Most recent update: December 6, 2019

# RightStart™ Mathematics

# Corrections and Updates for Geometric Approach Lessons, Worksheets, and Solutions

LESSON	I/WORKSHEET	CHANGE DATE	CORRECTION OR UPDATE
Lesson 23		10/05/2010	In chart toward bottom of page the last fraction should be 1/3.
Lesson 27	Worksheet 27-2	01/01/2012	The second sentence should say: Also find the area for the remaining rectangles
Lesson 30		06/01/2011	Answers: 1 digit: formula 1, 8 strokes; formula 2, 9 strokes; formula 3, 6 strokes.
Lesson 38	Solutions for Worksheet 38	12/25/2010	Problem 3: A clearer answer is: First flip 1 trapezoid vertically and put it next to other trapezoid. The 2 trapezoid make a parallelogram. The width of the parallelogram is $w1 + w2$ . The area is $(w1 + w2)h$ . So divide by 2 for the area of one trapezoid.
Lesson 45		12/06/2016	Second paragraph under Solving Angle Problems, about 3/4 of the way down the page, should read: To solve the problem, keep in mind the relationships you know about angles. Since you're missing ∠BAC and you know the measure of all the angles in a triangle is 180°, you can find the angle measurement. The answer is at the bottom of the page.
Lesson 48		03/01/2012	The equations in the middle of the page should be: $a < b + c$ , $b < a + c$ , and $c < b + a$ .
Lesson 55		11/21/2010	Convex and Concave: The first word in the second sentence should be <i>convex</i> , not concave.
Lesson 57	Solutions for Worksheet 57	01/01/2012	Problem 2A: The answer should say 32 = 9cm <sup>2</sup> .
Lesson 60	Solutions for Worksheet 60-1	12/17/2015	Problem 1: Second line of the calculations should read $c = 149$ cm.
Lesson 60	Solutions for Worksheet 60-2	02/07/2010	Problem 6: Second line of the equation should read $c^2 = a^2 + b^2$ = $20^2 + 25^2 = 1025$ , not 1050.

Lesson 62		10/05/2010	The picture is covering of the text. Should this:  I a point is considere you can perform. Planes. Sphere is point. '(plane is a circular sculpture in Detroit, MI
Lesson 63	Worksheet 63-1 and Solutions for Worksheet 63-1	03/26/2012	See attached PDF.
Lesson 64		06/01/2012	Second last sentence should say: Use your goniometer and straightedge for polygons with 5 and 7 sides.
Lesson 66		03/01/2012	Goal 3: To estimate the ratio of C to D (circumference to diameter) in a circle by using the perimeter of inscribed and circumscribed polygons.
Lesson 69		10/05/2010	Step 1 (at the bottom of page): The lines being drawn should be 1 cm. Start tick marks that are 1 cm away from lines, not .5 cm. The drawing at the bottom of the page should also be labled 1 cm and 1 cm.  Step 3: Should say "Look for a radius of 10 cm" instead of 5 cm.
Lesson 73	Worksheet 73 and Solution for Worksheet 73	10/05/2010	Midpoints were marked wrong. See attached PDF.
Lesson 74		10/05/2010	The picture is covering of the text. Should this:  Works! versions has strai  Works! second 1  Works! second 1  Works! second 1  Works! second 1  Warning left. It is line segi There are construct minimiz.
Lesson 76	Solutions for Worksheet 76	03/08/2017	The solution for Problem 4 should read as follows: Dist. = $1/4$ C = $1/4$ x $12,720,000$ $\pi$ = $9,985,200$ m = $9985$ km
Lesson 77	Worksheet 77-2 and Solutions for Worksheet 77-2	03/26/2012	See attached PDF.

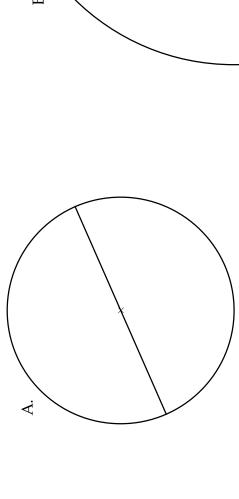
Lesson 79		10/05/2010	The graphic in the of the page wrong should $A = 30 = \pi r^2$ To find $r^2$ , divide by $\pi$ . $r^2 = \frac{30}{\pi} \approx 9.55$ To find $r$ , take the square root. $r \approx 3.09$
Lesson 79	Worksheet 79-2	09/01/2012	Problem 7: Find the radius (not diameter) of a circle that has twice the circumference of the circle below.
Lesson 80	Solutions for Worksheet 80	03/14/2017	Area for the 14" pizzas should be 153.9 in <sup>2</sup> , not 153.8 in <sup>2</sup> . Problem 5: answer should be \$25.99, not \$23.99.
Lesson 81	Worksheet 81-1 and Solutions for Worksheet 81-1	03/14/2017	Problem 1: Second sentence should read "The line below is the bottom of the tangram", not top of the tangram. Problem 13: Second sentence should read "Calculate the length of the other sides of the IgT." See attached PDF.
Lesson 82	Worksheet 82-1 and Solutions for Worksheet 82-1	03/21/2012	See attached PDF.
Lesson 82	Solutions for Worksheet 82-3	10/05/2010	See attached PDF.
Lesson 86		10/05/2010	The picture is covering of the text. Should this:  **Rug design in hotel.**  **The picture is covering of the text. Should this:  **Posture 3
Lesson 87	Solutions for Worksheet 87	01/01/2012	Answer 2: A' coordinate should be: A' (7,11)
Lesson 88	Worksheet 88	04/11/2017	The worksheet instructions have been significantly changed. See pdf. Solutions remain the same.
Lesson 91	Worksheet 91	04/20/2017	The worksheet instructions have been changed slightly to increase understanding. See pdf. Solutions remain the same.
Lesson 92		04/21/2017	Under the heading Analysis, the paragraph should start with "Compare your second image with the original object", not last image. Also, at the end of the that section, it should read "Then find the angle of rotation from the original to the second image and record it in the table."
Lesson 92	Worksheet 92-1 and 92-2	04/21/2017	Worksheets: Instructions have been been changed slightly. See pdf. Solutions remain the same.
Lesson 93		06/01/2012	The website address at the bottom of the page should be RightStartMath.com/geometry

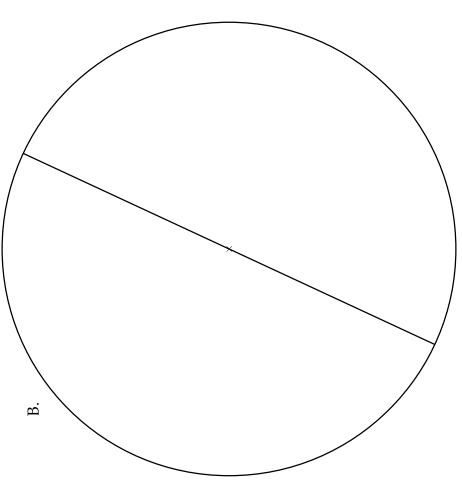
Lesson 94		06/01/2012	The top of the page should say to do problems 1-19 and then at the bottom of the page it should say problem 20 instead of problem 21. See attached PDF.
Lesson 95		05/02/2017	At the end of the second paragraph, add: The number of times it can be rotates is the <i>order of rotational symmetry</i> .  And at the end of the thir paragraph, add: The order of rotational symmetry is five.
Lesson 102	Solutions for Worksheet 102-1	06/01/2012	Problem 4: The code should read 3.3.3.4.4/3.4.6.4. (There was an extra 4 in the second half, 3.4.6.4.4)
Lesson 103		10/02/2017	Answer at the bottom of the page should say second.
Lesson 109	Worksheet 109-1 and Solutions for Worksheet 109-1	10/14/2010	The list of words on the lower right should be >1, 3, 3, 8, not 1, >1, 3, 8.  Solutions on the right side: "Number of different polygons with <6 sides" should be 3, not 1.
Lesson 109	Solutions for Worksheet 109-2	03/25/2011	Problem 2: Should be c, not a, b, c.
Lesson 116		10/05/2010	Graphic that show cross multiplying has the wrong symbols. It should look like this: $(3\times2) \qquad (5\times2)$ $6 \qquad 10$ $(3\times2) \qquad (5\times2)$ $6 \qquad 10$ $(3\times2) \qquad (5\times2)$ $6 \qquad 10$ $10 \qquad 18$ $3\times2 \times 5\times6 = 5\times2 \times 3\times6$
Lesson 118		09/10/2010	At the bottom of the page, the infinity sign should be $\Phi$ , the phi symbol.
Lesson 121		12/14/2017	Problem 2: The second and third paragraphs should read as follows. In the third column, you are comparing how much the ratios differ from the golden ratio. A quick way to find these differences is to use memory on your calculator. First put 1.61803 into memory using the M+ key. Then to subtract 1 from 1.61803, do the following: Press 1 then — MR = . The answer –0.61803 appears. (Ignore the negative sign when recording the answer.)  For the next answer: Press 2 then — MR = . The answer 0.38197 appears.  Note that some of the newer calculators have replaced the MR button with the MRC button. It will still work the same.
Lesson 122	Solutions for Worksheet 122	02/07/2010	Problem 1, Legal size paper: Should be 14/8.5, not 14/11, ratio is 1.65, and golden ratio is <b>yes</b> .
Lesson 125	Solutions for Worksheet 125-2	01/13/2014	Problem 20 has two more solutions: BDGA and BCDG.

