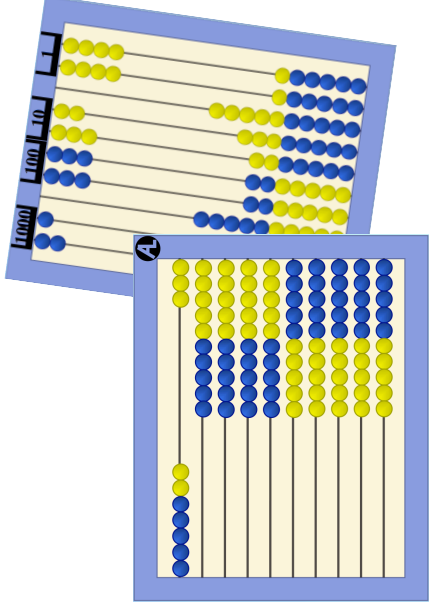


RIGHTSTART™ MATH:

Why it Works!



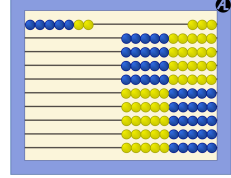
info@RightStartMath.com

Joan A. Cotter, Ph.D.

- Engineer from University of Wisconsin
- Montessori Educator
- Math Games Author
- Special Needs Teacher
- AL Abacus Designer
- Middle-School Teacher
- Ph.D. in Math Education
from University of Minnesota

© Activities for Learning, Inc. 2002

AL Abacus

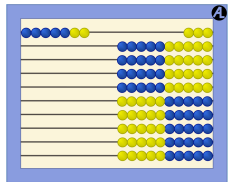


“The role of physical manipulatives is to help the child form those visual images and thus to eliminate the need for the physical manipulatives.”

—Ginsberg and others

© Activities for Learning, Inc. 2002

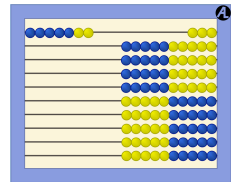
AL Abacus



- Visual and tactile manipulative
- Develops mental images of
Quantities
Strategies
Mathematical Operations

© Activities for Learning, Inc. 2002

AL Abacus



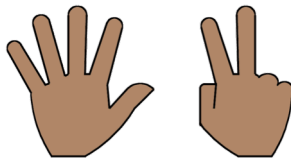
“Think in pictures, because the brain remembers images better than it does anything else.”

—Ben Pridmore
World Memory Champion, 2009

© Activities for Learning, Inc. 2002

Subitizing

- Subitizing is quick recognition of quantity without counting.



© Activities for Learning, Inc. 2002

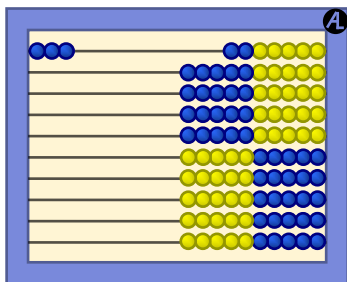
Grouping



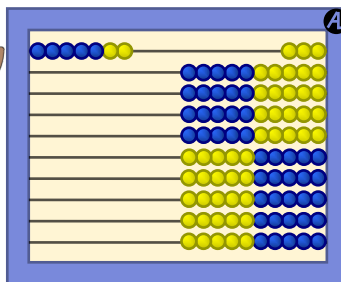
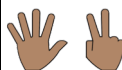
Try to visualize 8 apples without grouping.

© Activities for Learning, Inc. 2002

Entering Quantities

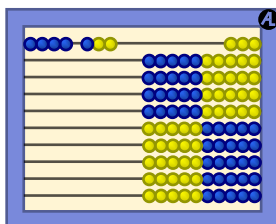


Entering Quantities

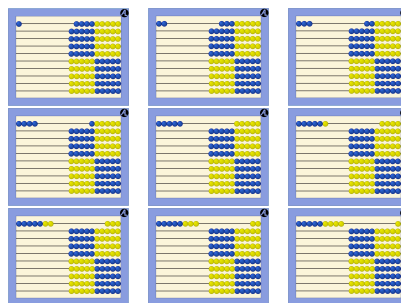


Adding

$$4 + 3 = \underline{\quad}$$



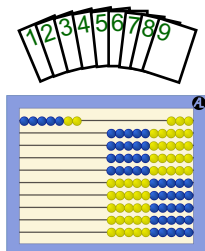
Facts Equaling 10



Go to the Dump Game

A “Go Fish” type of game where the pairs are:

- 1 & 9
- 2 & 8
- 3 & 7
- 4 & 6
- 5 & 5



Why Games

$$\frac{\text{Games}}{\text{Math}} = \frac{\text{Books}}{\text{Reading}}$$

Games provide interesting repetition needed for automatic responses in a social setting.

