACTIVITIES Line of symmetry. You probably know what a line of symmetry is.


From the garden of the Castle of Angers in Angers, France.


La Fleur de Lise, important symbol in France, in St. Malo, France.


Design in the floor at the Minneapolis-St. Paul Airport.

It divides the figure into two parts with one part being the reflection, or mirror image, of the other part. A good way to check the line of symmetry is to fold the figure in half. If the two halves match, the fold line is the line of symmetry.
Line symmetry is very common in nature, in art, and in logos, which are symbols identifying a business or institution. See examples in the figures below. Note that the line of symmetry may be at any angle. Also a figure can have more than one line of symmetry.


Comparing to line of reflection. Reflections are a transformation, which means an object is transformed, or changed, into something else. Mathematicians use the word map to describe this transforming of an object according to some rule.

To think how a line of reflection is related to a line of symmetry, take a figure with a line of symmetry and remove half along a line of symmetry. Then reflect the remaining part about the line of symmetry, which becomes the line of reflection. Now you're back to the original object.
So, a line of symmetry is a line within a figure. A line of reflection is usually outside the figure and is the line the figure is flipped over.
Some face fun. Faces are almost symmetric. It is fun to make a picture of someone completely symmetric. To do this, first put the picture into computer software, either Adobe Photoshop or similar software. Then remove half. Copy the remaining half and reflect it horizontally. Lastly place the reflected along side the original half. See an example on the next page.


From the botanical garden in Big Island, Hawaii


Building in Maryland.


A tulip from Monet's Gardens in Giverny, France.

> To write the infinity symbol, " $\infty$," start in the middle, draw a loop on one side and then the other side.


Original photo.


Photo with left side reflected.


Photo with right side reflected.

Worksheet. You can do the first 19 questions on the worksheet now or finish reading below before doing the worksheet.

Maximum and minimum. You may have seen the terms maximum and minimum on highway signs. The maximum speed is the highest speed a person may legally drive. The minimum speed is the lowest speed allowed on the road.
As an example for maximum and minimum number of lines of symmetry, let's take an isosceles triangle. See the left figure below, which shows the usual isosceles triangle. Clearly, it has one line of symmetry.


But the definition of a isosceles triangle says it has two equal sides. How about an equilateral triangle, shown above on the right with its three lines of symmetry. Doesn't it also have two equal sides? So, an equilateral triangle is a special case of an isosceles triangle. Therefore, the minimum number of lines of symmetry in an isosceles triangle is 1 and the maximum number is 3 .

Infinity symbol. Infinity is not a number, but a concept. For example, how many numbers are there. We say the answer is infinite because whatever number you say, I can say one higher. There is no limit.

You will need the symbol for infinity, " $\infty$ " for the table in problem 21. Sometimes the symbol is called "lazy eight."

Problem 20. You may need to look up some of the definitions. Remember the difference between a polygon and a regular polygon.

If you have a partner, compare your answers in the table before looking at the solution key. Discuss any answers where you disagree.
$\qquad$
2. If a figure has only even points, is it always an Euler path? $\qquad$
3. If a figure has four odd points, is it an Euler path? $\qquad$
4. If a figure has two odd points, is it an Euler path? $\qquad$
5. Where do you start when tracing an Euler path that has only even points?
$\qquad$
6. Where do you end when tracing an Euler path that has only even points?
7. Where do you start when tracing an Euler path that has only two odd points?
8. Where do you end after tracing an Euler path that has only two odd points?
9. Decide whether or not each figure has an Euler path. Indicate by writing yes or no below it. If it has different starting and ending points, show them with dots.

10. A family has a window they want to decorate with lights. Can they do it with one continuous string without doubling on any side? Explain.

11. Two children are delivering papers. Can they do the following route without retracing their steps? Explain.


## Basic Trigonometry

GOALS 1. To learn the meaning of trigonometry
2. To learn the terms opposite, adjacent, sine, cosine, and tangent
3. To construct trigonometry tables

MATERIALS Worksheets 131-1, 131-2
Calculator
Goniometer, ruler
ACTIVITIES A trigonometry problem. In the problem below, you are given $c$ and $\angle A$ and asked to find $a$. If you knew the ratio, $\frac{a}{c}$, you could solve the problem as you did on the previous worksheet.


