

Fractions: From Frightening to Fantastic!

1									
$\frac{1}{2}$					$\frac{1}{2}$				
$\frac{1}{3}$			$\frac{1}{3}$			$\frac{1}{3}$			$\frac{1}{3}$
$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$	
$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$	
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

based on the work of Dr. Joan A. Cotter

Fractions

- Fractions generally have a bad reputation.

PEANUTS CHARLES SCHULZ



- Often viewed as incomprehensible and unpredictable.
- Sometimes perceived as scary!

© Activities for Learning, Inc. 2023

Fractions

- This is a huge misunderstanding.
- Fractions are necessary and useful.
- Fractions are amazing!



© Activities for Learning, Inc. 2023

Fraction History

- Latin “frangere” meaning “to break.”
- Considered only as part of a whole.
- Could never be equal or greater than 1.

© Activities for Learning, Inc. 2023

Fraction History

- In the 1600s, the concept of fractions expanded.
- Now included a division perspective.
- Fractions could be equal to or more than 1.



$$\frac{1}{3}$$

© Activities for Learning, Inc. 2023

Fraction History

- In the 1600s, the concept of fractions expanded.
- Now included a division perspective.
- Fractions could be equal to or more than 1.



$$\frac{2}{2}$$

© Activities for Learning, Inc. 2023

Fraction History

- In the 1600s, the concept of fractions expanded.
- Now included a division perspective.
- Fractions could be equal to or more than 1.



$$\frac{2}{3}$$

© Activities for Learning, Inc. 2023

Fraction History

- In the 1600s, the concept of fractions expanded.
- Now included a division perspective.
- Fractions could be equal to or more than 1.



$$\frac{2}{3}$$

© Activities for Learning, Inc. 2023

Fraction History

Old Fractions

Always less than one

$$\frac{1}{3}$$

Whole is fractured

Familiar = “proper”

Limited view

New Fractions

Can be equal or greater

$$\frac{1}{3} \quad \frac{3}{3} \quad \frac{4}{3}$$

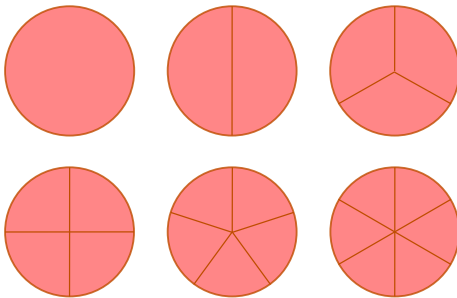
Viewed as division

Unfamiliar = “improper”

Supports understanding

© Activities for Learning, Inc. 2023

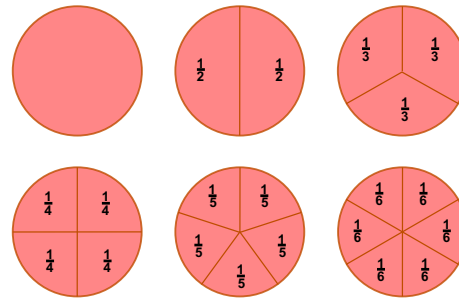
Fraction Model: Circles



Are we comparing angles, arcs, or area?

© Activities for Learning, Inc. 2023

Fraction Model: Circles



Try to compare $\frac{4}{5}$ and $\frac{5}{6}$ with this model.

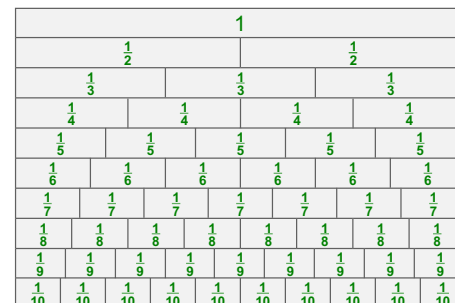
© Activities for Learning, Inc. 2023

Fraction Model: Circles

- Experts in visual literacy say that comparing quantities in pie charts is difficult because most people think linearly. It is easier to compare along a straight line than compare pie slices. askoxford.com
- Specialists also suggest refraining from using more than one pie chart for comparison. statcan.ca

