RightStart™ Tutoring

by Kathleen Cotter Clayton
and Joan A. Cotter, Ph.D.

MULTIPLICATION AND DIVISION
BOOK ONE

Activities for Learning, Inc.

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INTRODUCTION

Welcome! This manual is about understanding multiplication and division and becoming confident with the facts. It is intended for those who have a weak or incomplete grasp of multiplication and division and who are two or more grade levels behind.

Multiplication has been the mathematical downfall of many students, not so much because of the algorithms, but because they are required to memorize 100 facts. Before expecting the student to learn the facts, we need to teach the meaning of multiplication. Learning it as repeated addition develops a limited view, which doesn't work well for multiplying fractions or decimals.

All too often, the multiplication facts are taught by rote, often an overwhelming task, especially when 6 and 3 was learned as 9, but now is 18. Some programs increase the burden by extending the number of facts to be learned from 10 × 10 to 12 × 12. The 11s and 12s are not basic facts and increase the amount of facts to be memorized by 44%. The 11s and 12s can easily be calculated as a sum of 10 times the factor plus 1 or 2 times the factor. For example, 12 × 3 is 10 × 3 plus 2 × 3, or 30 + 6 = 36.

Sometimes students learn songs or rhymes to memorize the facts. One drawback is that they often need to sing the song from the beginning until the desired fact is reached. A second drawback is the additional time the brain needs to transfer the information from the language section of the brain to the mathematics section.

Another faulty approach to learning the facts incorporates pictures, one for each fact. For instance, to remember 4 × 4, one image shows a 4-wheel drive truck with the caption that the driver needs to be 16 to drive it. The legal age to drive in North Dakota used to be 14. Does that mean 4 × 4 might be equal to 14? These types of pictures cause a delay in fact retrieval because unrelated pictures need to be translated into mathematical concepts.

What about using skip counting for teaching the facts? It seems to make sense to teach the facts through skip counting; however, students often resort to counting on their fingers to find the desired fact. They become fast counters but are slow in mastering the facts. This simply becomes another rote procedure.

Attempts have been made to solve these struggles by focusing on rote memorization without much comprehension. For many, the burden of memorization is overwhelming, never mind the frequent need to review. Students who have memorization without understanding struggle to apply their skills to new situations. This results in frustration, confusion, and an aversion to math.

On the contrary, we now know that a deep understanding of concepts removes anxiety, lessens the burden of memorizing, makes advanced math easier to grasp, and makes math more enjoyable.

It does not matter if the student is 12 or 112 years old; these lessons will approach multiplication and division with a new perspective that follows the RightStart Mathematics approach and philosophy.

There are a number of things in this manual that will be different from the traditional way multiplication and division are taught. Although they will be explained in greater detail during the appropriate lessons, here is a quick overview.

Counting versus Subitizing

From a very young age, children are taught to count before they begin their formal education. This counting process is the traditional approach for adding and subtracting, yet quickly becomes a problem, especially with multiplication and division.

Rather than relying on counting, we will have the student see quantities in groups of fives and tens. This allows for quantities to be quickly recognized, or subitized. It also allows the quantities to become visualizable, that is, to be seen mentally.

The primary tool used throughout this manual is the AL Abacus, which is grouped in fives and tens. Strategies will also incorporate and utilize grouping. With frequent and consistent use, the student will develop a mental image of the abacus and strategies, thereby removing the need for the physical manipulative.

An additional reason to use the abacus is that the area of the brain that controls the fingers is adjacent to the math area of the brain. The motion of the fingers while using the abacus will stimulate the surrounding areas of the brain.

If a child struggles or reverts to counting or skip counting, tell them to use their abacus. It will not become a crutch; rather, with repeated use, the child will develop a mental image of the abacus they can rely on.

Place Value

In many Asian languages, numbers are said as ten-1 for eleven, ten-2 for twelve, ten-3 for thirteen, and so on. The twenties are read as 2-ten 1, 2-ten 2, 2-ten 3, and the thirties are read as 3-ten 1, 3-ten 2, 3-ten 3, and so on up to 9-ten 9. This way of saying numbers makes place value readily understood, in other words, transparent. Therefore, transparent number naming, also called the math way of saying numbers, will be referenced briefly in these lessons.

In many European languages, including English, names for numbers from 11 to 99 are confusing. The words eleven, twelve, thirteen, and so on, do not help the child understand tens and ones, which is the foundation of place value. Many English-speaking children do not realize that 13 is 10 and 3 more ones. Without understanding place value, it is more challenging to work with larger numbers.

Lessons will identify how the math way of saying the numbers connects to the English way of saying the numbers. This makes place value clear and easy to use. Understanding place value makes the strategies for multiplication and division effective and powerful.

Older students will likely catch on to the pattern of transparent number names quickly. If they understand the two ways of saying the numbers, they can use both the traditional names and the transparent way of number naming during the lessons.

Math Card Games

Most students get overwhelmed with math worksheets. Students who are not understanding something will not benefit from more and more worksheets. Flashcards just reinforce what a student doesn't know. They can become another source of frustration and feelings of failure. Rather than worksheets or flashcards, games will be used in this manual.

These math card games will allow the student to learn and practice new skills. Games keep math time enjoyable. Emotions are stored along with what has been learned. If a student has an enjoyable time learning, then positive emotions will replace past negative emotions.

A game will be assigned in each lesson. Some are solitaire games, and some are for two or more players. Include other family members in the games. There is nothing more motivating than a student playing a game against their parent—and winning!

Instructions are given for each game. Adapt as necessary to fit the student and the situation. For example, turn the games into one-person games or modify them to fit more than one player. Please contact RightStart Math if you need ideas for modifying the games.

It is impossible to overplay the games. The games will hone skills and help the student become more confident and fluid in their thinking. The more games are played, the more the student learns. If a concept is not solid, play the game again. Also, playing previously-played games will allow the student to see their growth and master their facts.

Multiple Approaches

Multiple approaches will be presented to solve multiplication and division problems. These are not given to confuse a student; rather, they provide options. One strategy might become the student's favorite, but the next day's strategy could be even better. Multiple approaches give the student additional perspectives to expand their understanding.

If a strategy or approach does not resonate with you as the teacher, that does not mean it won't be important to the student. Follow the lessons because it may be critical to the student's understanding.

Frequently, the lesson will ask the student to say equations out loud. This reinforces the learning of the facts and enhances auditory memory. It will also minimize skip counting, which research now shows is just another variation of counting.

Summary

The lessons, activities, and games in this program are from the RightStart™ Mathematics curriculum and *Math Card Games, 5th edition*, both written by Dr. Joan A. Cotter. This manual can be used alongside any math program; knowledge of the RightStart™ Mathematics program is not required.

This manual will provide the teaching guide and make learning interesting with games and activities. If a student struggles, slow down the lesson and concentrate on the activities and games. Make sure they are using the AL Abacus.

In these 59 days of lessons, a solid foundation of multiplication and division will be laid while proceeding step by step to develop a clear understanding. There are no worksheets; rather, daily games will provide practice and review.

This book is Multiplication and Division, Book One. This manual covers multiplication up to 10 × 10 and division of numbers 100 or less, including remainders. RightStart Tutoring Multiplication and Division, Book Two, scheduled for release in Fall of 2023, will address multi-digit multiplication along with short division and traditional long division.

We believe that through these lessons and games, students will develop a renewed interest in and enjoyment of mathematics, thereby enriching their lives. We also hope many of them will become tomorrow's mathematicians, scientists, and engineers.

We want you and your students to have great success in learning and discovering multiplication and division. Let us know how this tutoring program benefits you and your students. Please share your experience and keep in touch!

Kathleen Cotter Clayton

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DAILY LESSONS

Needed Materials

Materials needed for the day's activities will be identified at the beginning of the lesson. Paper and pencil or a dry erase board and marker will often be needed. If an appendix page is needed, it will be listed and found in the back of the book. Some will need to be copied.