Solving problems using triangle ratios involves a branch of mathematics called trigonometry (TRIG-ah-NOM-ah-tree). The name is often shortened to trig. The derivation of the word is quite simple. Tri means three, gon means angle, and metry means measure.
Trig terms. Basic trig uses six terms, one you already know, two are simple, and three will be new. Look at the triangle below; the hypotenuse is opposite the right angle. $\overline{B C}$ is the leg opposite $\angle A$ and $\overline{A C}$ is the leg adjacent to $\angle A$.

```
You might like this for
remembering the trig
ratios: soh-cah-toa.
(For example, "soh" is
sin, opp, hyp.)
```



$$
\begin{aligned}
& \text { sine }(A)=\frac{\text { opposite }}{\text { hypotenuse }} \\
& \operatorname{cosine}(A)=\frac{\text { adjacent }}{\text { hypotenuse }} \\
& \text { tangent }(A)=\frac{\text { opposite }}{\text { adjacent }}
\end{aligned}
$$

$$
\begin{aligned}
& \sin (A)=\frac{\text { opp }}{\text { hyp }} \\
& \cos (A)=\frac{\text { adj }}{\text { hyp }} \\
& \tan (A)=\frac{\text { opp }}{\text { adj }}
\end{aligned}
$$

The trig names for the ratios are given in the first rectangle above. They are usually abbreviated as given in the second rectangle. However, the abbreviations are pronounced like the original words, $\sin$ (SIGN), cos (KOH-sign), and tan (TAN-jent).
Trigonometry history. The Babylonians, Greeks, Egyptians, Indians, Chinese, and Arabs all contributed to the field of trigonometry. Surveyors and astronomers use it extensively for finding distances.
Worksheets. On the worksheets, you will be making a trig table for angles between $5^{\circ}$ and $85^{\circ}$. On Worksheet 1 use the Pythagorean theorem and calculate the ratios to 3 decimal places. On Worksheet 2 measure the sides with a ruler and the angles with a goniometer. Then calculate the ratios and enter them in the table on Worksheet 1.
$\qquad$
$\qquad$
1．Use the Pythagorean theorem and the 45 triangle on the right to find the $\sin (45), \cos (45)$ ，and $\tan (45)$ ．

2．Use the Pythagorean theorem and the 30－60 triangle to find the $\sin (60), \cos (60), \tan (60), \sin (30), \cos$ （30），and $\tan (30)$ ．

3．Measure the angles on the next worksheet to calculate the other values．

Write the values in the table to 3 decimal places．


| Angle | $\sin =\frac{\text { opp }}{\text { hyp }}$ | $\cos =\frac{\mathrm{adj}}{\mathrm{hyp}}$ | $\tan =\frac{\mathrm{opp}}{\mathrm{adj}}$ |
| :---: | :---: | :---: | :---: |
| 5 |  |  |  |
| 10 |  |  |  |
| 15 |  |  |  |
| 20 |  |  |  |
| 25 |  |  |  |
| 30 |  |  |  |
| 35 |  |  |  |
| 40 |  |  |  |
| 45 |  |  |  |
| 50 |  |  |  |
| 55 |  |  |  |
| 60 |  |  |  |
| 65 |  |  |  |
| 70 |  |  |  |
| 75 |  |  |  |
| 80 |  |  |  |
| 85 |  |  |  |

$\qquad$
$\qquad$


Use these triangles below to find the sin, cos, and tan of the angles in the
table on Worksheet 1 . Measure the angles with the goniometer.