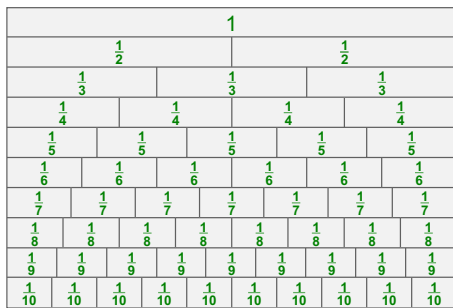
© Activities for Learning, Inc. 2023

Fraction Chart



© Activities for Learning, Inc. 2023

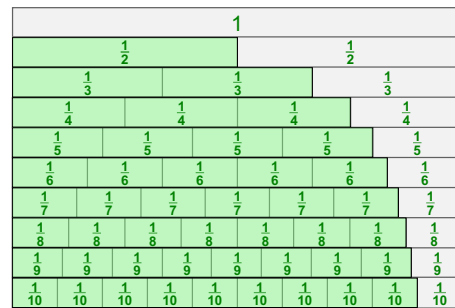
Fraction Chart



Ask the children to put it together like a puzzle.

© Activities for Learning, Inc. 2023

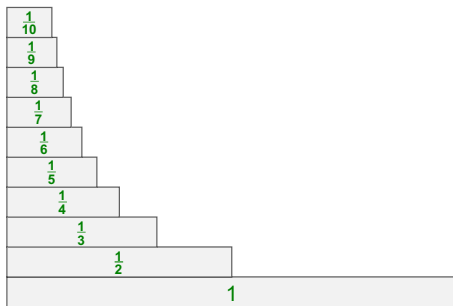
Fraction Chart



An interesting pattern.

© Activities for Learning, Inc. 2023

Fraction Stairs



© Activities for Learning, Inc. 2023

Games

$$\frac{\text{Games}}{\text{Math}} = \frac{\text{Books}}{\text{Reading}}$$

Games provide instant feedback.

Games provide interesting repetition needed for automatic responses in a social setting.

More importantly, games provide an application for the new information!

© Activities for Learning, Inc. 2023

Unit Fraction War

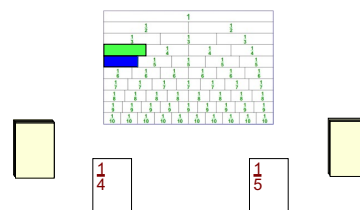
Purpose: Practice for naming and comparing unit fractions.

To help the children realize a unit fraction decreases as the denominator increases.

Goal: To collect all, or most, of the cards by comparing unit fractions.

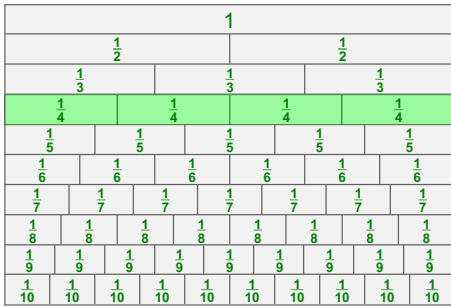
© Activities for Learning, Inc. 2023

Unit Fraction War



© Activities for Learning, Inc. 2023

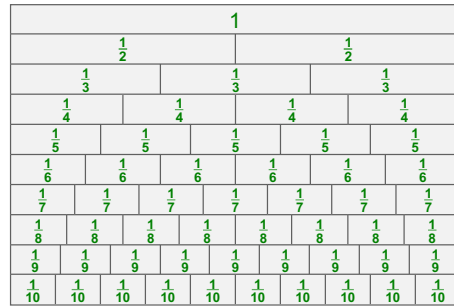
Fraction Chart



How many fourths in a whole?

© Activities for Learning, Inc. 2023

Fraction Chart



How many fourths in a whole? How many fifths? Eighths?

© Activities for Learning, Inc. 2023

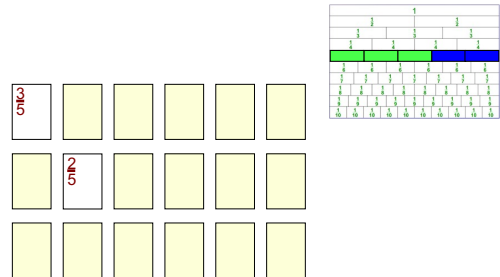
Concentrating on One

Purpose: To help the children realize that 5 fifths, 8 eighths, and so forth, make a whole.

