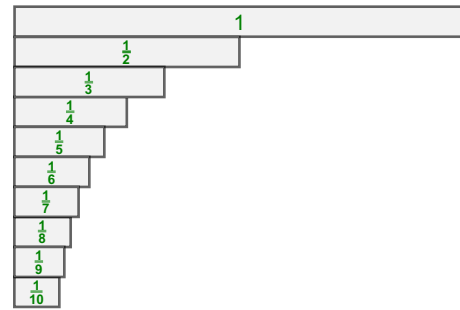# Fractions of the Future

info@RightStartMath.com

Fraction History

- Latin "frangere" meaning "to break."
- Considered only as part of a whole.
- Could never be equal or greater than 1.

Fraction History

Fraction History

- In the 1600s, the concept of fractions expanded.
- Now included a division perspective.
- Fractions could be equal to or more than 1.

$$\frac{1}{3}$$

Fraction History

- In the 1600s, the concept of fractions expanded.
- Now included a division perspective.
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$$\frac{2}{2}$$

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- In the 1600s, the concept of fractions expanded.
- Now included a division perspective.
- Fractions could be equal to or more than 1.

$$\frac{2}{3}$$

Fraction History

| Old Fractions	New Fractions
Always less than one	Can be equal or greater
$\frac{1}{3}$	$\frac{1}{3}$ $\frac{3}{3}$ $\frac{4}{3}$
Whole is fractured	Viewed as division
Familiar = "proper"	Unfamiliar = "improper"
Limited view	Supports understanding

Fraction Model: Circles
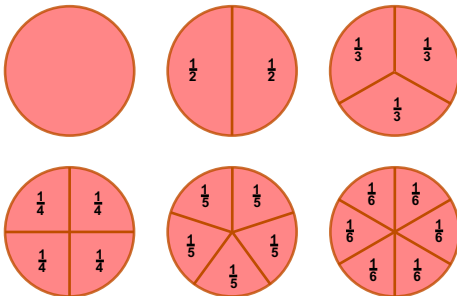
Try to compare 4/5 and 5/6 with this model.

Fraction Model: Circles

- Experts in visual literacy say that comparing quantities in pie charts is difficult because most people think linearly. It is easier to compare along a straight line than compare pie slices.   askoxford.com
- Specialists also suggest refraining from using more than one pie chart for comparison.   statcan.ca
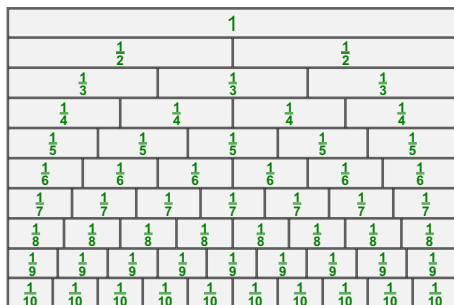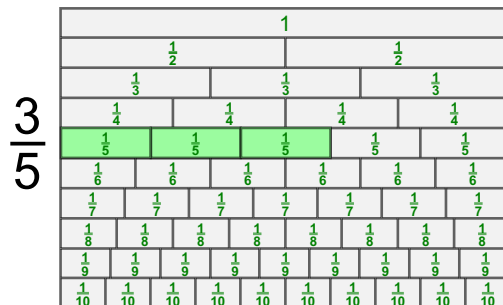
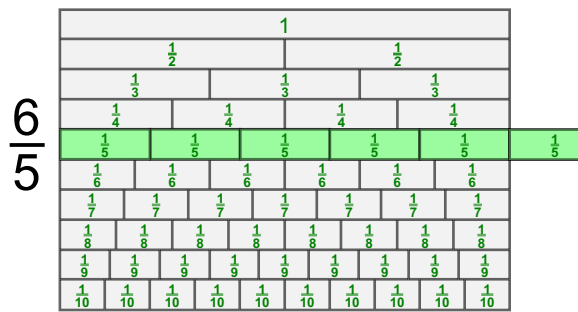## Fraction Model: Linear Chart

Fraction Model: Linear Chart

$$\frac{3}{5}$$

Fraction Model: Linear Chart

$\dfrac{6}{5}$

Fraction Stairs
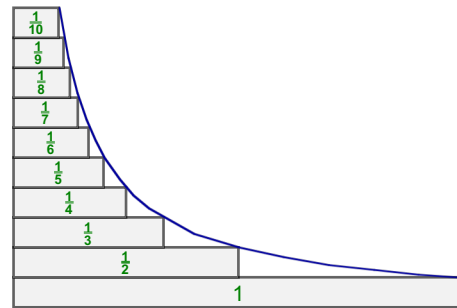
A hyperbola.