More importantly, games provide an application for the new information!

Place Value

- The author of *Treviso Arithmetic of 1478*, written over 500 years ago, considered place value so important that it was listed first among the “five” operations of arithmetic.
- Place value organizes numbers into neat packets.
- Without place value, computational algorithms make little sense.

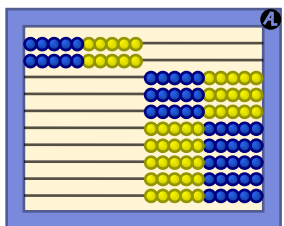
Place Value

Children often think of 14
as 14 ones,
not ten and 4 ones.

The pattern that is needed to make
sense of tens and ones is hidden
because of the English language!

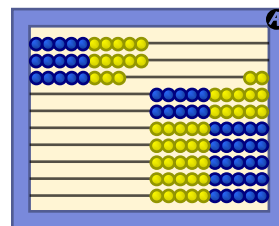
Transparent Place Value

2-ten



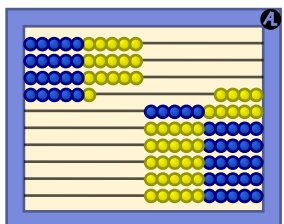
Transparent Place Value

2-ten 8



Transparent Place Value

3-ten 6



Transparent Number Naming

10 = ten	20 = 2-ten
11 = ten 1	21 = 2-ten 1
12 = ten 2	22 = 2-ten 2
13 = ten 3	23 = 2-ten 3
14 = ten 4
....
19 = ten 9	99 = 9-ten 9

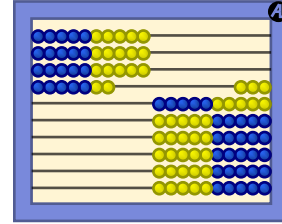
Transparent Number Naming

- Use this for two reasons:
 1. Patterning
 2. Place value

© Activities for Learning, Inc. 2002

Transparent Number Naming

3-ten 7 37



© Activities for Learning, Inc. 2002

Transparent Number Naming

30

3 - ten

300

3 hun-dred

3000

3 th-ou-sand

© Activities for Learning, Inc. 2002

Transparent Number Naming

3000

600

50

8

3000

600

50

8

3658

© Activities for Learning, Inc. 2002

Transparent Number Naming

- Just as reciting the alphabet doesn't teach reading, counting doesn't teach arithmetic.
- Just as we first teach the *sound* of the letters, we first teach the *name* of the quantity (math way).

© Activities for Learning, Inc. 2002

Transparent Number Naming

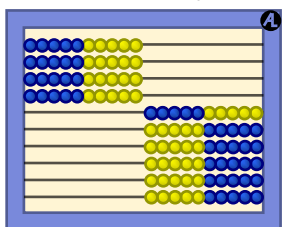
- Asian children learn mathematics using the math way of number naming.
- They understand place value in first grade; only half of U.S. children understand place value at the end of fourth grade.
- Mathematics is the science of patterns. The patterned math way of number naming greatly helps children learn number sense.