Goal: To find the pairs that make a whole.

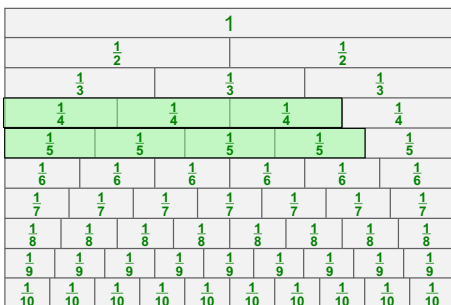
© Activities for Learning, Inc. 2023

Concentrating on One



© Activities for Learning, Inc. 2023

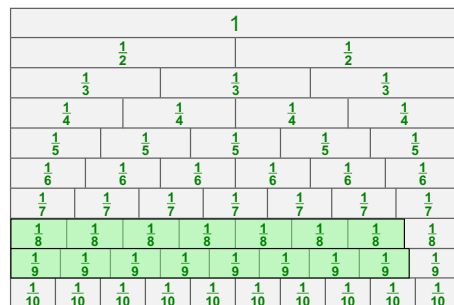
Fraction Chart



Which is more, $\frac{3}{4}$ or $\frac{4}{5}$?

© Activities for Learning, Inc. 2023

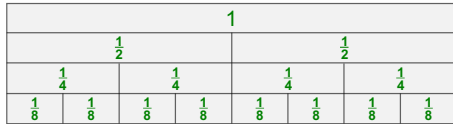
Fraction Chart



Which is more, $\frac{7}{8}$ or $\frac{8}{9}$?

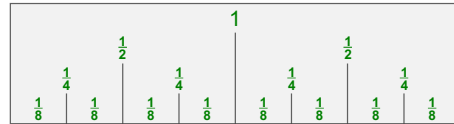
© Activities for Learning, Inc. 2023

Partial Chart



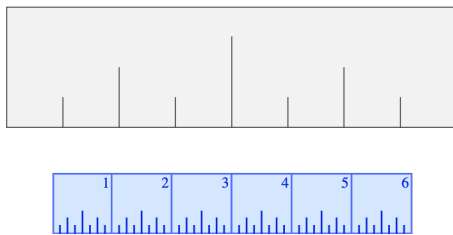
© Activities for Learning, Inc. 2023

Partial Chart



© Activities for Learning, Inc. 2023

Partial Chart



© Activities for Learning, Inc. 2023

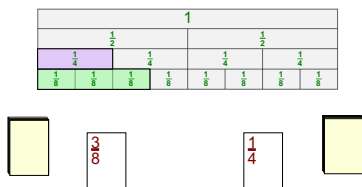
Fraction War

Purpose: To practice comparing ones, halves, fourths, and eighths in preparation for reading a ruler.

Goal: To capture all the cards.

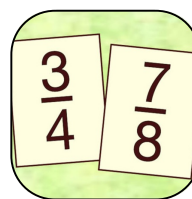
© Activities for Learning, Inc. 2023

Fraction War



© Activities for Learning, Inc. 2023

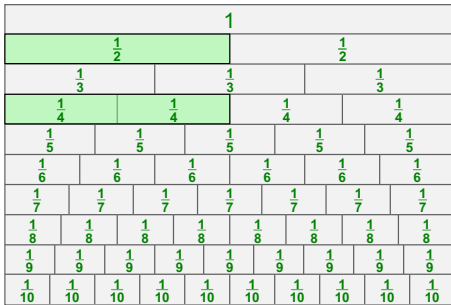
Game App



Fraction War		
$\frac{1}{2}$	Beginner	$\frac{7}{8}$
$\frac{3}{4}$	Easy	$\frac{1}{6}$
$\frac{4}{8}$	Medium	$\frac{5}{8}$
20%	Hard	$\frac{4}{4}$

© Activities for Learning, Inc. 2023

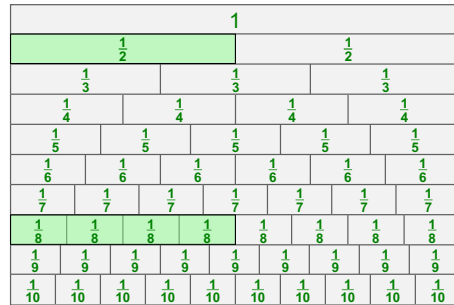
Equivalent Fractions



How many fourths equal a half?