The AL Abacus will enable the student to build a mental model necessary for concept formation. Even if a student knows a fact, say 5 × 5, it is important they also see it physically on the abacus. This helps with basic number sense, as well as develops an understanding of the relationships between numbers and the operations that can modify them.

Manipulatives are not to be regarded as crutches, but rather as tools for learning. In practice, the student will refer to them less and less and finally not at all. Sometimes just the security of having them nearby helps, even if they are not used.

Activities

This section is the heart of each day's lesson. These are the instructions for teaching the lesson. The expected answers from the student are given in square brackets.

Research shows that the quiet time for math fact responses should be about three to five seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This time gives the student an opportunity to think, visualize, and solve a problem. Encourage the student to develop persistence and perseverance. Avoid giving hints or explanations too quickly. Students, and people in general, tend to stop thinking once they hear the answer.

Notes are included in the lessons to help the teacher understand why something is done or not done. These are not directed towards the student, but to provide additional information for the teacher.

Games

Daily games, not worksheets or flashcards, provide practice of the new skills. The games can be played as many times as necessary until proficiency takes place. They are as important to learning math as books are to reading. Reviewing previously-played games lets the student see their progress while reinforcing familiar concepts.

Worksheets

There are no worksheets for this tutoring manual. Practice will come from the games.

There will be situations where equations may be written out. Paper and pencil or a dry erase board and marker will be needed. Some students may struggle with using paper and pencil yet will find a dry erase board and marker smoother and easier to work with. Use the student's preferred medium. If you need or want to record work from a dry erase board, take a picture, then save it for your records.

There are some students who find the simple act of writing uncomfortable, painful, or just overwhelming. In this case, we recommend the teacher becomes the scribe, writing exactly what the student says, even if it's a wrong answer.

THE MATH GAMES

The games develop the players' math skills while they play. The players do not need to know their facts before playing. They will learn and practice their facts as they play, using the manipulatives for support. More importantly, the games give the players a reason to learn their facts.

Strategies provided in the daily lessons will give students confidence and independence. What is a simple step to someone who knows multiplication or division often takes additional steps for a struggling learner. The variety of games and activities will support the process. Often a concept can be learned in more than one way, resulting in several games for the same concept.

Do not be in a hurry to get to the next lesson and game. Frequently go back to games already learned; the student will often play them from a new perspective. Game Day lessons will provide this review, although additional game play is strongly encouraged. Ideally, additional math card games should be played outside of the lesson time.

Description of the Cards

To play the daily games, you need two decks of special cards, which are available from Activities for Learning, Inc. The descriptions are as follows:

Basic Number Cards

These 132 cards are numbered from 0 through 10. There are 12 of each number.

Multiplication Cards

Each card in the multiplication deck corresponds to a number in the multiplication table from 1 × 1 through 10 × 10. Thus, it has 100 cards. Some numbers, such as 1, are found only once and others, such as 6, are repeated four times.

Some find it helpful to have two multiplication decks; one complete deck and a second deck sorted into the appropriate envelopes.

Where to play

For many players, the preferred place to play the games is on the floor. Children are more comfortable on the floor, and the games seem more informal.

The player with learning challenges

Often, those with learning challenges find memorizing unrelated facts very difficult and paperwork tedious. These games eliminate both issues and give the student a new approach to practicing their facts. Work in a place free from overwhelming noise and visual distractions. Repeat the games many times. The best way to end a game is by saying, "Let's play it again."

DR. JOAN A. COTTER'S BACKGROUND

Dr. Joan A. Cotter's love of children and their ability to learn goes well with her love of math and her desire to make it understandable and create a successful experience for all people. Adults who teach the RightStart program have been known to exclaim how much it has helped them better understand mathematics.

Dr. Cotter's educational background includes a Bachelor's in Electrical Engineering, a Master's in Curriculum and Instruction, and a Ph.D. in Mathematics Education. Her research was on primary children learning mathematics, especially place value.

She also earned a Montessori diploma and taught children, ages 3 to 6 years, in her own Montessori school. She also taught middle-school mathematics and tutored special education students. Dr. Cotter wrote the RightStart™ Mathematics program for school and home educators.

An interesting fact that Dr. Cotter finds fascinating: researchers have recently found that when people discover beauty in math, their brains light up in the same regions as that of artists when they find beauty in art. Understanding math brings out the beauty of math.

Dr. Cotter continues to write and speak across the US and internationally. She lives in Minnesota where she continues to run the family business, Activities for Learning, Inc. Joan and her husband have three adult children and five grandchildren.

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book one

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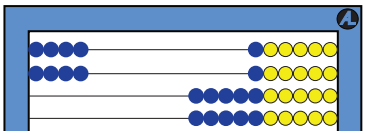
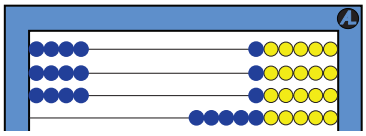
DAY 8 – Repeated Addition and Multiplication

Needed Materials. AL Abacus, paper and pencil or dry erase board and marker, Math Balance, and Basic Number card deck

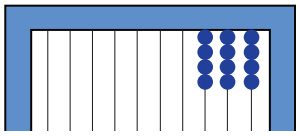
Creating an array on the abacus. Tell the student to enter 4 on the abacus. Then tell them to enter another 4 on the second row, as shown in the left figure below. Tell the student this array is read as 4 by 2. Ask: How many beads are in the array? [8]



4 by 2 array on the abacus.



4 by 3 array on the abacus.



3 by 4 array on the abacus.

Tell them to enter another 4 beads on the third row. See the middle figure above. Ask the student to read the array. [4 by 3]

*NOTE: Naming arrays with the word **by** will help the student connect arrays with multiplication and area.*

Say: There are several ways we can write an equation to show how many beads are in the array. Write the following for the student to see:

$$4 + 4 + 4 = \_\_\_\_\_$$

Ask: What is the total amount of beads? [12] How did you find the total? [adding] Write the sum.

Writing multiplication equations. Say: There is a simpler way to write this array. Rather than writing 4 by 3, let's use the multiplication sign for the word "by." Write for the student to see:

$$4 \text{ by } 3 \text{ is } 12 \quad \text{and} \quad 4 \times 3 = 12$$

Say: Now we have 4 *times* 3.

NOTE: Using arrays is a better model for multiplication rather than thinking of multiplication as repeated addition. Arrays are easier to visualize and can be used for multiplying fractions.

More equations. Tell the student to turn their abacus clockwise, as shown in the last figure above. Ask them to write the array name, the addition equation, and the multiplication equation.

3 by 4 is 12

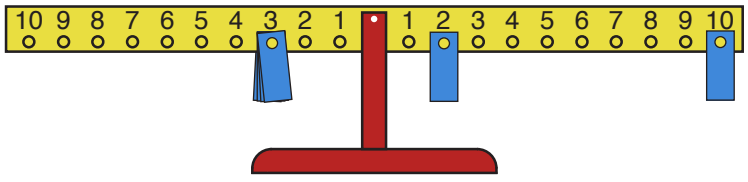
3 + 3 + 3 + 3 = 12

3 × 4 = 12

NOTE: Some students will need to have the three beads in the row pointed out to them. Once they see the three beads in the first row, they will recognize the four rows of the array.

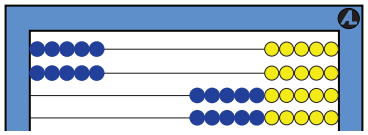
Ask: What is the total amount of beads? [12] Which equation is easier for you to find the total number of beads? [answers will vary]

Checking with the math balance. Tell the student they can put this equation on the math balance. Tell them to enter four weights on the left 3-peg. See the figure below. Ask: Using one weight on the 10, where do you need to put another weight to make it balance? [2] Is this the same answer you found on the abacus? [yes, 10 and 2]

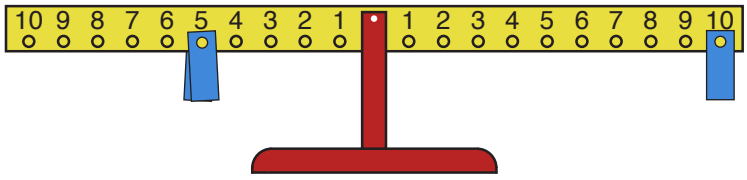


3 × 4 = 12

Creating another array. Tell the student to enter a 5 by 2 array on the abacus. See the first figure below. Then ask them to write the array name, the addition equation, and the multiplication equation. [5 by 2, 5 + 5 = 10, and 5 × 2 = 10]



5 by 2 array; 5 × 2



5 × 2 = 10

Finally, ask the student to check their solution on the math balance. See the second figure above.