Lesson 125	Solutions for Worksheet 125-2	12/06/2019	Problem 11: On the left side, the third one down, <i>ABG</i> , should be <i>POG</i> . On the right side, the fifth one down, <i>IPH</i> , should be <i>BMG</i> . Problem 18. Additional answers are <i>BCG</i> and <i>BMG</i> .
Lesson 127		12/29/2017	In the middle of the page below the circle, the calculation for the area of the circle, second line, is missing the pi symbol. It should read: $A = \pi(2)^2$ .
Lesson 128		10/05/2010	The figure in the center of the page is using the wrong symbol in the last line, it should look like this: $= \sqrt{10} \approx 3.162$
	Solutions for Worksheet 128	12/06/2019	The answer for Problem 2, the square root of 26, should be 5.099, not 5.100.
Lesson 129	Worksheet 129-2 and Solutions for Worksheet 129-2	01/08/2018	Question 7 should read: Where do you start when tracing an Euler path that has only two odd points?  Question 8 should read: Where do you end when tracing an Euler path that has only two odd points? See attached PDF.
			The three graphics shown had the wrong symbols.
Lesson 130		10/05/2010	$a = 10 \times .60$ $\frac{2}{4} = \frac{3}{6} \rightarrow \frac{2}{3} = \frac{4}{6}$ $12 = .75 \times b$
Lesson 131		09/01/2012	See attached PDF.
Lesson 131	Worksheet 131-1 and Solutions for Worksheet 131-1	09/01/2012	See attached PDF.
Lesson 131	Worksheet 131-2 and Solutions for Worksheet 131-2	09/01/2012	See attached PDF.
Lesson 133		01/23/2018	Replace the lesson with the attached PDF.
Lesson 133	Worksheet 133	01/23/2018	Problem 4: Change the question to read "What is the hypotenuse of a right triangle when the legs"
Lesson 135		10/05/2010	Graphic in the middle of the page is using the wrong symbol. This is what it should look like: $n = 154 \times \tan 14.5$
Lesson 145		04/26/2017	Under Problem 6 heading, second paragraph should read, "You need to use trigonometry to fin the pile radius", not pile height.
Lesson 145	Solutions for Worksheet 145-2	04/01/2013	Problem 6: The 4816 should be 4796 in both places.
Lesson 146	Solutions for Worksheet 146-1	10/25/2010	Problem 1: $H$ should be .707, not .5 and $V$ should be, .24 dm <sup>3</sup> and 236 cm <sup>3</sup> , not .17 dm <sup>3</sup> and 167 cm <sup>3</sup> .
Lesson 147	Worksheet 147-4	01/13/2014	Problem 13:: Second sentence should be Cut it out and fit it into your tetrahedron.

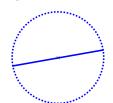
Lesson 152		10/05/2010	The symbols are $V(\text{cylinder}) = \pi r^2 h$ And $h = 2r$ So $V(\text{cylinder}) = \pi r^2 2r = 2\pi r^3$ $V(\text{sphere}) = \frac{2}{3}V(\text{cylinder}) = \frac{2}{3} \times 2\pi r^3 = \frac{4}{3}\pi r^3$
Lesson 154		03/25/2011	Above the second figure, it should say "Lastly, there are six axes." rather than "Lastly, there are three axes".
Lesson 159		10/05/2010	The figure at the bottom of the should look like this: $H = \frac{1}{2} \sqrt{2} \times a$
Lesson 159	Worksheet 159 and Solutions for Worksheet 159	01/13/2014	See attached PDF.
Lesson 162	Worksheet 162-2	01/13/2014	Problem 21: Formula should be $V(\text{rh}) = \frac{1}{3} (12 + 10 \sqrt{2}) a^3$
Lesson 162	Solutions for Worksheet 162-2	01/13/2014	Problem 21: Answer should be 8.7 dm <sup>3</sup> .
Lesson 163	Solutions for Worksheet 163-2	04/01/2013	Calculation for A(dod) should be 12 x 5 x A(triangle ABO) not ABC
Lesson 165		10/05/2010	Angles in a polyhedron: In the third sentence, the degree symbol was wrong or missing. It should read: Each triangle has 180° so the suface angles total 180 x 4 = 720°.
	Final test	09/01/2012	Heading for Questions 9-12: Angle 1 is 139.1, not 139.9.
	Final test Worksheets & Solutions	04/15/2016	Heading for Questions 13-15 is missing. Should read: Triangle ABC is similar to triangle XYZ. What is the perimeter of each triangle? What is the ratio of their perimeters? PDF is attached.

2. Measure the diameters (D) and circumferences (C) for both circles. You choose the units for measuring. Record your answers in the table below and on the graph on the next worksheet. Also measure the diameters and circumferences for five other circles. Date\_ 1. How many times do you think the diameter will fit around the circle?



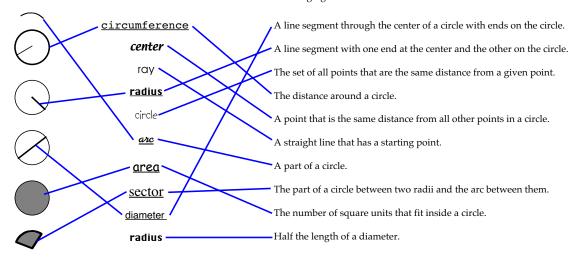


$\frac{C}{D}$					
$\overline{\mathcal{C}}$					
$\overline{D}$					
<u>Object</u>	A	В			



- 2. How many points is it possible to draw? \_\_infinite
- 3. What figure do you get when you connect the points? \_\_\_\_circle
- 4. What is the length of the longest line segment you can draw inside your figure? 4 cm

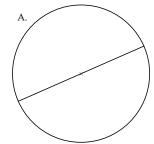
Read Lesson 43. Draw lines from the underlined words to the matching figure. Then draw lines from each word to its definition.



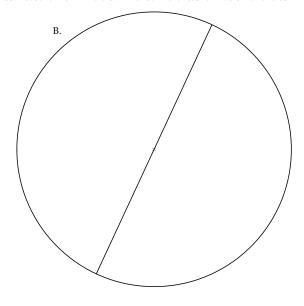
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Name \_\_\_\_\_\_
Date

- 1. How many times do you think the diameter will fit around the circle?
- 2. Measure the diameters (*D*) and circumferences (*C*) for both circles. You choose the units for measuring. Record your answers in the table below and on the graph on the next worksheet. Also measure the diameters and circumferences for five other circles.