Write the values in the table to 3 decimal places．

 $\sin (30), \cos (30)$ ，and $\tan (30)$ ．
 triangle to find the $\sin (45), \cos (45)$ ，and $\tan (45)$
2．Use the Pythagorean theorem and the $30-60$


$$
\begin{aligned}
& \text { (•KY甘 TITM SGDETA }
\end{aligned}
$$



Use these triangles below to find the sin，cos，and tan of the angles in the table on Worksheet 1 ．Measure with a ruler and goniometer to two decimal places．Give your answers to three decimal places．

（THE TENTHS PLACES WILL VARY．）


## Comparing Calculators

GOALS 1. To learn some differences between a basic and scientific calculator 2. To learn some features of the scientific calculator

MATERIALS Basic calculator (Casio SL-450S)
Scientific calculator (Casio fx-300MS)
Worksheet 133
ACTIVITIES Calculators. There are dozens of different types of calculators. A scientific calculator does the basic operations that a simple calculator does, but in a slightly different way. In addition to basic arithmetic, a scientific calculator does trigonometric functions, exponents, means (averages), has () and pi, and many more advanced topics.

```
The Casio fx-300MS
may be used on the SAT
and PSAT/NMSQ tests.
```


## If your calculator was used for advanced work, press MODE (next to $O N$ ) and 1 and $=$.



Basic calculator


Scientific calculator

The best way to learn to use it a new calculator is to experiment with it. Glance at the instructions when you need help with more advanced features.

Order of operations. Turn on the scientific calculator by pressing the ON button in the upper right corner. Perform the following operations in the order it is written on both calculators

$$
2 \oplus 3 \triangle 4 \Theta
$$

One calculator says 14 and other says 20 . The correct answer is 14 , according to the order of operations, which says multiply and divide before adding. Scientific calculators and spreadsheets follow the standard order of operations.
How did you like seeing all the numbers you entered?
Rounding. Enter $2 \div 3$ on both calculators. Both calculators have a a string of 6 s . Now multiply that by 3 . Answer should be 2 , right? The scientific calculator says 2; however, the basic calculator gives the answer as 1.9999998 . The scientific calculator used the actual answer for $2 \div 3$ whereas the basic calculator multiplied 0.6666666 by 3 .
As another example, try

$$
1 \div 3+2 \div 3 \Xi_{-}
$$

What should the answer be? The scientific calculator sees it as two fractions, or division, added together, $1 / 3+2 / 3$. Try it. On the other hand, the basic calculator does $1 \div 3$, adds 2 , and then divides all of that by 3 . To get the correct answer on the basic calculator, you must
use memory. First do $1 \div 3$ and add to memory. Next do $2 \div 3$ and add to memory. Try it and you'll find the answer still isn't exactly right; it says .99999999 , instead of 1.
Square root. On the basic calculator you found a square root by entering the number and $\downarrow$. On this scientific calculator, you first press $\sqrt{ }$, then enter the number, and press $\Xi$.
Enter 13 and find its square root on both calculators.

> Basic: $13 \triangle \sqrt{ }=3.6055512$
> Scientific: $\sqrt{ } 13=3.605551275$

Note that the basic calculator just quits the answer. If it had rounded the answer, the last digit would have been 3, not 2 .
Now square your results. On the basic calculator, press $x$ and $\Theta$. On the scientific calculator, press $x^{2}$ and $\Theta$. Do they both say 13 ?
Pi. On your basic calculator, you had to enter several digits to find pi. On a scientific calculator, $\pi$ is built in to 10 places.
You will find the $\pi$ symbol in the middle of the bottom row, above the EXP key. To use any function printed in gold letters, first press the SHIFT key in the upper left hand corner. Therefore, to access pi, press SHIFT and $\pi$; press $\equiv$ to see the actual value.
Memory. To place the value on the display into Memory, press SHIFT STO M+. To multiply by the value in Memory, press the number and then $x R C L=($. The values in Memory remain even after the power is off.
Worksheet. The 10 problems are actually geometry review problems. Do them on your scientific calculator without writing anything down. Be sure to estimate the answer and check that it makes sense. Solve them now. When you have finished or need help, continue reading below.
Problem 1-2. These are plain problems involving $\pi$. Don't confuse diameter and radius.