Games

$$\frac{\text{Games}}{\text{Math}} = \frac{\text{Books}}{\text{Reading}}$$

Games provide instant feedback.

Games provide interesting repetition needed for automatic responses in a social setting.

More importantly, games provide an application for the new information!
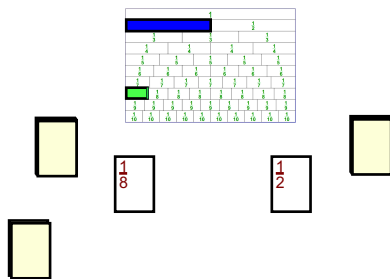
Unit Fraction War

Purpose: Practice for naming and comparing unit fractions.
To help the children realize a unit fraction decreases as the denominator increases.

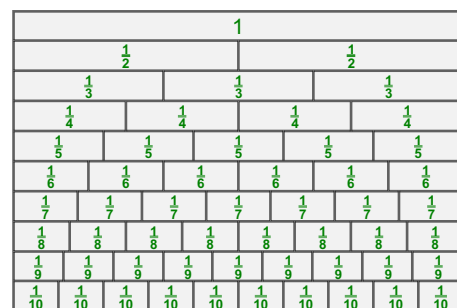Goal: To collect all, or most, of the cards by comparing unit fractions.

Unit Fraction War

$\dfrac{1}{8}$ $\dfrac{1}{2}$

Fraction Chart

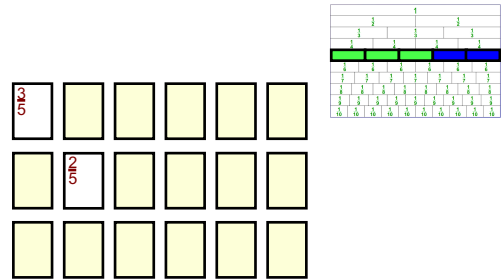How many fourths in a whole? How many fifths? Eighths?

Concentrating on One

Purpose: To help the children realize that 5 fifths, 8 eighths, and so forth, make a whole.
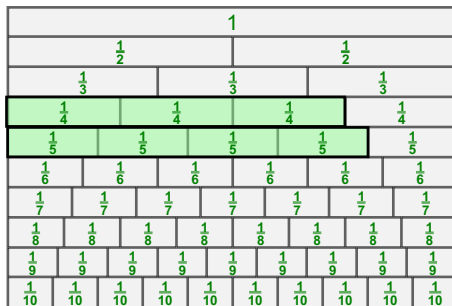
Goal: To find the pairs that make a whole.

Concentrating on One

Fraction Chart
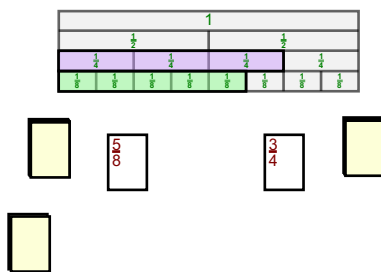

Which is more, 3/4 or 4/5?

Fraction War

Purpose: To practice comparing ones, halves, fourths, and eighths in preparation for reading a ruler.
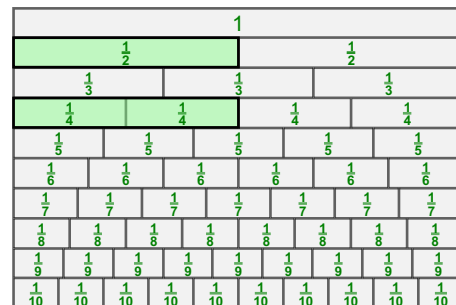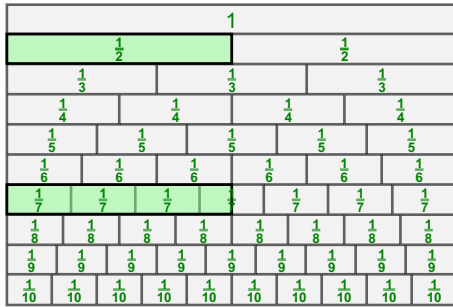
Goal: To capture all the cards.