Object	<u>D</u>	<u>C</u>	<u>C</u> D
A	6 cm*	18.8 cm*	3.15
В	12 cm*	37.7 cm*	3.14



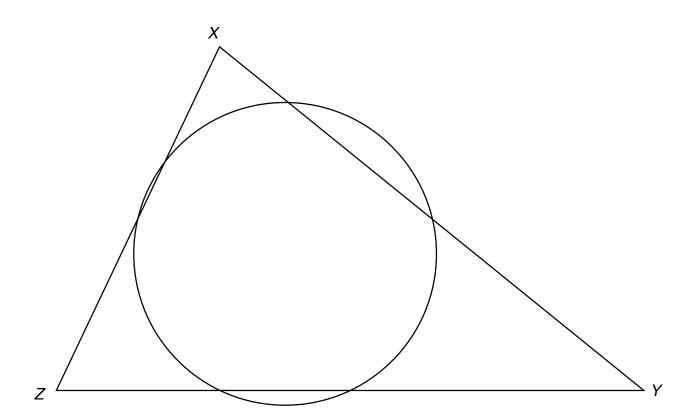
Worksheet 63-1, Ratio of Circumference to Diamet

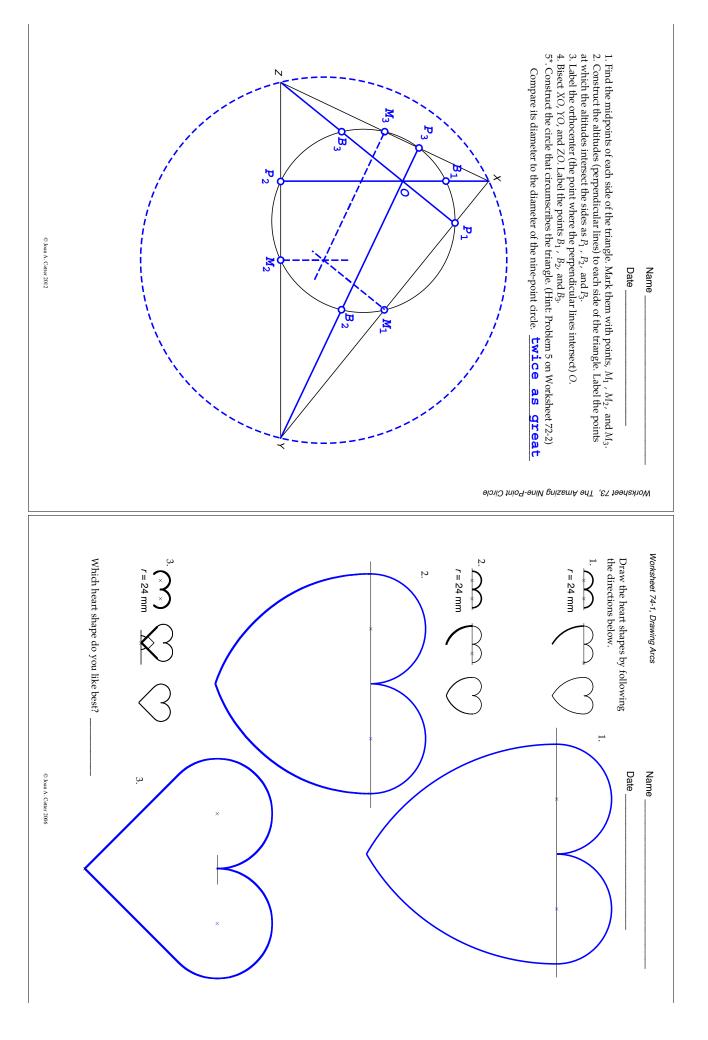
Worksheet 62, Circle Basics

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Date			

- 1. Find the midpoints of each side of the triangle. Mark them with points,  $M_1$ ,  $M_2$ , and  $M_3$ .
- 2. Construct the altitudes (perpendicular lines) to each side of the triangle. Label the points at which the altitudes intersect the sides as  $P_1$ ,  $P_2$ , and  $P_3$ .
- 3. Label the orthocenter (the point where the perpendicular lines intersect) *O*.
- 4. Bisect XO, YO, and ZO. Label the points  $B_1$ ,  $B_2$ , and  $B_3$ .
- 5\*. Construct the circle that circumscribes the triangle. (Hint: Problem 5 on Worksheet 72-2)

Compare its diameter to the diameter of the nine-point circle.

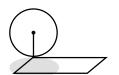


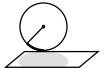


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7. Use your drawing tools to divide the circle below into 12 equal sectors. Cut out the circle.

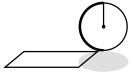
8. On the back side of the circle, draw a radius. (It needn't be exact.) Roll the circle along the top edge of the parallelogram below as shown. The radius of the circle is r, what is the width of the parallelogram. \_\_\_\_\_

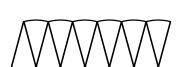








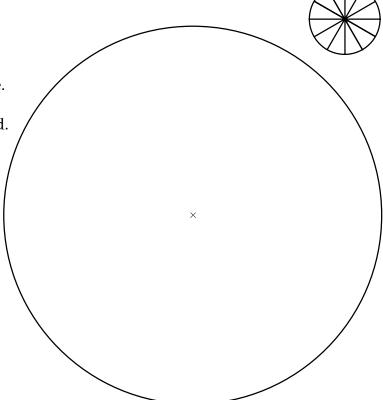


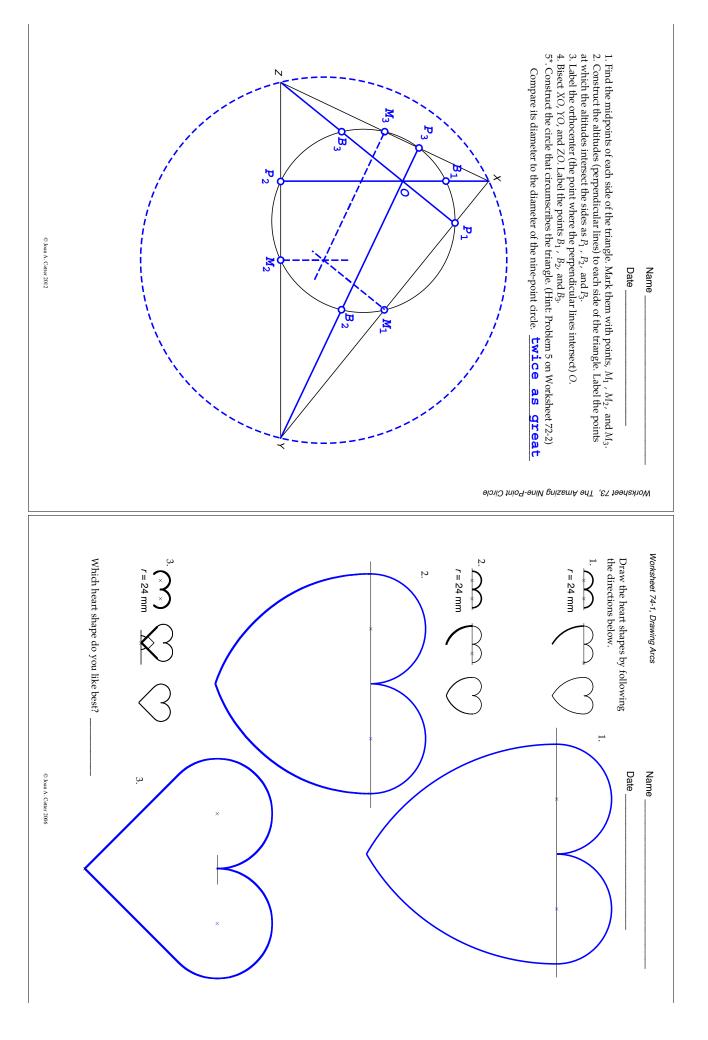


9. Cut apart the 12 sectors of your circle. Arrange them on the parallelogram above as shown. Glue or tape, if desired.

Imagine dividing the circle into a million parts, instead of 12, and arranging the sectors the same way. Then the sides of the parallelogram would look straight.