Problem 3. There are two ways to solve this problem. Try both.
Problem 4. Here is where you'll appreciate the scientific calculator.
Problem 5. You really don't need the calculator for this one.
Problem 6. This would be a tough problem without a calculator. After finding the ratio, store it in Memory. Then enter each side of the polygon and press $x$ RCL $M+\equiv$.
Problem 7. Remember something special about isosceles triangles.
Problem 8. The easy way to do this problem is to recognize the relationship between the two squares. Mentally fold a corner of the larger square unto the smaller square.
Problem 9. Using parentheses makes this problem easier.
Problem 10. This is a two-step problem, not hard if you remember what you're doing.
in an octahedron.

2. Guess how many cubic decimeters would fit into your truncated octahedron made with panels.
3. Answer the following questions about the volume of the
polyhedrons in the figure above. (An edge is $a$.) 3a. The volume of the octahedron is ___ times
the volume of the large pyramid.
3b. For the large pyramids, $a=$ ___
3c. For the small pyramids, $a=$.
3d. The volume of the truncated octahedron is
equal to the volume of the octahedron -___
times the volume of the small pyramids. 3a. The volume of the octahedron is ___ times
the volume of the large pyramid.
3b. For the large pyramids, $a=$.__
3c. For the small pyramids, $a=$
3d. The volume of the truncated octahedron is
equal to the volume of the octahedron - __
times the volume of the small pyramids. 3a. The volume of the octahedron is ___ times
the volume of the large pyramid.
3b. For the large pyramids, $a=$.__
3c. For the small pyramids, $a=$
3d. The volume of the truncated octahedron is
equal to the volume of the octahedron - __
times the volume of the small pyramids. 3a. The volume of the octahedron is ___ times
the volume of the large pyramid.
3b. For the large pyramids, $a=$.__
3c. For the small pyramids, $a=$
3d. The volume of the truncated octahedron is
equal to the volume of the octahedron - __
times the volume of the small pyramids.
times the volume of the small pyramids.
4. Calculate the volume of your truncated octahedron using decimeters. Give your answer to two decimal places.
$\qquad$

1. Circumscribe the truncated octahedron in an octahedron.

2. Guess how many cubic decimeters would fit into your truncated octahedron made with panels. $\qquad$
3. Answer the following questions about the volume of the polyhedrons in the figure above. (An edge is $a$.)

3a. The volume of the octahedron is $\mathbf{2}$ times the volume of the large pyramid.
3 b . For the large pyramids, $a=3$.
3c. For the small pyramids, $a=1$.
3d. The volume of the truncated octahedron is
equal to the volume of the octahedron - $\mathbf{6}$
times the volume of the small pyramids.
4. Calculate the volume of your truncated octahedron using decimeters. Give your answer to two decimal places.

Date $\qquad$
$V(\operatorname{LgPy} r)=\frac{1}{3} B H$
$B=a^{2}=3^{2}=9$
$H=\frac{1}{2} \sqrt{2} \times a$
$H=\frac{1}{2} \sqrt{2} \times 3=2.12$
$V($ LgPyr $)=\frac{1}{3} \times 9 \times 2.12$
$V(L g P y r)=6.36$
$V($ Oct $)=2 \times 6.36=12.73$
Small pyr:
$B=a^{2}=1^{2}=1$
$H=\frac{1}{2} \sqrt{2} \times a=\frac{1}{2} \sqrt{2}=.71$
$V($ SmPyr $)=\frac{1}{3} \times 1 \times .71=.24$
$V($ TrOc $)=V($ Oct $)-6 \times V(S m P y r)$
$V($ TrOc $)=12.73-6 \times .24$
$V($ TrOc $)=11.31 \mathrm{dm}^{3}$

## Worksheet 160, Truncated Icosahedron

1. Truncate each vertex of the icosahedron below by connecting the $\times$ 's around the vertex. Color or shade the pentagons. Then cut out your new figure.
2. What is the figure? truncated icosahedron (soccer ball)
3. Each pentagon is surrounded by 5 hexagons.
4. Each hexagon is surrounded by 3 hexagons
and 3 pentagons