NOTE: Sometimes 5 × 2 is thought of as "5 groups of 2." However, consistency with other arithmetic operations requires a second look. When adding 5 + 2, we start with 5 and transform it by adding 2. When subtracting 5 – 2, we start with 5 and transform it by removing 2. When dividing 5 ÷ 2, we start with 5 and transform it by dividing it into 2 groups or into groups of 2s. Likewise, 5 × 2 means we start with 5 and transform it by duplicating it 2 times.

If needed, repeat this process with arrays of 2 by 4 [8] and 6 by 3. [18]

Game 8 Short Sum Rummy with the Math Balance

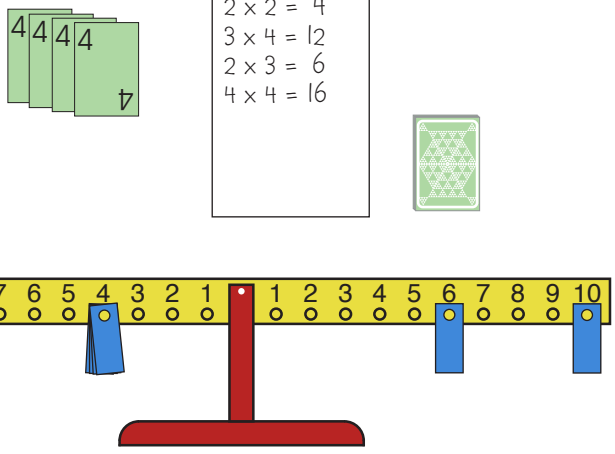
This game is similar to the previous day's game, with a few changes to the rules and an interesting inclusion of the math balance.

The objective of the game is to help players further understand multiplication as a shortcut to addition. Rather than forming groups of three or more identical cards, this game will allow for groups of **two** or more identical cards.

Two to four players will use the basic number cards with numbers 1 through 5; six of each for two players and nine of each for three or more players. Shuffle and deal five cards to each player. The remaining cards form the stock. Turn over the top card to start the discard pile.

Players take turns picking up a card or cards, laying down sets of cards if possible, and discarding one card. A player may either pick up the top card from the stock or take, in order, as many cards as desired from the discard pile.

In this version of the game, players check their hand for groups of two or more identical cards, which are placed face up on the table. Using multiplication equations, record the total of each set of cards. The equation is then displayed on the math balance. See the example shown.



Once a set of cards has been recorded, they are put aside. No additional cards may be added to that set, although a new group of the same number may be played, such as the 2s on the score sheet shown with 2 × 2 and 2 × 3.

A player concludes their turn by laying down a discard, overlapping the previous card so that all numbers can be seen. If a player runs out of cards, they may take five more from the stock. The game is over when the stock is exhausted, and all possible cards have been played. When done, pick up the cards in random order.

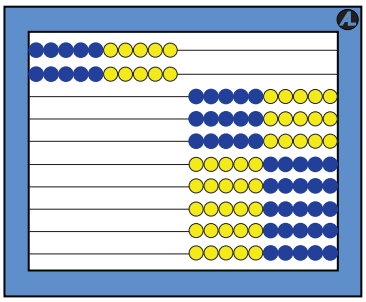
The winner is the player with the highest total score of all their sets.

DAY 18 – Multiples of Ten and Five

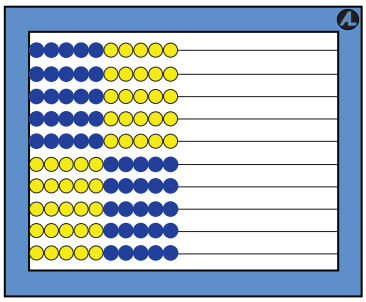
Needed Materials. AL Abacus, paper and pencil or dry erase board and marker, Clock and 55-card cut out (Appendix p. 4), and Multiplication cards in their envelopes

Review. Ask: What is 2 × 2? [4] What is 4 × 2? [8] What is 8 × 2? [16] What is 2 × 8? [16] What is 2 × 4? [8] What is 4 × 4? [16] What is 8 × 4? [32] What is 4 × 8? [32] What is 4 × 1? [4] What is 1 × 4? [4] What is 1 × 7? [7] What is 0 × 7? [0] What is 8 × 1? [8] What is 8 × 0? [0]

Multiplying by tens. Give the student the abacus and say: Now, we will multiply tens. Tell them to enter 10 one time on the abacus and say the equation. [10 × 1 = 10] Enter another 10 and say the equation. [10 × 2 = 20] See the first figure. Continue up to 10 × 10, [100] as shown in the second figure below.



10 × 2 = 20



10 × 10 = 100

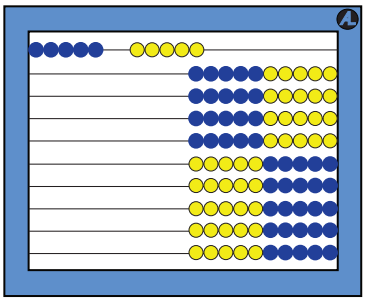
Tell the student to clear the abacus, then enter the tens again, writing each equation in two rows as shown below.

10 × 1 = 10	10 × 2 = 20	10 × 3 = 30	10 × 4 = 40	10 × 5 = 50
10 × 6 = 60	10 × 7 = 70	10 × 8 = 80	10 × 9 = 90	10 × 10 = 100

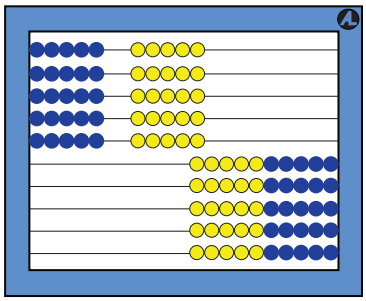
Then ask: What other multiples are similar to the tens multiples? [the ones] Look at 10 × 6 = 60; how do the multiplier, 6, and the product, 60, compare? [They are similar, but the product has a zero after the 6.] Say: Putting a zero after a number is called *annexing* a zero.

> *NOTE: To multiply 10 by a number (or to multiply by 10), we do not add a zero; rather, we annex a zero. Adding zero to a number does not change the number; 18 + 0 is still 18, whereas annexing a 0 onto 18 makes it 180.*

Multiplying by fives. Say: Now, we will work with the multiples of five. Enter 5 times 1 and say the equation. [5 × 1 = 5] Enter another 5 in the same row leaving a space between the groups. Say the equation. [5 × 2 = 10] See the first figure below.



5 × 2 = 10



Multiples of 5.

Tell them to continue entering fives while saying the equations. Stop after 5 × 10. See the second figure above.

Tell the student to write the products for multiplying five in two columns, as shown on the next page.

5 × 1 = 5	5 × 2 = 10
5 × 3 = 15	5 × 4 = 20
5 × 5 = 25	5 × 6 = 30
5 × 7 = 35	5 × 8 = 40
5 × 9 = 45	5 × 10 = 50

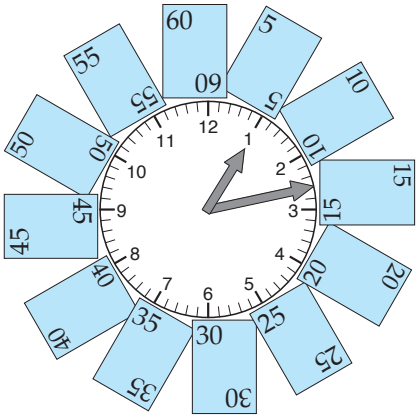
NOTE: Make sure the equations are written in two columns. This layout allows the pattern of the multiples to be visible, which will be addressed later.