© Activities for Learning, Inc. 2002

Traditional Names

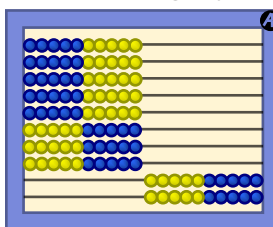
4-ten = forty

ten = ty



Traditional Names

8-ten = eighty



Traditional Names

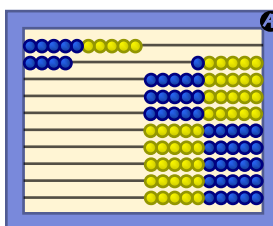
A word game

fireplace → place-fire
 newspaper → paper-news
 box-mail → mailbox

Traditional Names

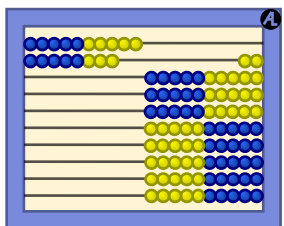
ten 4 → teen 4 → fourteen

ten = teen



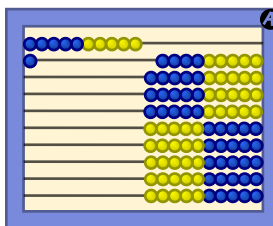
Traditional Names

ten 8 → teen 8 → eighteen



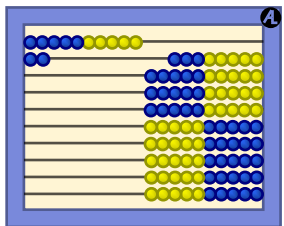
Traditional Names

a one left → a left-one → eleven



Traditional Names

two left → twelve



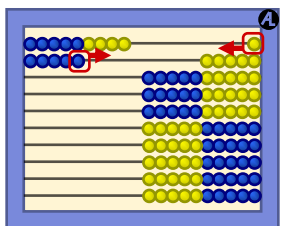
Strategies

A strategy is a way to learn a new fact or recall a forgotten fact.

A visual representation is a powerful strategy.