10. The height of the parallelogram is *r*. Using Question 8, what is the area of the parallelogram? \_\_\_\_\_\_
What is the area of a circle? \_\_\_\_\_



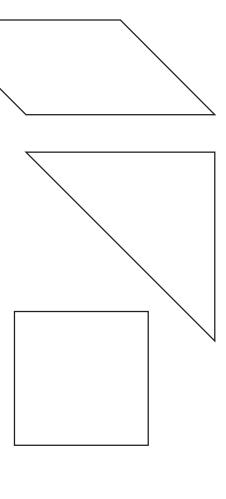


Use your drawing tools to draw all lines.

1. Draw a tangram as shown. The line below is the *bottom* of the tangram. Then either use a tangram set or cut out your tangram.



one line.
in half with or
half
e below
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Note: lgT is the large triangle, mdT is the medium-size triangle, and smT is the small triangle.

- 7. How does the area of each figure above compare to the area of the smT?
- 8. What is the ratio of the mdT to the lgT?
- 9. What is the ratio of the lgT to the smT? \_\_\_\_\_
- 10. If the area of the smT is 1, what is the area of the lgT?
- 11. If the area of the smT is 1, what is the area of the entire
  - tangram? \_\_\_\_
- 12. What fraction of the area of the tangram is triangles? \_\_\_\_
- 13. The hypotenuse of the lgT is 10 cm. Calculate the length of the other sides of lgT.

5. Which applies to all the triangles in the tangram?

4. What fraction of the pieces are triangles?

2. How many pieces are in a tangram?

3. How many different shapes?

Right Isosceles Equilateral Congruent

same day in December. the charts below. These were the prices at the same pizza chain on the Prices for cheese pizza in the United States and in Canada are listed in

Date

Name

2. Find the price per square in. in tenths of a cent for each pizza. 1. Find the area to the nearest tenth of a square in. for each size of pizza in both charts



	Cheese pizza in Montreal, Canada	Montreal, Can	<u>ada</u>		Cheese pizza	Cheese pizza in Florida, U.S.A.	
Size	10"	12"	14"	Size	12"	14"	16"
Price*	\$10.49	\$14.99	\$18.99	Price*	\$9.99	\$12.99	\$14.99
Area	78.5 in <sup>2</sup>	$113 in^2$	$78.5  \text{in}^2    113  \text{in}^2    153.9  \text{in}^2  $	Area	$113 in^2$	113	201 in <sup>2</sup>
Price/in <sup>2</sup>	13.4¢	13.3¢	12.3¢	Price/in <sup>2</sup>	8.8¢	8.4¢	7.5¢
*.	1. 1.11			*	1 11		

<sup>\*</sup>Prices in U.S. dollars

12"

- 4. For the Florida pizza, which pizza size is the cheapest per square in.? 14"

3. For the Montreal pizza, which pizza size is the cheapest per square in.?

Prices in Canadian dollars

- 5. If the Montreal store sold a 16" pizza, what should be the price? \$20.99 \$23.99 **×** \$25.99
- 6. If the Florida store sold a 10" pizza, what should be the price? \$8.99 **x** \$7.49 \_\$6.99
- both Montreal and Florida. Use the chart below to help you organize your work. 14" 7. You are ordering pizza for a group of friends. Should you order seven 12" or five 14" pizzas? Check prices in <u>ը</u> cheaper.

\$59.96	\$64.95	\$69.93	Total Cost	\$104.93   \$94.95   \$104.90	\$94.95	\$104.93	Total Cost
804	769	791	Total Area	785	769	791	Total Area
4	U	7	Number	10	UI	7	Number
16"	14"	12"	Size	10"	14"	12"	Size
	Florida, U.S.A.	Cheese pizza in Florida, U.S.A.		la	Montreal, Canac	Cheese pizza in Montreal, Canada	

<sup>\*8.</sup> In Montreal, how many 10" pizzas would you need to have the same amount? What is the total cost? What size is best? Repeat for 16" pizza in Florida. **14" is best in Montreal. 16" is best in Florid** 

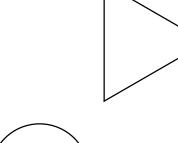
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Align Top

Draw the three figures aligned to the top, right, bottom, and left lines as marked.

Name\_

Align Bottom



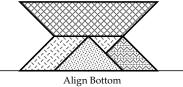
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Name\_

Date\_

Shapes are shown aligned to two sides. Draw them in the

(FIGURES WILL VARY ACCORDING TO HOW THEY'RE STACKED.)



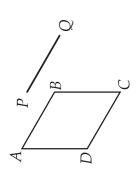
		Name			
Shapes are shown aligned to	and the December 1 and 1	Date			
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Worksheet 83-1, Reflecting					
	ine of reflection, shown as a cent Check your work with a reflectir	er			
line. Use your drawing tools. tool, if available.	Check your work with a reflectir  2.	ng Date	3.	<del></del>	
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Cube. a. Translate the rhombus ABCD (6, 0).

b. Translate the rhombus (6, 0), then reflect it about the right side.

c. Translate  $\overline{PQ}$  (6, 1)

d. Translate  $\overline{PQ}$  (6, 1), then rotate it  $-120^{\circ}$  about point P'.



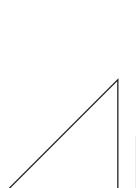
Chair. a. Translate the large triangle (5, -2.5).

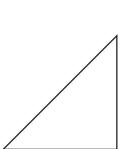
b. Translate the large triangle (6.5, -4), then reflect it across the hypotenuse.

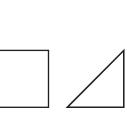
c. Translate the small triangle (8, 0).

d. Translate the small triangle (5, 4.5), reflect vertically in place.

e. Translate the square (5, -2)

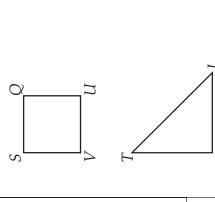


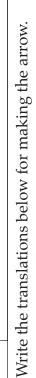


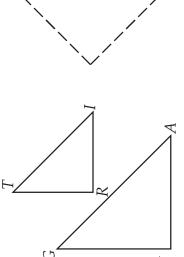


a. Translate SQUV (6, 0) Ship. b. Translate  $\Delta TRI$  (4.5, 2), rotate 45° about R'.

c. Translate  $\Delta TRI$  (7.5, 2), rotate 45° about R'.







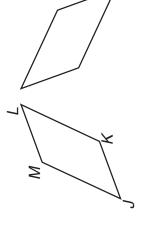
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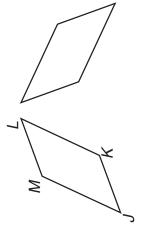
 $\mathbb{Z}$ 

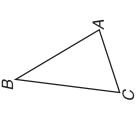
- 1. Connect each point of the triangle with its corresponding image. Use a light line. Then construct the perpendicular bisector of each line. Where they intersect, label the point P.
- 2. Draw circles with center P and radii  $\overline{PC}$ ,  $\overline{PA}$ , and  $\overline{PB}$ .
- ZBPB' ZAPA 3. Measure: ZCPC'

(no fractions) of images around the point of rotation! What is the angle of rotation between  $\triangle ABC$  and  $\triangle A'B'C'$ ? Make sure the rotation angle gives a whole number

images, continuing the rotation around P. Label each new image. 4. Using the same center and angle of rotation, draw four new







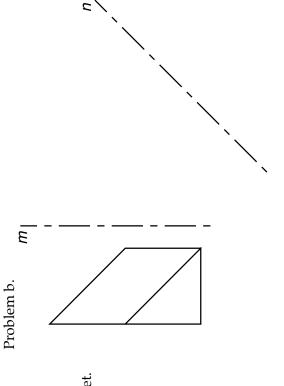
Draw circles and two more Angle of rotation is rotated objects.