Ask: How is five times a number related to ten times a number; 5 × 4 and 10 × 4, for example? [5 × 4 is 20 and 10 × 4 is 40, so 5 times a number is half 10 times that number.] Continue with other equations until they are confident with the relationship; 5 × 5 = 25 and 10 × 5 = 50, 5 × 8 = 40 and 10 × 8 = 80, 5 × 3 = 15 and 10 × 3 = 30, 5 × 7 = 35 and 10 × 7 = 70, 5 × 10 = 50 and 10 × 10 = 100, 5 × 9 = 45 and 10 × 9 = 90, and so on.

Minutes on a clock. Show them the clock from the appendix. Say: The shorter hand is the hour hand and indicates the hour. The longer hand is the minute hand and indicates the minutes. How many minutes are there between each number of the clock? [five]

*NOTE: If needed, discuss that each **space** between the lines is one minute, not the small lines between the numbers.*

Give them the cards from the 5s envelope, a 60-card (from the 10s envelope), and the 55-card cut from the appendix page. Tell the student to place the minute numbers around the clock, using the cards in hand. The finished clock is shown on the right.



Multiples of five around the clock.

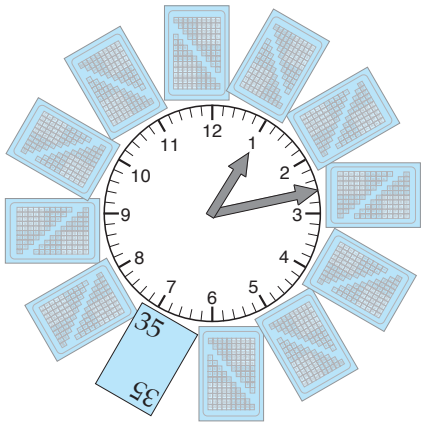
Tell the student to look at the hour numbers, starting at 1. Point to the 1 and ask: What is the hour number multiplied by five? [5] What is the minute number? [5]

Point to the 2 and ask: What is the hour number multiplied by five? [10] What is the minute number? [10] Continue around the clock, pointing to the hour number, asking what the hour number is when multiplied by five, then verifying the product with the card in place. Ask: How are the minute numbers related to the fives? [Five times an hour number equals the minute number.]

Mystery Minute

This game will help the players work on their multiplication facts of five while gaining confidence with the minutes around the clock. Cards from the 5s envelope, the 55-card (from the appendix page), and 60-card (from the 10s envelope) will be used.

The objective of the game is to focus on the minutes without counting by fives. Two players are needed, but three could play.



Shuffle the cards, then players turn over one card at a time and place it in the correct position around the clock.

When all 12 cards are in place, turn them face down. The first player names a random hour number, for example 7, and the second player calculates the number of minutes, 35, and turns over the card to check. If she names it correctly, it is her turn to call the hour, and the first player calculates and checks the minute. If she does not name the minute correctly, the first player receives another turn. Encourage the players to use the abacus as needed.

DAY 24 – The Multiplication Table

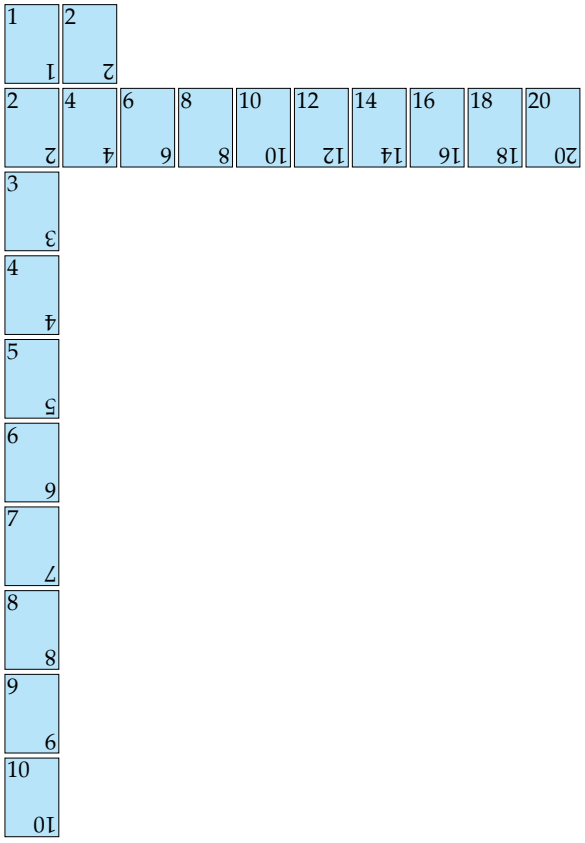
Needed Materials. Multiplication cards in their envelopes and Basic Number card deck

Building a table. Tell the student they will be building a table today, which will take up a large area. The full deck of 100 multiplication cards will be used.

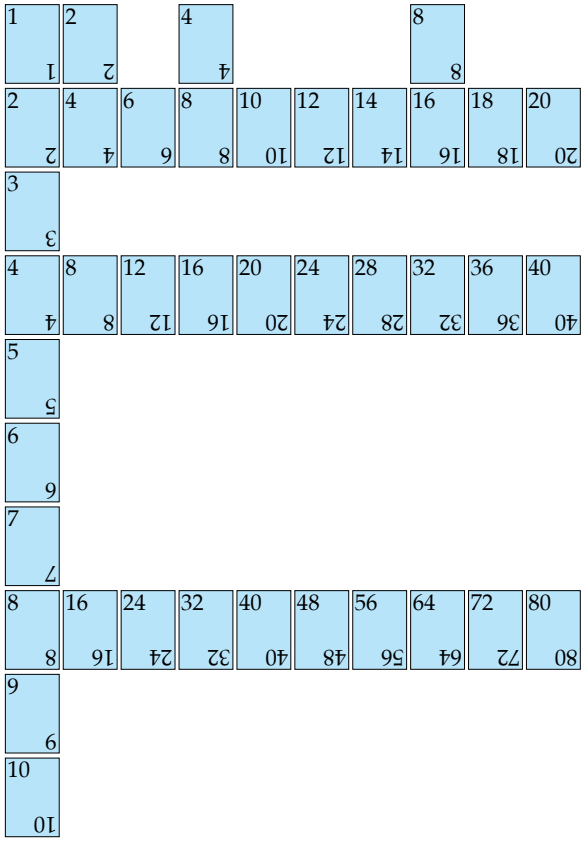
NOTE: The completed table will be 24 by 36 inches (60 by 90 cm), so a large table or floor space is needed.

Have the student take the cards out of the 1s envelope and lay them neatly in order in a **column** on the left side of the open area.

Next, tell them to take the cards from the 2s envelope and set aside the 2-card. Tell the student to lay the remaining nine cards in order in the second **row** while stating the equations. Then say: Lay the 2-card in the top row next to the 1-card. See the first figure below.



Multiples of ones and twos in place.



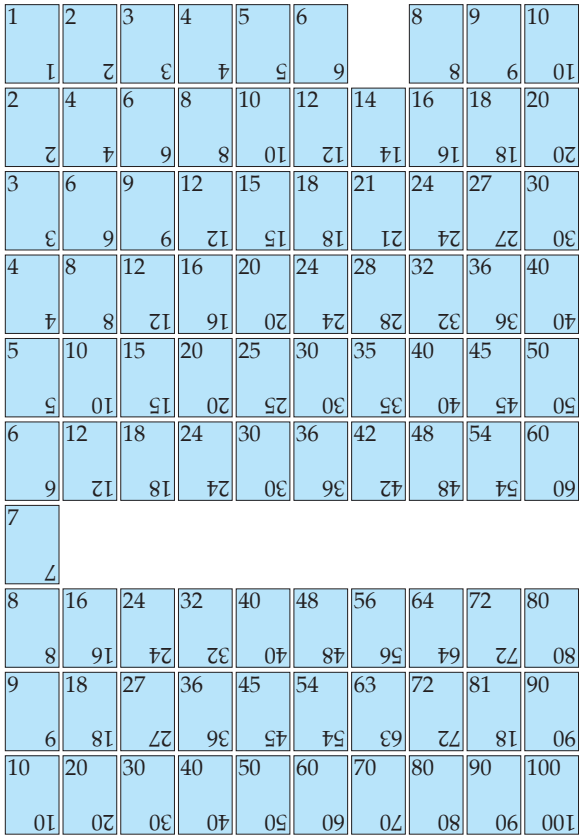
Multiples of ones, twos, fours, and eights in place.