Fraction War

Equivalent Fractions

How many fourths equal a half?

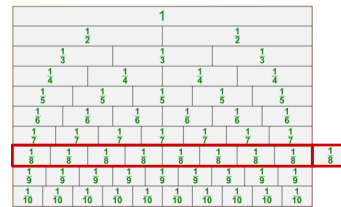## Equivalent Fractions

How many fourths equal a half? Eighths? Sevenths?

Proper and Improper Fractions
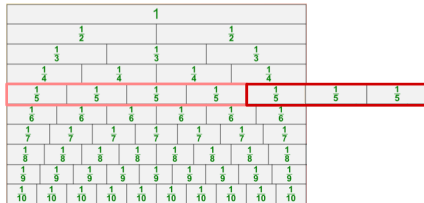

$$\frac{9}{8} = 1\frac{1}{8}$$

Rewrite the improper fractions using a whole number.

Proper and Improper Fractions
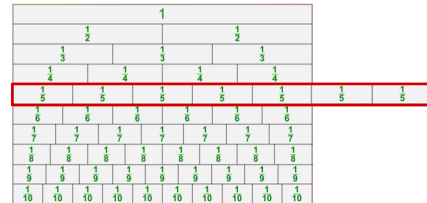

$$\frac{4}{5} + \frac{3}{5} =$$

Proper and Improper Fractions

$$\frac{4}{5} + \frac{3}{5} = \frac{7}{5} = 1\frac{2}{5}$$

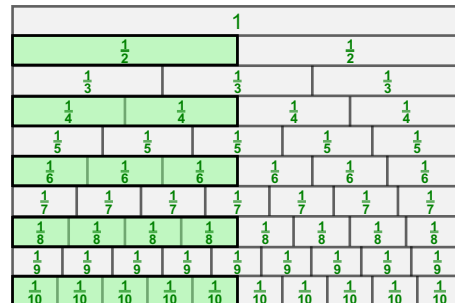Rewrite the improper fraction using a whole number.

Help your student **discover** the algorithm.

"What you have been obliged to discover by yourself leaves a path in your mind which you can use again when the need arises." — *G.C. Lichtenberg*
18th century physicist

Simplifying Fractions

Simplifying Fractions

Simplifying Fractions

$$\frac{21}{28}$$

Simplifying Fractions

$$\frac{45}{72}$$

Why does this work?

Remember, let your child **discover** the algorithm.
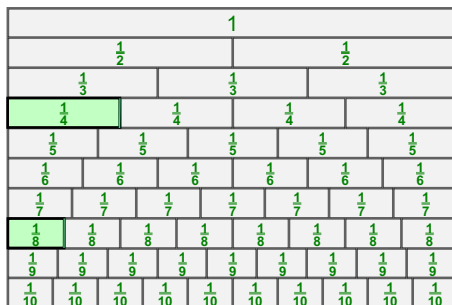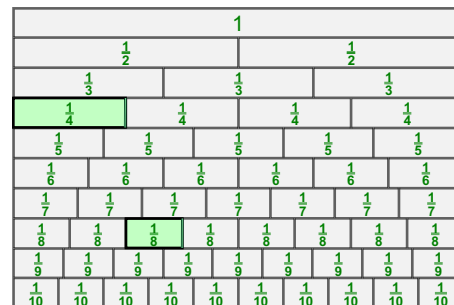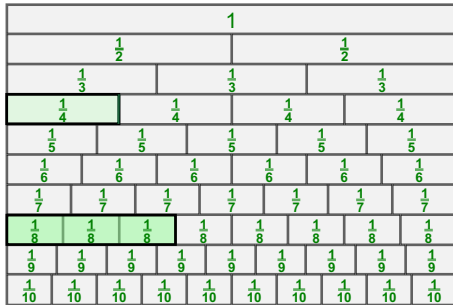
Fraction Chart

What is $\frac{1}{4} + \frac{1}{8}$?

Fraction Chart

What is $\frac{1}{4} + \frac{1}{8}$?