6. Repeat for RST and its image, TUV. Angle of rotation is

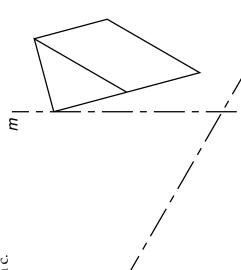
1. Reflect the object about line m. Then reflect the image about line n. Find the center of rotation.

CONTINUE READING THE LESSON.

figure above and label it P. Record your answer on the second worksheet. 2. Find the center of rotation from the original to the second image of the



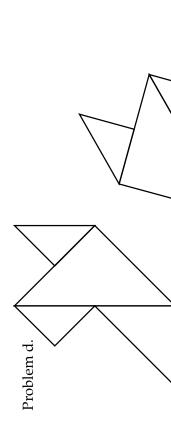
Problem c.



5. What do you observe about the angles between lines m and n

and the angle of rotation?

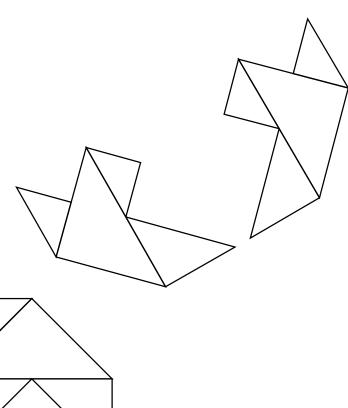
3. Draw the lines of reflection. Find the center of rotation.



6. In an object reflected twice, where is the center of rotation? If necessary, extend lines in the figures to see the relationship.

	\

7. Do these results about the lines of reflection work with the objects and images in Worksheet 89-2 and 89-3? Explain.



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Angle of Rotation				
Angle Between Lines $m$ and $n$				
Problem	a.	b.	c.	d.

#### More Double Reflections

**GOALS** 1. To explore double reflections with different angles

2. To discover the location of the center of rotation

**MATERIALS** Worksheets 92-1, 92-2

Drawing board, T-square, 45 triangle, 30-60 triangle

Ruler

A set of tangrams (optional)

Goniometer

**ACTIVITIES** 

**Other double reflections.** In Lesson 89, you discovered what happens when an object is reflected vertically and horizontally. The results give the same image as if the object had been rotated 180. This lesson explores reflecting through other angles.

**Problem 1.** On the worksheet, the first problem shows a square formed with tangram pieces. First reflect it around line *m*. Before you can use your drawing tools, you will need to measure to find a starting point. Repeat the reflection over line *n*.

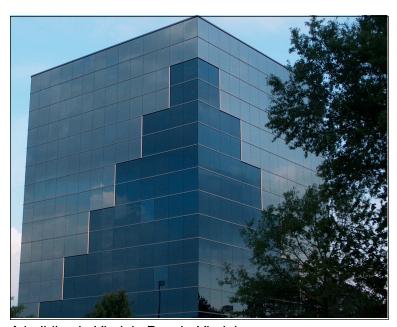
Do this before reading further.

**Analysis.** Compare your second image with the original object. Is it a reflection or a rotation? The answer is at the bottom of the page.

Measure the angle between the two lines of reflection, m and n. Consider line m rotating into line n to find whether the angle is positive or negative. Remember that if the rotation is counterclockwise, the rotation is positive. Write the angle in the table on the second worksheet.

Next find the center of rotation. Then find the angle of rotation from the original to the second image and record it in the table.

**Worksheet.** Complete the worksheets. (Answer: rotation)



A building in Virginia Beach, Virginia.

You may find it helpful to label the points on your figure.

## **Lines of Symmetry**

- 1. To review *line of symmetry*
- 2. To compare a line of symmetry to a line of reflection
- 3. To find the lines of symmetry in a figure
- 4. To learn the terms *maximum* and *minimum*
- 5. To learn about the infinity symbol, "∞"

**MATERIALS** 

Worksheets 94-1, 94-2

Drawing board, T-square, 30-60 triangle, 45 triangle

A set of tangrams

**ACTIVITIES** 



From the garden of the Castle of Angers in Angers, France.



La Fleur de Lise, important symbol in France, in St. Malo, France.



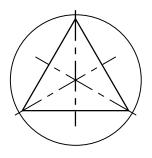
Design in the floor at the Minneapolis-St. Paul Airport.

**Line of symmetry.** You probably know what a line of symmetry is. It divides the figure into two parts with one part being the reflection, or mirror image, of the other part. A good way to check the line of symmetry is to fold the figure in half. If the two halves match, the fold line is the line of symmetry.

Line symmetry is very common in nature, in art, and in logos, which are symbols identifying a business or institution. See examples in the figures below. Note that the line of symmetry may be at any angle. Also a figure can have more than one line of symmetry.







**Comparing to line of reflection.** Reflections are a transformation, which means an object is transformed, or changed, into something else. Mathematicians use the word map to describe this transforming of an object according to some rule.

To think how a line of reflection is related to a line of symmetry, take a figure with a line of symmetry and remove half along a line of symmetry. Then reflect the remaining part about the line of symmetry, which becomes the line of reflection. Now you're back to the original object.

So, a line of symmetry is a line within a figure. A line of reflection is usually outside the figure and is the line the figure is flipped over.

**Some face fun.** Faces are almost symmetric. It is fun to make a picture of someone completely symmetric. To do this, first put the picture into computer software, either Adobe Photoshop or similar software. Then remove half. Copy the remaining half and reflect it horizontally. Lastly place the reflected along side the original half. See an example on the next page.



From the botanical garden in Big Island, Hawaii



Building in Maryland.



A tulip from Monet's Gardens in Giverny, France.

To write the infinity symbol, "∞," start in the middle, draw a loop on one side and then the other side.



Original photo.



Photo with left side reflected.

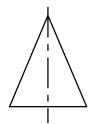


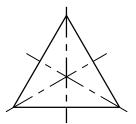
Photo with right side reflected.

**Worksheet.** You can do the first 19 questions on the worksheet now or finish reading below before doing the worksheet.

**Maximum and minimum.** You may have seen the terms *maximum* and *minimum* on highway signs. The maximum speed is the highest speed a person may legally drive. The minimum speed is the lowest speed allowed on the road.

As an example for maximum and minimum number of lines of symmetry, let's take an isosceles triangle. See the left figure below, which shows the usual isosceles triangle. Clearly, it has one line of symmetry.





But the definition of a isosceles triangle says it has two equal sides. How about an equilateral triangle, shown above on the right with its three lines of symmetry. Doesn't it also have two equal sides? So, an equilateral triangle is a special case of an isosceles triangle. Therefore, the minimum number of lines of symmetry in an isosceles triangle is 1 and the maximum number is 3.

*Infinity symbol.* Infinity is not a number, but a concept. For example, how many numbers are there. We say the answer is infinite because whatever number you say, I can say one higher. There is no limit.

You will need the symbol for infinity, " $\infty$ " for the table in problem 21. Sometimes the symbol is called "lazy eight."

**Problem 20**. You may need to look up some of the definitions. Remember the difference between a polygon and a regular polygon.

If you have a partner, compare your answers in the table before looking at the solution key. Discuss any answers where you disagree.

		Date		
2. If a figure has only ev	ven points, is it al	ways an Euler path	?	
3. If a figure has four or	dd points, is it an	Euler path?		
4. If a figure has two od	ld points, is it an	Euler path?		
5. Where do you start w	vhen tracing an E	uler path that has o	nly even points?	
6. Where do you end w	hen tracing an E	uler path that has or	nly even points?	
7. Where do you start w	vhen tracing an E	uler path that has o	nly two odd points	?
8. Where do you end af	ter tracing an Eu	ler path that has onl	y two odd points?	
9. Decide whether or no If it has different startin	- C		,	or no below it.
10. A family has a wind decorate with lights. Ca continuous string with side? Explain.	nn they do it with	one they do th	nildren are deliverin e following route w ? Explain.	0 1 1

## **Basic Trigonometry**

**GOALS** 1. To learn the meaning of *trigonometry* 

2. To learn the terms opposite, adjacent, sine, cosine, and tangent

3. To construct trigonometry tables

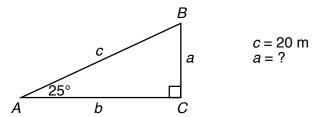
MATERIALS Worksheets 131-1, 131-2

Calculator

Goniometer, ruler

**ACTIVITIES** 

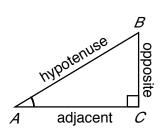
**A trigonometry problem.** In the problem below, you are given c and  $\angle A$  and asked to find a. If you knew the ratio,  $\frac{a}{c}$ , you could solve the problem as you did on the previous worksheet.