Tell the student to continue building the table by finding the 4s envelope, setting aside the 4-card, then laying the cards in order in the fourth row while they state the equations. Put the 4-card on the top row, above the 8-card. Repeat with the 8s set, laying the 8-card above the 16-card. See the second figure above.

NOTE: The sets are laid out in the same order the multiples were taught in previous lessons.

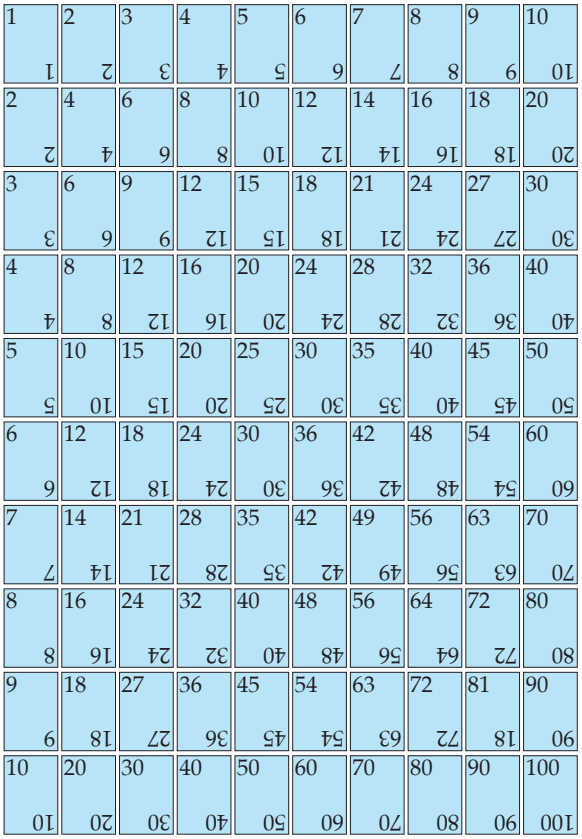
Continue, in order, with the multiples of 10, 9, 5, 3, and 6. Remind the student to state the equations as they lay down the cards. See the first figure on the next page.

With only the 7s row missing, ask the student to point to the second **row** and read the multiples of 2. [2, 4, 6, 8, 10, 12, 14, 16, 18, 20] Then ask the student to point to the second **column** and read the multiples of 2. [2, 4, 6, 8, 10, 12, missing card, 16, 18, 20] See the first figure on the next page.



1	2	3	4	5	6		8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7									
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

All the multiples except the sevens in place.



1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Completed table.

Give the student the 7s envelope and tell them to put the cards in order. Set aside the 7-card. Put the next card, the 14, in the missing spot in the 2s column.

Next, ask the student to read the multiples of 3 in the third **row**. [3, 6, 9, 12, 15, 18, 21, 24, 27, 30]. Then have them read the multiples of 3 in the third **column** and fill in the missing card from the stack of cards in their hand. [21] Continue with the 4s, 5s, and 6s. Skip the 7s and finish with the 8s, 9s, and 10s. Say: You have been working with all but one of the multiples of seven!

Tell the student to read the multiples of 7 in the seventh row. [7, 14, 21, 28, 35, 42, missing card, 56, 63, 70] Ask: What card do you think goes in the missing spot? [49] What card goes in the top row? [7] Tell them to place the cards, then read the multiples of 7 in the seventh column. [7, 14, 21, 28, 35, 42, 49, 56, 63, 70]

Tell the student that they just made their own multiplication table! It will be used in today's game.

Multiplication Wipe Out

Use the multiplication table already created and one each of the basic number cards 1 through 10. Shuffle the ten basic number cards and place them face down near the multiplication table.

The first player takes the 2s envelope, turns over a basic number card, multiplies that number by 2, then collects that number from the 2s **column**. The player continues turning over the basic number cards until all the multiples of 2 are collected from the column. Place the collected cards in the 2s envelope and set the envelope aside.

The next player takes the 4s envelope, reshuffles the basic number cards, then turns over the top card and collects that multiple from the 4s column. Continue taking turns collecting the columns in this order: 8s, 10s, 9s, 5s, 3s, 6s, 7s, and finally, the 1s. All cards will be picked up and put in their respective envelopes by the end of the game.

DAY 32 – Multiplication Problems

Needed Materials. Day 32 Problems (Appendix p. 5), paper and pencil or dry erase board and marker, AL Abacus, and Basic Number card deck

Review. Say: We have learned several ways to solve multiplication equations. Today, we are going to read and work through some situational problems using multiplication to find the answers.

Problem 1. Give the student the Day 32 Problems and read the first scenario:

> A small restaurant has 9 tables with each table having 4 chairs. How many chairs does the restaurant have?

Read the problem again. Tell them to see the room in their mind. Encourage the student to draw a picture if it helps. Ask: What are you looking for? [number of chairs] How can you find the number of chairs? [Multiply the number of tables by the number of chairs at each table, 9 × 4 = 36]

> *NOTE: Some students may need to read the word problem themselves to understand and retain the details. Additionally, some students will approach this as 4 × 9, which will produce the same product.*

Tell them to write the equation like this:

c = number of chairs
c = 9 × 4
c = 36 chairs

9 × 4 = 36

Finally, tell them to show the problem on their abacus as shown.

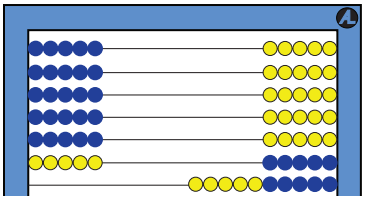
Problem 2. Read the second problem to the student:

> A park has an eating area with 5 picnic tables. Each table can seat 6 people. If every seat is taken, how many people can be seated at one time?

Read the problem again. Tell them to see the park area in their mind and draw a picture if needed. Ask: What are you looking for? [number of people who can be seated] How can you find the total number of seats? [Multiply the number of tables by the number of seats, 5 × 6 = 30]

Tell them to write the equation and show it on their abacus:

s = number of seats
s = 5 × 6
s = 30 seats



5 × 6 = 30

Problem 3. Read the next problem to the student, reading it twice:

> Quinn is paid $3 for walking a dog. How much money will Quinn receive altogether after he has walked 7 dogs?

Ask: What do we need to find? [amount of money] Give them several minutes to solve the problem independently before sharing their answer. [$21] The equation is:

d = dollars
d = 7 × 3
d = $21

Problem 4. Read the next situation:

> Carlos goes to bed every night at 10 o'clock and sleeps nine hours. How many hours does he sleep in a week?

Tell the student to solve this story problem. [63 hours of sleep] The equation is:

h = hours
h = 9 × 7
h = 63 hours

Ask: What did you need to know in order to solve this problem? [number of days in a week] Does the 10 o'clock time help you solve the problem? [no] Does your answer make sense? How much sleep do you get a week? [answers will vary]

Problem 5, optional. Read this optional problem to the student:

> Zoey is buying 4 gifts that cost $5 each. Aspen is buying 6 gifts that cost $3 each. Who is spending more money?

Read the story again and tell the student to solve it as before. [Zoey is spending more money.] The equations are:

Z = cost of Zoey's gifts
Z = 4 x 5 = $20

A = cost of Aspen's gifts
A = 6 x 3 = $18

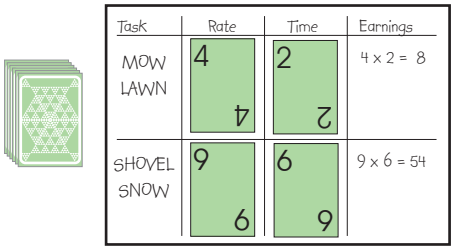
Zoey spent more than Aspen.