Fraction Chart

What is $\frac{1}{4} + \frac{1}{8}$? **$\frac{3}{8}$**

Fraction Chart
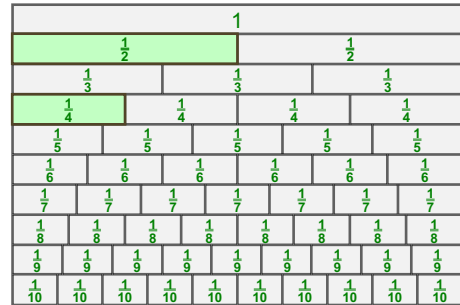
What is $\frac{1}{2}$ of $\frac{1}{2}$?

Multiplying Fractions

Multiplying is not exclusively repeated addition.

$$4 \times 4 = 4 + 4 + 4 + 4$$

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{2} + \ ?$$

Multiplying Fractions

Multiplying is not exclusively repeated addition.

Area is a better model.

$$4 \times 4 =$$
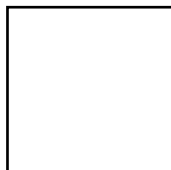
Multiplying Fractions
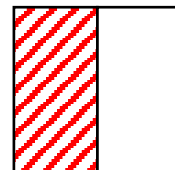
$$\frac{1}{2} \times \frac{1}{2} =$$

One half
  of one half

Multiplying Fractions

$$\frac{1}{2} \times \frac{1}{2} =$$

One half
  of one half
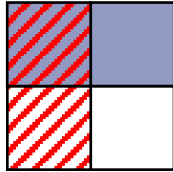
Multiplying Fractions

$\frac{1}{2} \times \frac{1}{2} =$

One half
of one half

Multiplying Fractions

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

One half
of one half
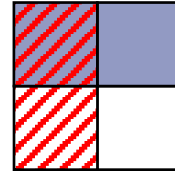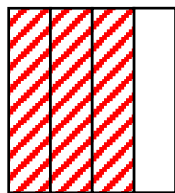
Multiplying Fractions

$\frac{2}{3} \times \frac{3}{4} =$

Three fourths
of two thirds

Multiplying Fractions

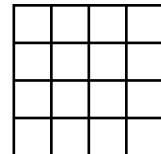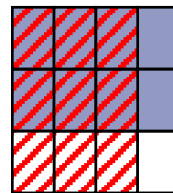$\frac{2}{3} \times \frac{3}{4} =$

Three fourths
of two thirds

Multiplying Fractions

$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$

Three fourths
of two thirds
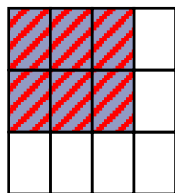
Multiplying Fractions

$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$

The total number of rectangles is 3 × 4.

Multiplying Fractions

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

The total number of rectangles is 3 × 4.
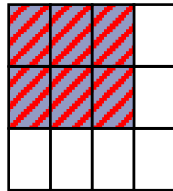
The number of colored crosshatched rectangles is 2 × 3.

Dividing Fractions

$$1 \div \frac{1}{2} = 2$$

How many $\frac{1}{2}$s in 1?

	1		
	$\frac{1}{2}$		$\frac{1}{2}$
$\frac{1}{3}$		$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Dividing Fractions
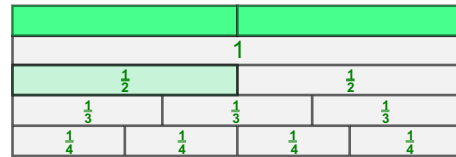
$$1 \div \frac{1}{2} = 2$$

$$1 \div \frac{1}{3} = 3$$

How many $\frac{1}{3}$s in 1?
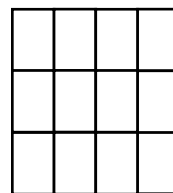
	1		
	$\frac{1}{2}$		$\frac{1}{2}$
$\frac{1}{3}$		$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Dividing Fractions

$$1 \div \frac{1}{2} = 2 \qquad 1 \div \frac{2}{3} = 1\frac{1}{2} = \frac{3}{2}$$

$$1 \div \frac{1}{3} = 3$$

How many $\frac{2}{3}$s in 1?

$$1 \div \frac{1}{4} = 4$$

$$1 \div \frac{1}{5} = 5$$

$$1 \div \frac{1}{6} = 6$$

Dividing Fractions

$$1 \div \frac{1}{2} = 2 \qquad 1 \div \frac{2}{3} = \frac{3}{2}$$

$$1 \div \frac{1}{3} = 3 \qquad 1 \div \frac{3}{4} = 1\frac{1}{3} = \frac{4}{3}$$

$$1 \div \frac{1}{4} = 4$$

How many $\frac{3}{4}$s in 1?