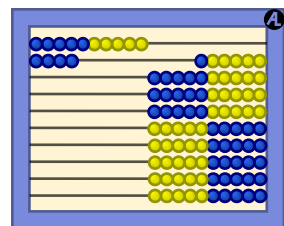
Strategy: Complete the Ten

$$9 + 5 = \underline{\quad}$$



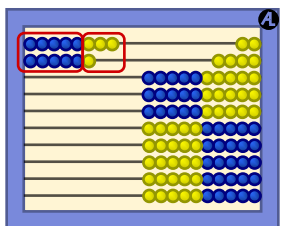
Strategy: Complete the Ten

$$9 + 5 = \underline{14}$$



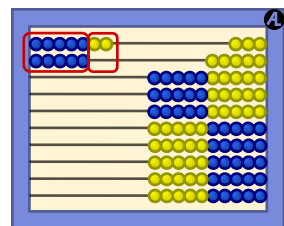
Strategy: Two Fives

$$8 + 6 = \underline{14}$$



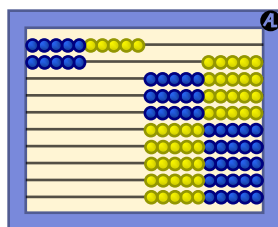
Strategy: Two Fives

$$7 + 5 = \underline{12}$$



Strategy: Part from Ten

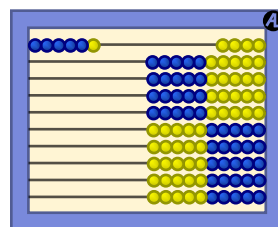
$$15 - 9 = \underline{\quad}$$



Subtract 5,
then 4

Strategy: Part from Ten

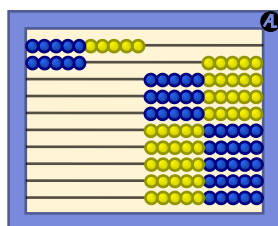
$$15 - 9 = \underline{6}$$



Subtract 5,
then 4

Strategy: All from Ten

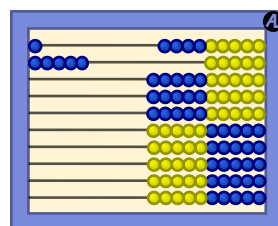
$$15 - 9 = \underline{\quad}$$



Subtract 9
from the 10

Strategy: All from Ten

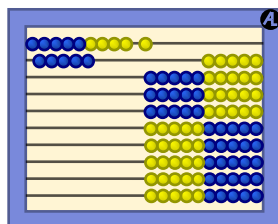
$$15 - 9 = \underline{6}$$



Subtract 9
from the 10

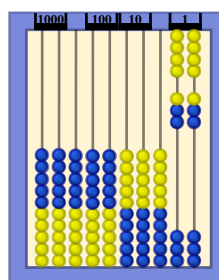
Strategy: Going Up

$$15 - 9 = \underline{6}$$



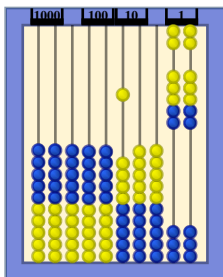
Start at 9;
go up to 15

AL Abacus: Side 2



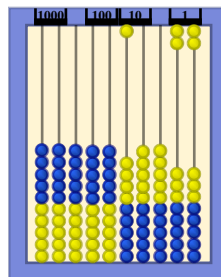
$$\begin{array}{r} 8 \\ + 6 \\ \hline \end{array}$$

AL Abacus: Side 2



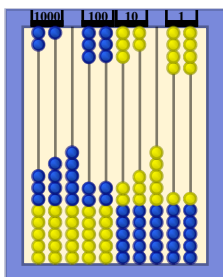
$$\begin{array}{r} 8 \\ + 6 \\ \hline 14 \end{array}$$

AL Abacus: Side 2



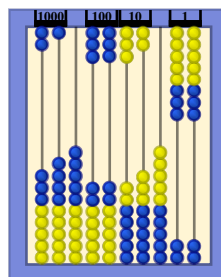
$$\begin{array}{r} 8 \\ + 6 \\ \hline 14 \end{array}$$

Adding 4-Digit Numbers



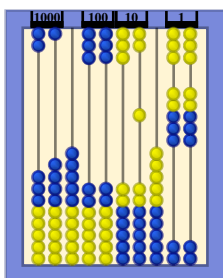
$$\begin{array}{r} 3658 \\ + 2738 \\ \hline \end{array}$$

Adding 4-Digit Numbers



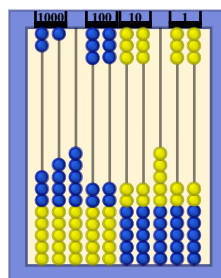
$$\begin{array}{r} 3658 \\ + 2738 \\ \hline \end{array}$$

Adding 4-Digit Numbers



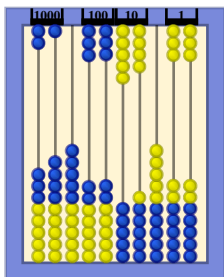
$$\begin{array}{r} 3658 \\ + 2738 \\ \hline \end{array}$$

Adding 4-Digit Numbers



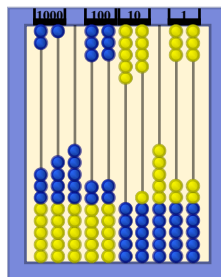
$$\begin{array}{r} 3658 \\ + 2738 \\ \hline 6 \end{array}$$

Adding 4-Digit Numbers



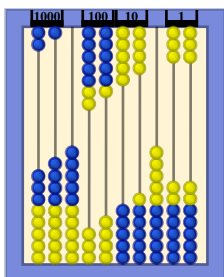
$$\begin{array}{r} \overset{1}{3}658 \\ + 2738 \\ \hline 6 \end{array}$$

Adding 4-Digit Numbers



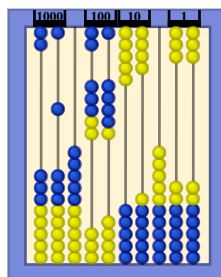
$$\begin{array}{r} \overset{1}{3}658 \\ + 2738 \\ \hline 96 \end{array}$$

Adding 4-Digit Numbers



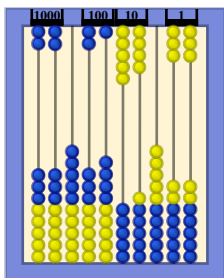
$$\begin{array}{r} \overset{1}{3}658 \\ + 2738 \\ \hline 96 \end{array}$$

Adding 4-Digit Numbers



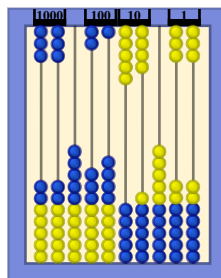
$$\begin{array}{r} \overset{1}{3}658 \\ + 2738 \\ \hline 96 \end{array}$$