Would You Rather...?

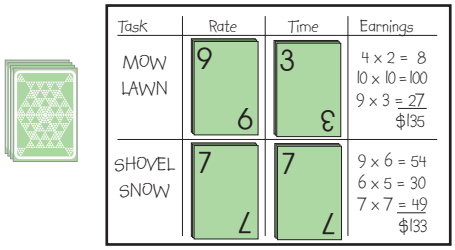
Today's game is a variation of the popular Would You Rather game. This game can be played with one to ten players. Each player will need a grid with two task options.

To create the grid, use a full sheet of paper or a dry erase board and turn it to the landscape view. Across the top, write "Task," "Rate," "Time," and "Earnings." In the first column under "Task," write two tasks. See the first figure below.

> *NOTE: Some task ideas are mowing the lawn, shoveling snow, babysitting, weeding the garden, cleaning the garage, or washing the car. Encourage the players to use tasks that are realistic to their lives, although silly tasks might be good for a second round of the game.*



Game starting with the first set of cards being played and calculated.



End of the game with three sets of cards played, calculated, and totaled.

Shuffle the basic number cards and place them face down to form the stock. From the stock, place two cards under "rate" and two cards under "time." The first column of cards represents the rate of pay and the second column represents the amount of time for the task.

To calculate the earnings, multiply the rate by the time and record the earnings. In the first example shown, lawn mowing pays $4 per hour. The job takes 2 hours. The recorded earnings of 4 × 2 is $8. Snow shoveling pays $9 per hour. Six hours were worked for a total of $54.

Repeat the process at least two more times, replacing the rate and time with new cards from the deck. When there are at least three calculations, total the amount earned and discuss which was the more profitable task.

> *NOTE: Although one task may earn more, some students might dislike one of the chosen tasks, which may create some interesting discussions.*

Play the game two more times, changing the tasks each time.

DAY 36 – Area on the Multiplication Table

Needed Materials. Multiplication table (Appendix p. 6), AL Abacus, two blank sheets of paper, and Multiplication card deck

Multiplication table. Say: A few lessons back, you made a multiplication table using the multiplication cards. Rather than getting out all 100 cards and making the table, we have a multiplication table for you to use.

Give the student the multiplication table. Let them take a moment to study it.

NOTE: Sometimes multiplication tables go to 12 by 12. However, the 11s and 12s are not basic facts. There is no reason to burden the student with memorizing 44% more facts. The 12s can be quickly calculated when 12 is understood as 10 plus 2. 12 × 9 is the same as 10 × 9 plus 2 × 9, which is 90 + 18 = 108.

Say: To find 2 × 4 on this table, move along the top row until you come to the 2, then go down 4 rows. See the figure on the right. Ask: What is 2 × 4? [8]

NOTE: Find the first factor across the top row, then the second factor by dropping down the column. This aligns with the abacus. Encourage the student to use their finger to track across the top row then down the columns.

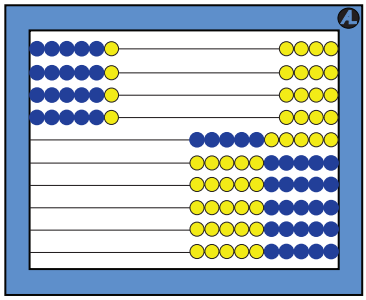
Some students may need to review rows and columns: rows have a horizontal alignment, and columns have a vertical alignment. Some remember columns as being up and down, like the columns of a building.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

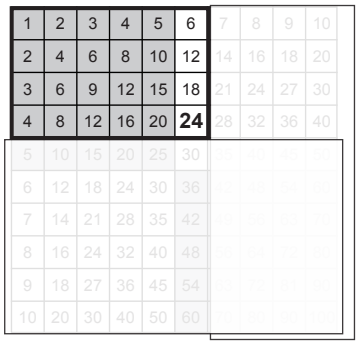
2 × 4 and 4 × 2 on the multiplication table

Ask: How could you find 4 × 2? [On the top row, go to the 4, then down 2 rows.] What is 4 × 2? [8] See the figure on the right.

Array area. Ask the student to enter 6 × 4 on the abacus. See the first figure below. Then ask: How could you find 6 × 4 on the multiplication table? [Find 6 on the top row, then go down to the fourth row, 24]



6 × 4 displayed on the abacus



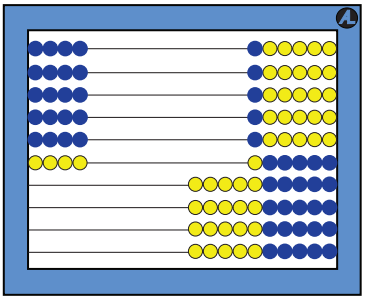
6 × 4 displayed on the table

Using the two blank sheets of paper, tell them to cover the columns and rows that are not needed. See the second figure above. Ask: What is 6 × 4? [24]

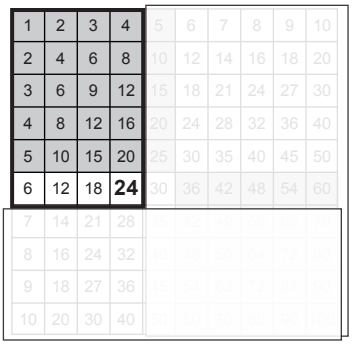
Ask: How many blue beads are entered on the abacus? [20] How many dark shaded squares are showing on the multiplication table? [20] How many yellow beads are entered on the abacus? [4] How many white squares are showing on the multiplication table? [4] So how many total squares are there in the 6 by 4 array on the table? [24]

NOTE: Many people are astonished to discover that the 6 × 4 array on the multiplication table contains 24 squares.

Now tell the student to enter 4 × 6 on the abacus. See the first figure below. Then ask: How could you find 4 × 6 on the multiplication table? [Find 4 on the top row, then go down to the sixth row.] Tell them to cover the columns and rows that are not needed with the two sheets of paper. See the second figure below.



4 × 6 displayed on the abacus



1	2	3	4
2	4	6	8
3	6	9	12
4	8	12	16
5	10	15	20
6	12	18	**24**

4 × 6 displayed on the table

Ask: What is 4 × 6? [24] Does 4 × 6 on the abacus look similar to 4 × 6 outlined on the multiplication table? [yes] How many total squares are there in the 4 by 6 array on the table? [24]

Ask: How could you tell it was the sixth row without counting by looking at the color of the squares on the table? [five are dark, and one more is light colored] How can you tell it was the sixth row using the numbers on the multiplication table? [by seeing the number 6 on the far left]

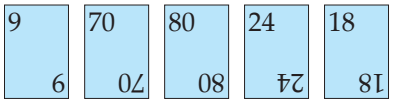
Ask them to find other products on both the abacus and the multiplication table, such as 7 × 8, [56] 8 × 7, [56] 6 × 6, [36] 9 × 7, [63] and 7 × 9. [63]

Stack the Row Solitaire

This game will provide practice in identifying multiples. Occasionally, a player may come to a point where it seems like there are no options, but with more evaluation, they may be winning again.

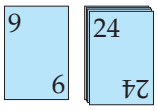
Use the multiplication table and any five multiplication card sets for a total of 50 cards. After thoroughly shuffling the cards together, lay five cards face up in a row.

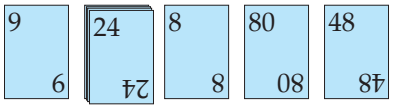
The basic rule for play is that a card may be stacked on another card if they belong to the same set. The object of the game is to have no more than two stacks in the row at the end of the game.



In the game shown on the right using the 6, 7, 8, 9, and 10 sets, the figures show four stages of the game. The first row has the five starting cards laid out. The 24 and 18 may be stacked because they belong to the 6s set. Whether the 24 is placed on 18 or the 18 on 24 is a point of strategy. Tell them to state the equations.