Name $\qquad$
Date $\qquad$

In the icosahedron:
5. Since a triangle has $\qquad$ 3 vertices, on an icosahedron the number of vertices in all 20 triangles is 60
6. Since a vertex on an icosahedron is composed of 5 triangles, it has 12 vertices. ( $60 \div 5$ )
7. Since each triangle has 3 sides, the total number of edges for the 20 triangles is 60 .
8. Since you put $\quad \mathbf{2}$ sides of a triangle together with a
rubber band, the icosahedron has 30 edges. $(60 \div 2)$
In the truncated icosahedron:
9. Since an icosahedron has 20 faces and you added 12 more faces, the truncated icosahedron has 32
faces.
10. Since an icosahedron has 30 edges and you added

5 more for each pentagon, the truncated icosahedron
has 90 edges. $(30+5 \times 12)$
11. Since an icosahedron has 12 vertices and for each pentagon you added $\quad 5$ vertices but removed 1 , the truncated icosahedron has $\frac{60}{12}$ vertices.
$(12+4 \times 12)$

Circle the letter in front of all correct answers. Some questions may have more than one answer.

1. What is the longest line in a circle?
a. diameter
b. diagonal
c. radius
d. circumference
2. The length of a side of a square is $w$.
a. The area of the square is $4 w$.
b. The volume of the square is $3 w$.
c. The perimeter of the square is $4 w$.
d. The diagonal of the square $>w$.
3. Which regular figures will tessellate?
a. octagon
b. hexagon
c. cube
d. equilateral triangle
e. pentagon

4. How many diagonals are in the heptagon?
a. 14
b. 26
c. 28
d. none

5. What is the perimeter of $D E F G$ ?
a. 2.7 cm
b. 67 mm
c. $3.35 \mathrm{~cm}^{2}$
d. 6.7 cm

6. Angle $X W Y$ is $40^{\circ}$. What is angle $W Y Z$ ?
a. $70^{\circ}$
b. $40^{\circ}$
c. $110^{\circ}$
d. $120^{\circ}$

7. $P Q R S$ is a rhombus. Line $Q S$ is $\frac{2}{3}$. Angle $R$ is $60^{\circ}$. What is the perimeter of the rhombus?
a. 2
b. $\frac{8}{3}$
c. $2 \frac{2}{3}$
d. $\frac{10}{3}$

8. What is the area of triangle $R S T$ ?
a. $R T \times S O$
b. $R T+T S+S R$
c. $\frac{S T \times R N}{2}$
d. $\frac{1}{2}(R S \times M T)$

No drawing tools or calculator for this page.
Date


Lines $a$ and $b$ are parallel. Angle 1 is $139.1^{\circ}$.
9. What is $\mathrm{m} \angle 2$ ? $\qquad$
10. What is $\mathrm{m} \angle 3$ ? $\qquad$
11. What is $\mathrm{m} \angle 4$ ? $\qquad$
12. What is $\mathrm{m} \angle 5$ ? $\qquad$

13-15. Triangle $A B C$ is similar to triangle $X Y Z$. What is the perimeter of each triangle? What is the ratio of their perimeters?


16-19. What are the coordinates of cities $C$ and $D$ ? A car is driven from $C$ to $D$. How far is the drive? How much farther if you drive through city $E$ on your way from $C$ to $D$ ?

20. Morgan has some blocks that are cubes with a side measuring 2 cm . How many blocks will fit into a box that measures 6 cm by 10 cm by 4 cm ?


21-22. Reflect the design horizontally about the vertical line. Then reflect the original and your reflection vertically about the horizontal line. Do not do any measuring.

23. Find the area of $\triangle T U V$. Measure


24-25. A wading pool is circular with a square deck around it. The designer wants the deck to be twice the area of the pool. Draw the deck around the pool (the square around the circle).







[^0]:    Name
    Date __________________

[^1]:    Name
    Date __________________