Adding 4-Digit Numbers



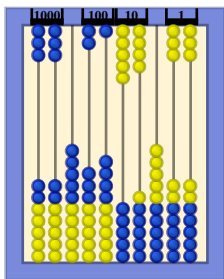
$$\begin{array}{r} \overset{1}{3}658 \\ + 2738 \\ \hline 396 \end{array}$$

Adding 4-Digit Numbers



$$\begin{array}{r} \overset{1}{3}658 \\ + 2738 \\ \hline 396 \end{array}$$

Adding 4-Digit Numbers



$$\begin{array}{r} ^1 ^1 \\ 3658 \\ + 2738 \\ \hline 6396 \end{array}$$

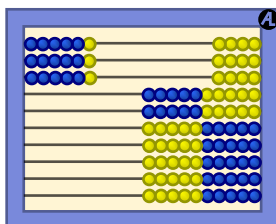
Adding 4-Digit Numbers

Most children who learn to add on the AL Abacus transition to the paper and pencil algorithm without further instruction.

$$\begin{array}{r} ^1 ^1 \\ 3658 \\ + 2738 \\ \hline 6396 \end{array}$$

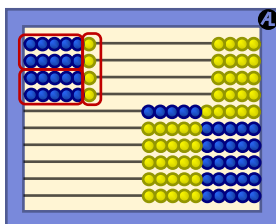
Multiplication

6 taken 3 times; 6×3



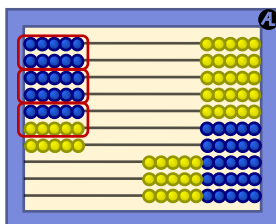
Multiplication

6 taken 4 times; 6×4



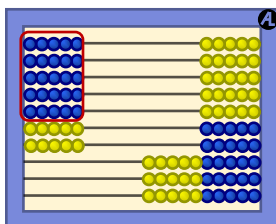
Multiplication

5×7



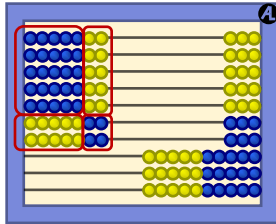
Multiplication

5×7



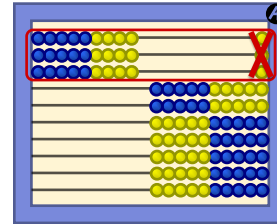
Multiplication

$$7 \times 7 = 49$$



Multiplication

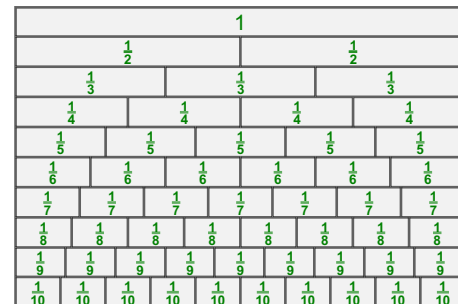
$$9 \times 3 = 30 - 3 = 27$$



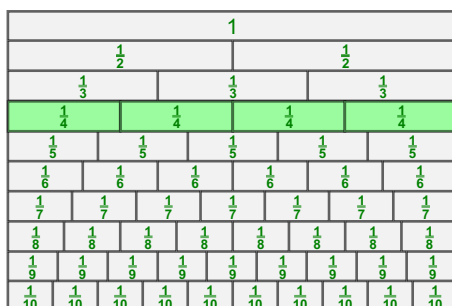
Ring Around the Product Game

2	6	3	5	1
8	54	12	42	7
5	50	15	9	8
9	1	6	8	3

Fraction Model: Linear Chart

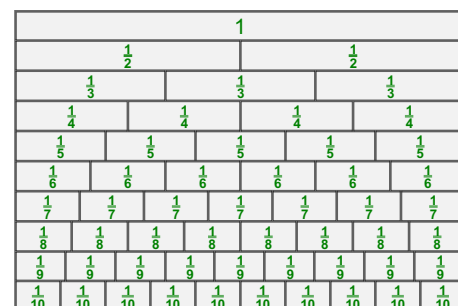


Fraction Model: Linear Chart



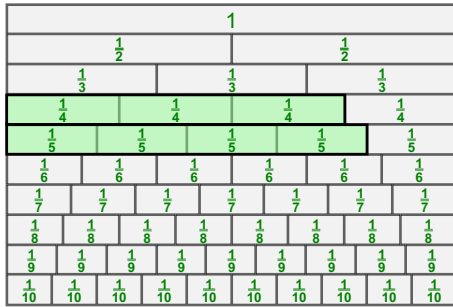
How many fourths in a whole?