In the second stage of the game, as shown in the second row, 24 is placed on 18, and 24 could then be combined with 80 because 8 × 3 is 24 and 8 × 10 is 80. But first, 80 is placed over 70 because 10 × 7 is 70 and 10 × 8 is 80 and the 80 will be available for play.





In the third stage, the third row, 24 (including the card beneath it) is stacked on 80. Since there are no additional moves to make, fill in the empty spaces from the stock, shown in the fourth row, and proceed as before.

Whenever a stack of cards becomes too big, remove some of the cards below it and set them aside. Identical cards may be stacked together.

DAY 49 – Division Strategies on the AL Abacus

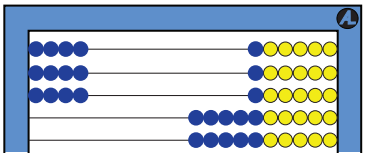
Needed Materials. Paper and pencil or dry erase board and marker, AL Abacus, Short Multiplication table (Appendix p. 8), Multiplication card deck, and Basic Number card deck

Multiplication and division. Ask: What is 2 × 3? [6] What is 6 ÷ 2? [3] What is 6 ÷ 3? [2] What is 5 × 4? [20] What is 20 ÷ 5? [4] What is 20 ÷ 4? [5] What is the inverse, or opposite, of multiplication? [division] What is the inverse of division? [multiplication]

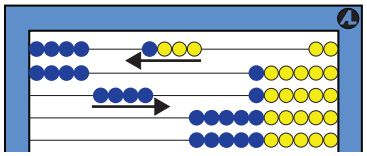
Finding the number of groups. Give the student the abacus.

Write: $4 \times 3 = \_\_\_\_$

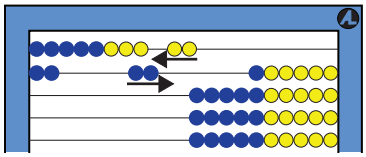
and tell them to show 4 × 3 on the abacus. See the first figure below.



4 × 3 = 12



Using Take and Give.



Using Take and Give; 4 × 3 = 12

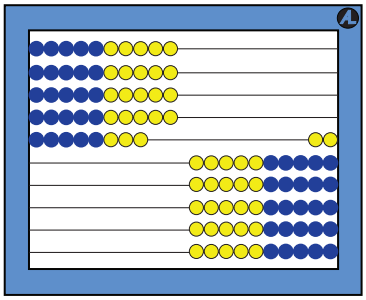
Discuss that the 4 in the written equation refers to the number of beads on a wire; it is the size of the group. Then discuss that the 3 in the equation is the number of groups.

Using the Take and Give strategy, tell the student to move the beads to find the product. [12] See the second and third figures above.

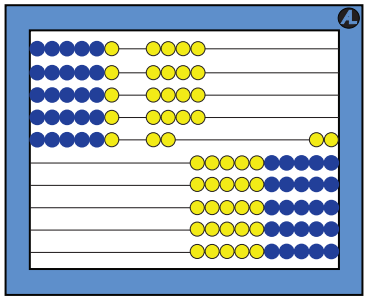
NOTE: The Take and Give strategy was introduced on Day 9.

Next, write: $6 \times n = 48$

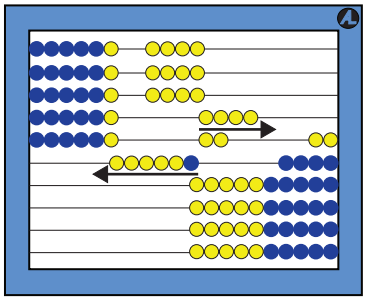
Tell them to enter 48 on their abacus and find *n*, the number of groups of 6s found in 48. Since 6 beads are needed on each wire, slide the amounts greater than 6 a short way away, as shown. Then use the Take and Give strategy to find the solution. [8 groups] See the figures below.



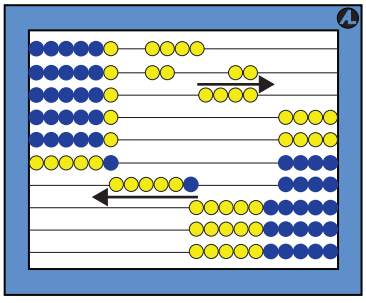
48 entered.



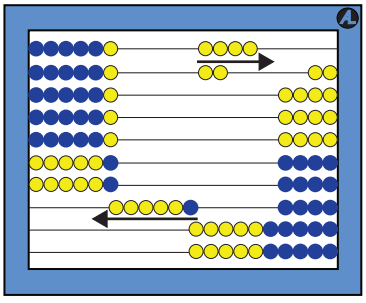
Forming groups of 6.



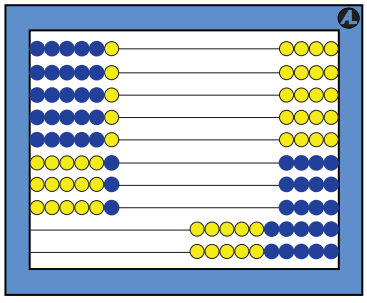
Using Take and Give.



Using Take and Give.



Using Take and Give.



6 × *n* = 48, *n* = 8

Ask: What is the size of the group? [6] How many groups are there? [8] Say: We can also write this as a division equation. Write: $48 \div 6 = n$

Ask: How many groups of 6s are in 48? [8 groups] What is *n*? [8]

The Short Multiplication table. Give the student the Short Multiplication table. Tell them to see if they can figure out how to find 48 ÷ 6 on the table. If needed, guide them to start at 6 and proceed until they reach 48, which is 8 cells. In the figure on the right, see the arrow starting at the 6, turning the corner, ending at 48.

Another method to find 48 ÷ 6 is to see that 48 is in the 8-row and is 6 cells in. See the second arrow starting at 48 and moving left.

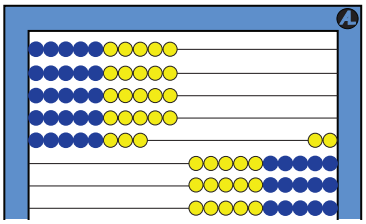
Tell them to find 72 ÷ 8 = *n* on the abacus and with the Short Multiplication table. [*n* = 9]

1									
2	4								
3	6	9							
4	8	12	16						
5	10	15	20	25					
6	12	18	24	30	36				
7	14	21	28	35	42	49			
8	16	24	32	40	48	56	64		
9	18	27	36	45	54	63	72	81	
10	20	30	40	50	60	70	80	90	100

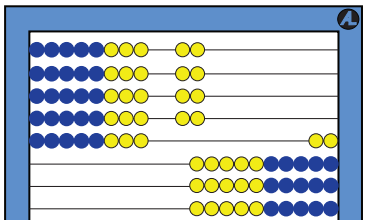
6 × *n* = 48 and *n* = 8

Finding the size of the group. Write: *s* × 6 = 48

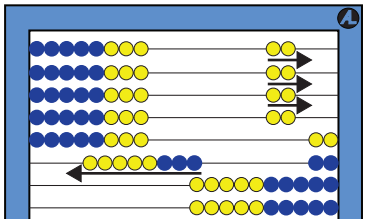
Tell them to enter 48 on their abacus again and find the **size** of the group, how many are in the group, when there are 6 groups in 48. See below.



48 entered.



Starting to form 6 equal groups.



Using Take and Give.

Then ask: What is the size of the group? [8] How many groups? [6] Say: The division equation is written the same as before: 48 ÷ 6 = *s*

Ask: What is *s*? [8] Tell them to find the equation on the Short Multiplication table.

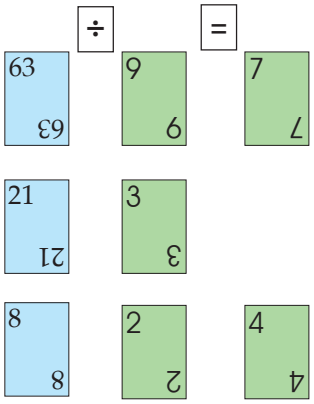
NOTE: These two division equations are written the same, 48 ÷ 6 = 8. As shown in the lesson, the 6 has two meanings: the number of groups or the size of the groups.