$$1 \div \frac{1}{5} = 5$$

$$1 \div \frac{1}{6} = 6$$

Dividing Fractions
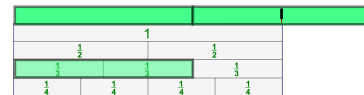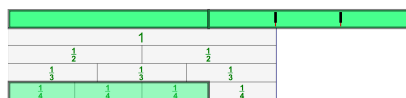
$$1 \div \frac{1}{2} = 2 \qquad 1 \div \frac{2}{3} = \frac{3}{2}$$

$$1 \div \frac{1}{3} = 3 \qquad 1 \div \frac{3}{4} = \frac{4}{3}$$

$$1 \div \frac{1}{4} = 4 \qquad 1 \div \frac{2}{5} = \frac{5}{2}$$

$$1 \div \frac{1}{5} = 5 \qquad 1 \div \frac{5}{8} = \frac{8}{5}$$

$$1 \div \frac{1}{6} = 6 \qquad 1 \div \frac{4}{7} = \frac{7}{4}$$

Guide the child to making the discovery that the answers are the inverted form of the divisor.

Fraction Division War

Purpose: Practice in dividing fractions.
To help the children realize the quotient is the inverted form of the divisor.

Goal: To collect all, or most, of the cards by having the greatest quotient.

Fraction Division War

$$1 \div \tfrac{1}{2} = 2$$
$$1 \div \tfrac{1}{4} = 4$$

Fraction Division War

$$1 \div \tfrac{3}{8} = \tfrac{8}{3}$$
$$1 \div \tfrac{2}{3} = \tfrac{3}{2}$$
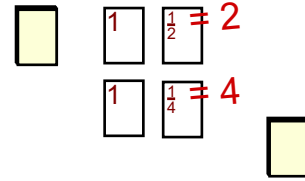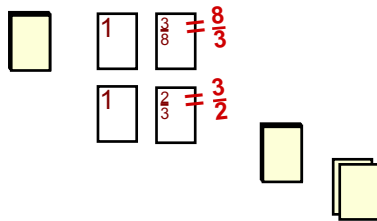
More Dividing Fractions

$$1 \div \tfrac{1}{4} = 4$$
$$2 \div \tfrac{1}{4} = 2 \times (1 \div \tfrac{1}{4})$$
$$= 2 \times \quad 4 \quad = 8$$

More Dividing Fractions

$$1 \div \tfrac{1}{4} = 4$$
$$2 \div \tfrac{1}{4} = 2 \times (1 \div \tfrac{1}{4}) = 2 \times 4 = 8$$

More Dividing Fractions

$$1 \div \tfrac{1}{4} = 4$$
$$2 \div \tfrac{1}{4} = 2 \times (1 \div \tfrac{1}{4}) = 2 \times 4 = 8$$
$$3 \div \tfrac{1}{4} = 3 \times (1 \div \tfrac{1}{4}) = 3 \times 4 = 12$$
$$\tfrac{1}{2} \div \tfrac{1}{4} = \tfrac{1}{2} \times (1 \div \tfrac{1}{4}) = \tfrac{1}{2} \times 4 = 2$$
$$\tfrac{1}{3} \div \tfrac{1}{4} = \tfrac{1}{3} \times (1 \div \tfrac{1}{4}) = \tfrac{1}{3} \times 4 = \tfrac{4}{3}$$
$$\tfrac{3}{4} \div \tfrac{1}{4} = \tfrac{3}{4} \times (1 \div \tfrac{1}{4}) = \tfrac{3}{4} \times 4 = \tfrac{12}{4} = 3$$

Fraction Chart

Allow the child to explore the whole picture and relationships within the whole using the linear perspective.

In Conclusion …

Math needs to be taught so 95 percent is understood and only 5 percent memorized.

Richard Skemp
-- major pioneer in
mathematics education

In Conclusion …

Our goal as a teacher of mathematics is to help our children transform, expand, and refine these beginning ideas into deeper mathematical thinking.

– Dr. Joan A. Cotter