Solving problems using triangle ratios involves a branch of mathematics called *trigonometry* (TRIG-ah-NOM-ah-tree). The name is often shortened to *trig*. The derivation of the word is quite simple. *Tri* means *three*, *gon* means *angle*, and *metry* means *measure*.

**Trig terms.** Basic trig uses six terms, one you already know, two are simple, and three will be new. Look at the triangle below; the <u>hypotenuse</u> is opposite the right angle.  $\overline{BC}$  is the leg *adjacent* to  $\angle A$ .

You might like this for remembering the trig ratios: soh-cah-toa. (For example, "soh" is sin, opp, hyp.)



sine 
$$(A) = \frac{\text{opposite}}{\text{hypotenuse}}$$
  
cosine  $(A) = \frac{\text{adjacent}}{\text{hypotenuse}}$   
tangent  $(A) = \frac{\text{opposite}}{\text{adjacent}}$ 

$$\sin(A) = \frac{\text{opp}}{\text{hyp}}$$

$$\cos(A) = \frac{\text{adj}}{\text{hyp}}$$

$$\tan(A) = \frac{\text{opp}}{\text{adj}}$$

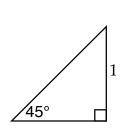
The trig names for the ratios are given in the first rectangle above. They are usually abbreviated as given in the second rectangle. However, the abbreviations are pronounced like the original words, sin (SIGN), cos (KOH-sign), and tan (TAN-jent).

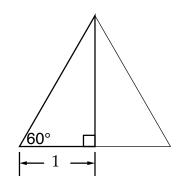
**Trigonometry history.** The Babylonians, Greeks, Egyptians, Indians, Chinese, and Arabs all contributed to the field of trigonometry. Surveyors and astronomers use it extensively for finding distances.

**Worksheets.** On the worksheets, you will be making a trig table for angles between 5° and 85°. On Worksheet 1 use the Pythagorean theorem and calculate the ratios to 3 decimal places. On Worksheet 2 measure the sides with a ruler and the angles with a goniometer. Then calculate the ratios and enter them in the table on Worksheet 1.

- 1. Use the Pythagorean theorem and the 45 triangle on the right to find the sin (45), cos (45), and tan (45).
- 2. Use the Pythagorean theorem and the 30-60 triangle to find the sin (60), cos (60), tan (60), sin (30), cos (30), and tan (30).
- 3. Measure the angles on the next worksheet to calculate the other values.

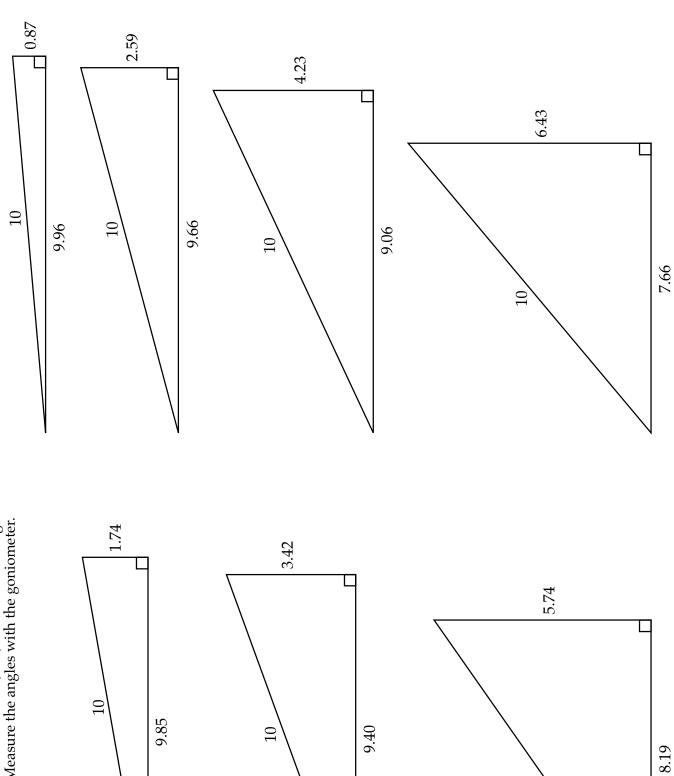
Write the values in the table to 3 decimal places.





Angle	$\sin = \frac{\text{opp}}{\text{hyp}}$	$\cos = \frac{adj}{hyp}$	tan = opp adj
5			
10			
15			
20			
25			
30			
35			
40			
45			
50			
55			
60			
65			
70			
75			
80			
85			

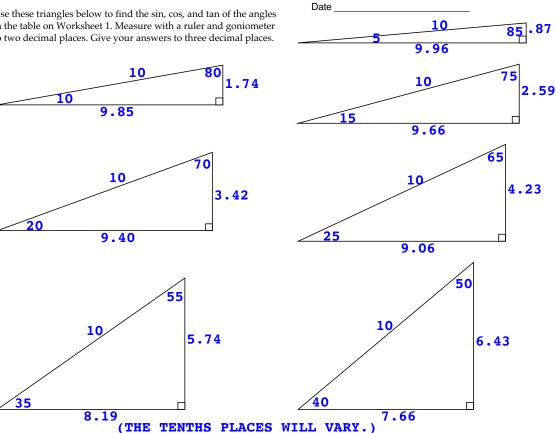
Use these triangles below to find the sin, cos, and tan of the angles in the table on Worksheet 1. Measure the angles with the goniometer.



10

Use these triangles below to find the sin, cos, and tan of the angles in the table on Worksheet 1. Measure with a ruler and goniometer to two decimal places. Give your answers to three decimal places.

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Name

Worksheet 131-2, Basic Trigonometry

## **Comparing Calculators**

**GOALS** 1. To learn some differences between a basic and *scientific calculator* 

2. To learn some features of the scientific calculator

**MATERIALS** Basic calculator (Casio SL-450S)

Scientific calculator (Casio fx-300MS)

Worksheet 133

**ACTIVITIES** 

**Calculators.** There are dozens of different types of calculators. A scientific calculator does the basic operations that a simple calculator does, but in a slightly different way. In addition to basic arithmetic, a scientific calculator does trigonometric functions, exponents, means (averages), has () and pi, and many more advanced topics.

The Casio fx-300MS may be used on the SAT and PSAT/NMSQ tests.



Basic calculator



Scientific calculator

If your calculator was used for advanced work, press MODE (next to ON) and 1 and =.

The best way to learn to use it a new calculator is to experiment with it. Glance at the instructions when you need help with more advanced features.

**Order of operations.** Turn on the scientific calculator by pressing the ON button in the upper right corner. Perform the following operations in the order it is written on both calculators

$$2 + 3 \times 4 = \_$$

One calculator says 14 and other says 20. The correct answer is 14, according to the order of operations, which says multiply and divide before adding. Scientific calculators and spreadsheets follow the standard order of operations.

How did you like seeing all the numbers you entered?

**Rounding.** Enter  $2 \div 3$  on both calculators. Both calculators have a a string of 6s. Now multiply that by 3. Answer should be 2, right? The scientific calculator says 2; however, the basic calculator gives the answer as 1.9999998. The scientific calculator used the actual answer for  $2 \div 3$  whereas the basic calculator multiplied 0.6666666 by 3.

As another example, try

What should the answer be? The scientific calculator sees it as two fractions, or division, added together,  $\frac{1}{3} + \frac{2}{3}$ . Try it. On the other hand, the basic calculator does  $1 \div 3$ , adds 2, and then divides all of that by 3. To get the correct answer on the basic calculator, you must

use memory. First do  $1 \div 3$  and add to memory. Next do  $2 \div 3$  and add to memory. Try it and you'll find the answer still isn't exactly right; it says .99999999, instead of 1.