Find the Quotient—Level 3

This game is an advanced version of the Day 48 game. In this game, as many cards as possible may be played during a player's turn. The Short Multiplication table, AL Abacus, or Math Balance may be used as needed.

One or two players can play. About half of the multiplication cards and all the basic number cards, except the 0s, are needed. Shuffle each deck separately. Start **three** rows with one multiplication card in each row. The rest of the deck will be the stock. Each player takes six basic number cards, drawing replacement cards after their turn, keeping six cards in hand.

The object of the game is to collect the most cards by completing rows. The multiplication card divided by the first basic number card in the row equals the second basic number card in the row, the quotient.



The first player plays a card that is a factor of the multiplication card in any row and, if possible, the resulting quotient to complete the division equation. Up to six cards may be played in one turn.

The next player plays either the quotient or a factor and quotient of another row. Start a new row whenever fewer than three rows are available for play. A player unable to play starts a new row and his turn ends. The players continue to take turns until either stock runs out.

NOTE: The next game will need the cards in their envelopes.

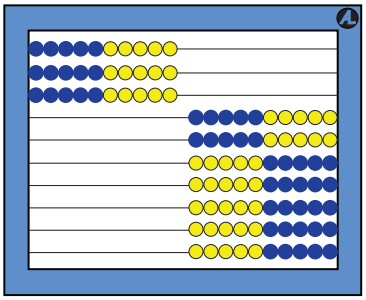
DAY 54 – Remainders on the AL Abacus

Needed Materials. Day 54 Problems (Appendix p. 12), AL Abacus, paper and pencil or dry erase board and marker, Short Multiplication table (Appendix p. 8), Multiplication card deck, and Basic Number card deck

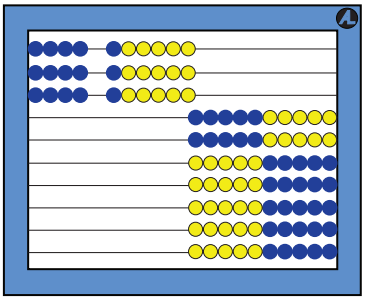
Remainders on the abacus. Give the student the Day 54 Problems and the abacus. Read the first situation, which is repeated from the previous lesson:

> A group of thirty people enters a restaurant. Only four people can sit around a table. How many tables does the group need?

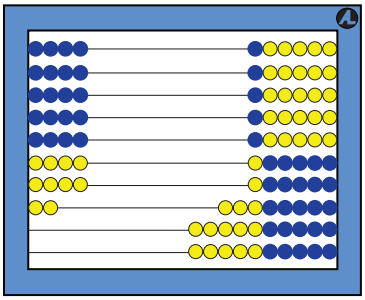
Tell them to solve the problem on the abacus. If needed, guide them by saying: Enter 30 on the abacus. Since only 4 people can sit at a table, slide the amounts greater than 4 a short distance away and use the Take and Give strategy to find the number of tables needed. [8] See the figures.



30 entered.



Amounts over 4 separated.



30 divided by 4 is 7 groups with 2 remaining.

Review with the student that the quotient for 30 divided by 4 is 7, and the 2 remaining beads is the remainder. Tell them to write it: 30 ÷ 4 = 7 r2

Ask: How many tables are needed for the 30 people? [8] Why are 7 tables not enough? [The remaining two people need to sit at a table.]

Remainders on the Short Multiplication table. Give the student the Short Multiplication table. Read the above scenario again. Tell them to use the Short Multiplication table to solve the problem.

If needed, guide them to start at row 4, then move over and down to the largest number that is not more than 30. [28] See the arrow in the figure on the right. Find the difference between 28 and 30 to calculate the remainder. [2] Tell them to write it again.

30 ÷ 4 = 7 r2

1									
2	4								
3	6	9							
4	8	12	16						
5	10	15	20	25					
6	12	18	24	30	36				
7	14	21	28	35	42	49			
8	16	24	32	40	48	56	64		
9	18	27	36	45	54	63	72	81	
10	20	30	40	50	60	70	80	90	100

30 ÷ 4 = 7 r2

Problem variation. Tell the student we are changing the problem:

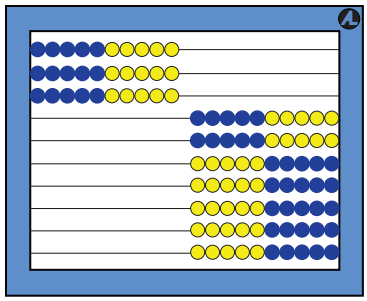
> The same group of thirty people is visiting another restaurant, but now **eight** people can sit around a table. How many tables do they need?

Tell them to solve the problem on the abacus or the Short Multiplication table. Ask them to write it:

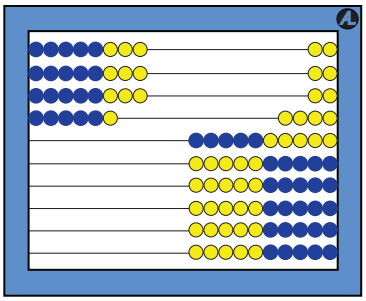
30 ÷ 8 = 3 r6
4 tables are needed

If needed, guide them to enter 30 on the abacus, then create 3 groups of 8 with a remainder of 6, as shown on the next page.

On the Short Multiplication table, go to row 8, then over to the largest number that is not more than 30. [24] The remainder is the difference between 24 and 30. See the third figure below.



30 entered.



30 divided by 8 is 3 groups with 6 remaining.

1									
2	4								
3	6	9							
4	8	12	16						
5	10	15	20	25					
6	12	18	24	30	36				
7	14	21	28	35	42	49			
8	~~16~~	24	32	40	48	56	64		
9	18	27	36	45	54	63	72	81	
10	20	30	40	50	60	70	80	90	100

30 ÷ 8 = 3 r6

Problem 2. Give the student the following situation:

> A box holds 6 two-pound bags of gecko food. Milo collected 53 bags of gecko food to donate to the pet shelter in Austin, Texas. How many boxes can be completely filled and delivered?

Tell them to solve the problem, using the abacus or the Short Multiplication table, and write the equation: 53 ÷ 6 = 8 r5

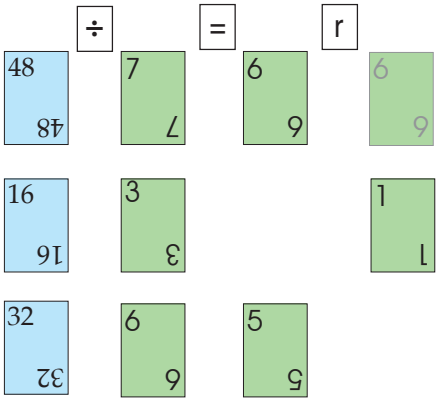
Ask: How many boxes can be donated? [8] Discuss with the students why only 8 boxes can be filled, not 9 boxes. [The remaining five bags do not completely fill a box.] Some students may want to discuss what to do with the remaining bags of gecko food.

Quotient and Remainder—Level Up

This game is a version of the previous game. In this game, all rows will include a remainder. The Short Multiplication table, AL Abacus, or Math Balance may be used as needed.

Two or three players can play. About half of the multiplication cards and all the basic number cards, except the 0s, will be used. Shuffle the decks separately. Start two rows with one multiplication card in each row. The rest of the deck will be the stock. Notes with "÷," "=," and "r" may be helpful.

To the right of each multiplication card, place a basic number card face up, making sure a remainder will be needed. Each player then takes six basic number cards, drawing a replacement card after their turn, keeping six cards in hand.



The object of the game is to collect the most multiplication cards by completing a row with the quotient and remainder.

The first player plays a card for either the quotient or remainder to any row. Only **one card** may be played in each turn. When a row is complete, the player keeps the multiplication card and sets the basic number cards aside.

Players take turns playing one card at a time, trying to complete a row. Start a new row whenever fewer than two rows are available for play. A player unable to play, starts a new row, and his turn ends. The players continue to take turns until either stock runs out, and no more play can be made.