Fraction Model: Linear Chart



How many fourths in a whole? How many fifths? Eighths?

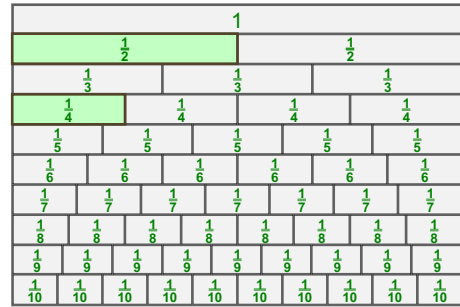
Fraction Chart



Which is more, $\frac{3}{4}$ or $\frac{4}{5}$?

© Activities for Learning, Inc. 2002

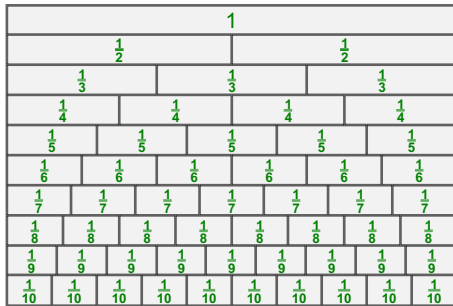
Fraction Chart



What is $\frac{1}{2}$ of $\frac{1}{2}$?

© Activities for Learning, Inc. 2002

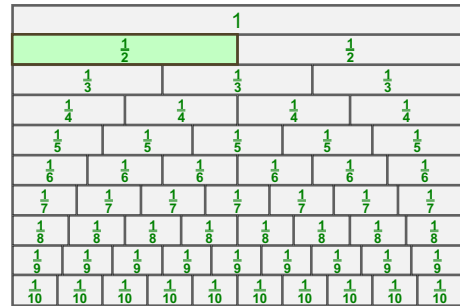
Fraction Chart



What is $\frac{1}{3}$ of $\frac{1}{2}$?

© Activities for Learning, Inc. 2002

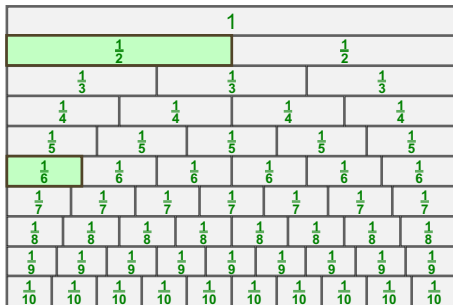
Fraction Chart



What is $\frac{1}{3}$ of $\frac{1}{2}$?

© Activities for Learning, Inc. 2002

Fraction Chart



What is $\frac{1}{3}$ of $\frac{1}{2}$? That's multiplying fractions!

© Activities for Learning, Inc. 2002

Effective Math Users

“...The now well established fact that those who are mathematically effective in daily life seldom make use ‘in their heads’ of the standard written methods which are taught in the classroom.”

W. H. Cockcroft, 1982
eminent mathematics educator in England

© Activities for Learning, Inc. 2002

[illegible]

RightStart™ Mathematics

- Uses the abacus to develop visualization.
- Teaches topics in different ways with different approaches.
- Fractions are presented in a linear format.
- Games are the practice and review.
- Uses over 20 different manipulatives.
- Arranged in levels rather than grades.