**Square root.** On the basic calculator you found a square root by entering the number and  $\sqrt{\ }$ . On this scientific calculator, you first press  $\sqrt{\ }$ , then enter the number, and press  $\boxed{\ }$ .

Enter 13 and find its square root on both calculators.

Basic:  $13 \sqrt{\ } = 3.6055512$ Scientific:  $\sqrt{\ } 13 = 3.605551275$ 

Note that the basic calculator just quits the answer. If it had rounded the answer, the last digit would have been 3, not 2.

Now square your results. On the basic calculator, press  $\times$  and =. On the scientific calculator, press  $(x^2)$  and (=). Do they both say 13?

**Pi.** On your basic calculator, you had to enter several digits to find pi. On a scientific calculator,  $\pi$  is built in to 10 places.

You will find the  $\pi$  symbol in the middle of the bottom row, above the EXP key. To use any function printed in gold letters, first press the SHIFT key in the upper left hand corner. Therefore, to access pi, press SHIFT and  $\pi$ ; press  $\equiv$  to see the actual value.

**Memory.** To place the value on the display into Memory, press (SHIFT)(M+). To multiply by the value in Memory, press the number and then (XRCL)(M+)(=). The values in Memory remain even after the power is off.

**Worksheet.** The 10 problems are actually geometry review problems. Do them on your scientific calculator without writing anything down. Be sure to estimate the answer and check that it makes sense. Solve them now. When you have finished or need help, continue reading below.

**Problem 1-2.** These are plain problems involving  $\pi$ . Don't confuse diameter and radius.

**Problem 3.** There are two ways to solve this problem. Try both.

**Problem 4.** Here is where you'll appreciate the scientific calculator.

**Problem 5.** You really don't need the calculator for this one.

**Problem 6.** This would be a tough problem without a calculator. After finding the ratio, store it in Memory. Then enter each side of the polygon and press  $\times$  RCL M+ =.

**Problem 7.** Remember something special about isosceles triangles.

**Problem 8.** The easy way to do this problem is to recognize the relationship between the two squares. Mentally fold a corner of the larger square unto the smaller square.

**Problem 9.** Using parentheses makes this problem easier.

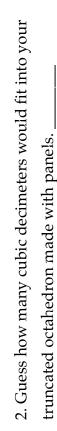
**Problem 10.** This is a two-step problem, not hard if you remember what you're doing.

When the calculator shows, "Syntax Error," press the red button, AC, and try again.

Date







3. Answer the following questions about the volume of the polyhedrons in the figure above. (An edge is a.)

3a. The volume of the octahedron is \_ the volume of the large pyramid.

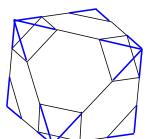
3b. For the large pyramids, a =\_\_

3c. For the small pyramids, a =\_

3d. The volume of the truncated octahedron is times the volume of the small pyramids. equal to the volume of the octahedron –

4. Calculate the volume of your truncated octahedron using decimeters. Give your answer to two decimal places. 2011





- 2. Guess how many cubic decimeters would fit into your truncated octahedron made with panels.
- 3. Answer the following questions about the volume of the polyhedrons in the figure above. (An edge is a.)
  - 3a. The volume of the octahedron is 2 times the volume of the large pyramid.
  - 3b. For the large pyramids, a = 3.
  - 3c. For the small pyramids, a = 1
  - 3d. The volume of the truncated octahedron is equal to the volume of the octahedron - 6 times the volume of the small pyramids.
- 4. Calculate the volume of your truncated octahedron using decimeters. Give your answer to two decimal places.

Name

$$V(LgPyr) = \frac{1}{3}BH$$

$$B = a^2 = 3^2 = 9$$

$$H = \frac{1}{2}\sqrt{2} \times a$$

$$H = \frac{1}{2} \sqrt{2} \times 3 = 2.12$$

$$V(LgPyr) = \frac{1}{3} \times 9 \times 2.12$$

$$V(LgPyr) = 6.36$$

$$V(\text{Oct}) = 2 \times 6.36 = 12.73$$

#### Small pyr:

$$B = a^2 = 1^2 = 1$$

$$H = \frac{1}{2}\sqrt{2} \times a = \frac{1}{2}\sqrt{2} = .71$$

$$V(\text{SmPyr}) = \frac{1}{3} \times 1 \times .71 = .24$$

$$V(\text{TrOc}) = V(\text{Oct}) - 6 \times V(\text{SmPyr})$$

$$V(TrOc) = 12.73 - 6 \times .24$$

$$V(TrOc) = 11.31 \, dm^3$$

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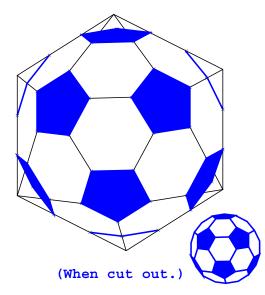
Worksheet 160, Truncated Icosahedron

- 1. Truncate each vertex of the icosahedron below by connecting the x's around the vertex. Color or shade the pentagons. Then cut out your new figure.
- 2. What is the figure? \_truncated

icosahedron (soccer ball)

- 3. Each pentagon is surrounded by **5 hexagons**.
- 4. Each hexagon is surrounded by **3 hexagons**

and 3 pentagons



Date



In the icosahedron:

- 5. Since a triangle has <u>3</u> vertices, on an icosahedron the number of vertices in all 20 triangles is 60.
- triangles, it has 12 vertices. (60 ÷ 5)
- 7. Since each triangle has 3 sides, the total number of edges for the 20 triangles is 60.
- 8. Since you put \_\_\_\_ sides of a triangle together with a rubber band, the icosahedron has 30 edges.  $(60 \div 2)$

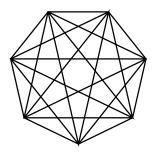
In the truncated icosahedron:

- 9. Since an icosahedron has 20 faces and you added 12 more faces, the truncated icosahedron has 32
- 10. Since an icosahedron has 30 edges and you added \_\_\_\_\_\_ more for each pentagon, the truncated icosahedron has 90 edges.  $(30 + 5 \times 12)$
- 11. Since an icosahedron has 12 vertices and for each pentagon you added \_\_\_\_\_ vertices but removed 1, the truncated icosahedron has  $\underline{60}$  vertices. (12 + 4 × 12)

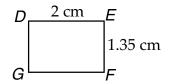
Date

Circle the letter in front of all correct answers. <u>Some questions may have more than one answer.</u>

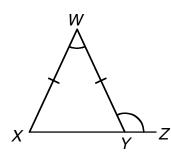
- 1. What is the longest line in a circle?
  - a. diameter
  - b. diagonal
  - c. radius
  - d. circumference
- 2. The length of a side of a square is w.
  - a. The area of the square is 4w.
  - b. The volume of the square is 3w.
  - c. The perimeter of the square is 4w.
  - d. The diagonal of the square > w.
- 3. Which regular figures will tessellate?
  - a. octagon
  - b. hexagon
  - c. cube
  - d. equilateral triangle
  - e. pentagon



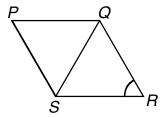
- 4. How many diagonals are in the heptagon?
  - a. 14
  - b. 26
  - c. 28
  - d. none



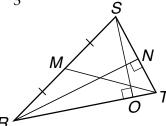
- 5. What is the perimeter of *DEFG*?
  - a. 2.7 cm
  - b. 67 mm
  - c.  $3.35 \text{ cm}^2$
  - d. 6.7 cm



- 6. Angle XWY is 40°. What is angle WYZ?
  - a. 70°
  - b. 40°
  - c. 110°
  - d. 120°



- 7. *PQRS* is a rhombus. Line *QS* is  $\frac{2}{3}$ . Angle *R* is 60°. What is the perimeter of the rhombus?
  - a. 2
  - b.  $\frac{8}{3}$
  - c.  $2\frac{2}{3}$
  - d.  $\frac{10}{3}$



- 8. What is the area of triangle *RST*?
  - a.  $RT \times SO$
  - b. RT + TS + SR
  - c.  $\frac{ST \times RN}{2}$
  - d.  $\frac{1}{2}(RS \times MT)$

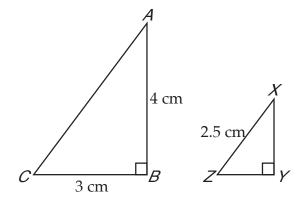
а	(5)
	4
b	2/3
	$\angle 1$

Date \_\_\_\_\_

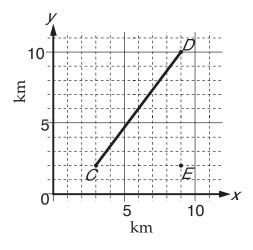
Lines a and b are parallel. Angle 1 is 139.1°.

- 9. What is m∠2? \_\_\_\_\_
- 10. What is m∠3? \_\_\_\_\_
- 11. What is m∠4? \_\_\_\_\_
- 12. What is m∠5? \_\_\_\_\_

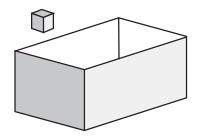
13-15. Triangle *ABC* is similar to triangle *XYZ*. What is the perimeter of each triangle? What is the ratio of their perimeters?



16-19. What are the coordinates of cities *C* and *D*? A car is driven from *C* to *D*. How far is the drive? How much farther if you drive through city *E* on your way from *C* to *D*?

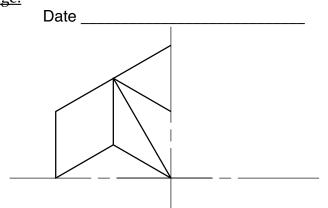


20. Morgan has some blocks that are cubes with a side measuring 2 cm. How many blocks will fit into a box that measures 6 cm by 10 cm by 4 cm?

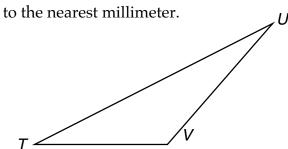


Use drawing tools and calculator for this page.

21-22. Reflect the design horizontally about the vertical line. Then reflect the original and your reflection vertically about the horizontal line. Do not do any measuring.

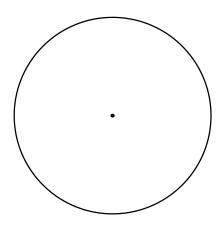


23. Find the area of  $\Delta TUV$ . Measure



24-25. A wading pool is circular with a square deck around it. The designer wants the deck to be twice the area of the pool. Draw the deck around the pool (the square around the circle).





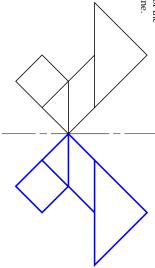
Test: Second Quarter, Page 3

Use your drawing tools (no calculator) for

Name

Date

figure about the vertical line. 22. Construct a reflection of the



23. The figure below is made with tangram pieces. Construct it on the line at the right rotated 90°.



of the four pieces?  $\frac{1}{2}$  unit<sup>2</sup>, what is the total area if the area of the square is

 $2\frac{1}{4}$  units<sup>2</sup>

square that circumscribes the circle. Find its area. 25-26. The radius of the circle is 3.5 cm. Construct a

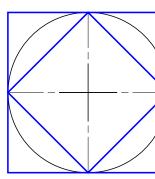
$$A = 49 \text{ cm}'$$

27-28. Inscribe a square in the circle. What is its area?

$$A=24.5$$
 cm

29. The area of the circle is greater than which inscribed

square? <u>circumscribed</u> 30. The area of the circle is less than which



No drawing tools or calculator for this page.

Name

Circle the letter in front of all correct answers. Some questions may have more than one answer

Test: Final, Page 1

- 1. What is the longest line in a circle?
- a. diameter b. diagonal
- d. circumference
- 2. The length of a side of a square is w.
- C. The perimeter of the square is 4w. d. The diagonal of the square > w. a. The area of the square is 4w.
  b. The volume of the square is 3w.
- 3. Which regular figures will tessellate?
- b. hexagon

24. In the figure in Problem 23,

dequilateral triangle e. pentagon

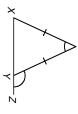


4. How many diagonals are in the heptagon?

- a. 14 b. 26 c. 28 d. none
- $2 \, \mathrm{cm}$  $1.35 \, \mathrm{cm}$

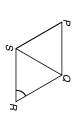
5. What is the perimeter of DEFG?

- a. 2.7 cm b 67 mm
- c.  $3.35 \text{ cm}^2$



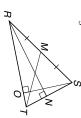
6. Angle XWY is 40°. What is angle WYZ?

- a. 70° b. 40° c. 110° d. 120°



7. PQRS is a rhombus. Line QS is  $\frac{2}{3}$ . Angle R is  $60^\circ$ . What is the perimeter of the rhombus?

- a. 2



8. What is the area of triangle RST?

- a.  $RT \times SO$
- b. RT + TS + SR
- $\bigcirc \underline{ST \times RN}$
- d.  $\frac{1}{2}(RS \times MT)$

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No drawing tools or calculator for this page.

Name

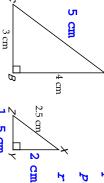
9. What is m \( 2? \) 139.1 Lines a and b are parallel. Angle 1 is 139.1.

10. What is m∠3? **40.9°** 

11. What is m \( \arr 4? \) 40.9°

12. What is m∠5? **139.1** 

is the ratio of their perimeters? 13-15. Triangle ABC is similar to triangle XYZ. What is the perimeter of each triangle? What

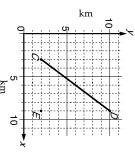


P(ABC)II **4** + ω + σı Ш 12 G

$$P(XYZ) = 2 + 1.5 + 2.5 = 6 \text{ cm}$$

1.5 cm 2 cm II N ..

drive? How much farther if you drive through city E on your way from C to D? 16-19. What are the coordinates of cities C and D? A car is driven from C to D. How far is the



 $\ddot{c}$ (3, 2)

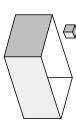
D: (9, 10)

$$CD = \sqrt{6^2 + 8^2} = 10 \text{ km}$$

$$7 + ED = 6 + 8 = 14 \text{ km},$$

km farther

20. Morgan has some blocks that are cubes with a side measuring 2 cm. How many blocks will fit into a box that measures 6 cm by 10 cm by 4 cm?



V(cube) =Ŋ П œ

$$V(\text{box}) = 6 \times 10 \times 4 = 240$$

$$\frac{240}{8} = 30 \text{ blocks}$$

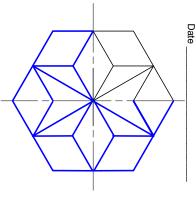
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Test: Final, Page 2 Use drawing tools and calculator for this page.

Name

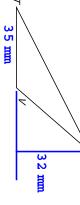
Test: Final, Page 3

21-22. Reflect the design horizontally about the vertical line. Then reflect the original and your reflection vertically any measuring. about the horizontal line. Do not do



23. Find the area of  $\Delta TUV$ . Measure

to the nearest millimeter.



M П

¥,

$$A = \frac{1}{2} \times 35 \times 32$$

24-25. A wading pool is circular with a square deck around it. The

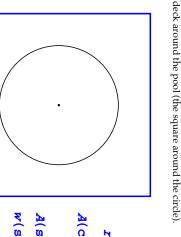
Ъ

II

560 mm<sup>2</sup>

designer wants the deck to be twice the area of the pool. Draw the





$$r=2.6\,\mathrm{cm}$$

$$A(cir) = \pi \times 2.6^2$$
  
= 21.24 cm<sup>2</sup>

$$A(sq) = 3A(cir) = 63.71$$
  
 $w(sq) = 8.0 cm$ 

$$w(sq) = 8.0 cm$$

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