© All rights reserved. © 1999-2005 by S. Parkes

RightStart™ Mathematics

128

LESSON 110: INTRODUCING MULTIPLICATION AS ADDS

OBJECTIVES:

1. To introduce multiplication
2. To learn the first table
3. To understand the relationship between multiplication and addition

MATERIALS:

1. RightStart™ Student Book
2. RightStart™ Teacher Book
3. Worksheet 110, Introducing Multiplication as Adds

ACTIVITIES FOR TEACHING:

Warm-up: Ask the class: "Do you know how to count the children?" (Who was usually, seated, in the class?) Ask: "How many were usually only one year old?"

Ask: "What is a half dollar value half of a dollar? Is a half of a dollar . . .? Why is a quarter called a quarter? Is a quarter half of a half?"

Have the children add together the number of 1's in the first row below. 2 + 1 = 3. Continue with writing the operation: $2 + 1 = 3$.

Ask: "What is half of 12? Is there a half of 12?"

Beginning multiplication: Use the child's time to do the first 10 problems. Ask: "How many 1's are there in 10? How many 2's are there in 10? How many 5's are there in 10? How many 10's are there in 10?"

Ask the class to add, one at a time, the way the right hand side of the table below. Ask: "How many 1's are there in 10? Is the result 10 times 1?"

Ask the class to add, one at a time, the way the left hand side of the table below. Ask: "How many 1's are there in 10? Is the result 10 times 10?"

The operation is tables 1 times 10 and 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

10 times 10

EXPLANATIONS:

Introduce $1 \times 1 = 1$ as thought of a 1 group of one. Ask: "How many 1's are there in 1?"

Introduce $2 \times 1 = 2$ as thought of 2 groups of one. Ask: "How many 1's are there in 2?"

Introduce $3 \times 1 = 3$ as thought of 3 groups of one. Ask: "How many 1's are there in 3?"

Introduce $4 \times 1 = 4$ as thought of 4 groups of one. Ask: "How many 1's are there in 4?"

Introduce $5 \times 1 = 5$ as thought of 5 groups of one. Ask: "How many 1's are there in 5?"

Introduce $6 \times 1 = 6$ as thought of 6 groups of one. Ask: "How many 1's are there in 6?"

Introduce $7 \times 1 = 7$ as thought of 7 groups of one. Ask: "How many 1's are there in 7?"

Introduce $8 \times 1 = 8$ as thought of 8 groups of one. Ask: "How many 1's are there in 8?"

Introduce $9 \times 1 = 9$ as thought of 9 groups of one. Ask: "How many 1's are there in 9?"

ACTIVITIES FOR TEACHING:

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

Using 10's and 10's to find 10's

EXPLANATIONS:

Using 10's and 10's to find 10's

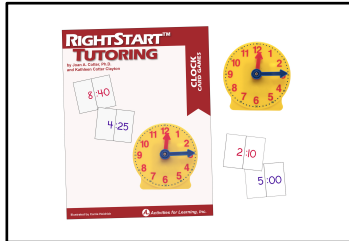
RightStart™ Tutoring

The image displays the RightStart Tutoring materials. On the left is the cover of the book "Activities for the AL Abacus: A Hands-On Approach to Arithmetic" by Joan A. Cutler Ph.D., published by A Activities for Learning, Inc. The cover features a circular diagram of abacus frames. To the right of the book are base ten blocks representing the number 30072, arranged in a spiral pattern. Below the blocks is a blue abacus with yellow beads, showing the number 30072.

RightStart™ Tutoring

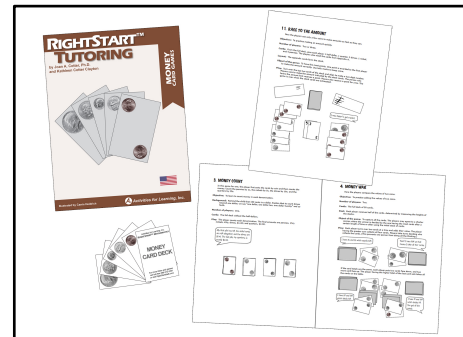
RightStart™ Tutoring

RightStart™ Tutoring



© Activities for Learning, Inc. 2002

RightStart™ Tutoring



© Activities for Learning, Inc. 2002

In Conclusion ...

Our goal as a teacher of mathematics is to help our children transform, expand, and refine these beginning ideas into deeper mathematical thinking.

— Dr. Joan A. Cotter

© Activities for Learning, Inc. 2002