© Activities for Learning, Inc. 2023

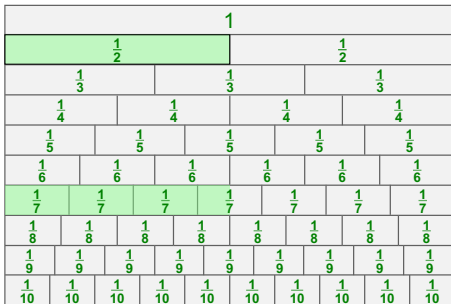
Equivalent Fractions



How many fourths equal a half? Eighths?

© Activities for Learning, Inc. 2023

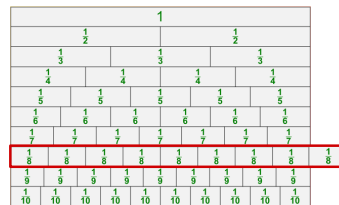
Equivalent Fractions



How many fourths equal a half? Eighths? Sevenths?

© Activities for Learning, Inc. 2023

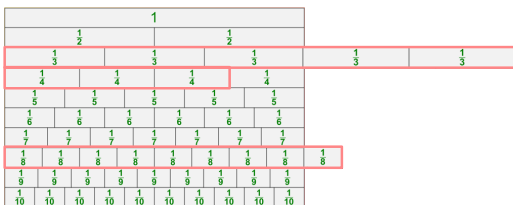
Proper and Improper Fractions



$\frac{6}{8}$

© Activities for Learning, Inc. 2023

Proper and Improper Fractions



$\frac{9}{8}$

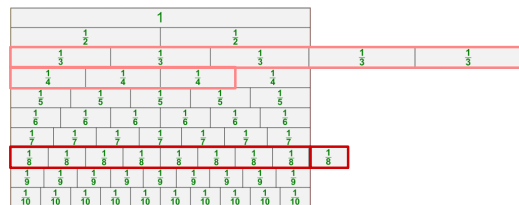
$\frac{5}{3}$

$\frac{3}{4}$

Which fraction(s) are improper?

© Activities for Learning, Inc. 2023

Proper and Improper Fractions



$\frac{9}{8} = 1\frac{1}{8}$

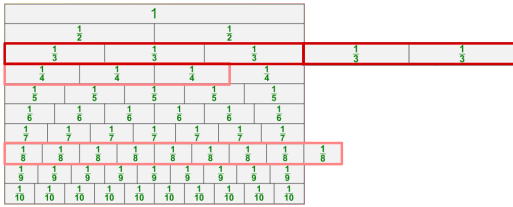
$\frac{5}{3}$

$\frac{3}{4}$

Rewrite the improper fractions using a whole number.

© Activities for Learning, Inc. 2023

Proper and Improper Fractions

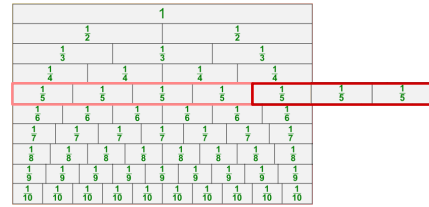


$$\frac{9}{8} = 1\frac{1}{8} \quad \frac{5}{3} = 1\frac{2}{3} \quad \frac{3}{4}$$

Rewrite the improper fractions using a whole number.

© Activities for Learning, Inc. 2023

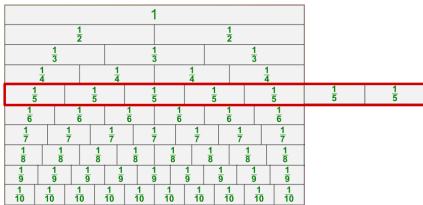
Proper and Improper Fractions



$$\frac{4}{5} + \frac{3}{5} =$$

© Activities for Learning, Inc. 2023

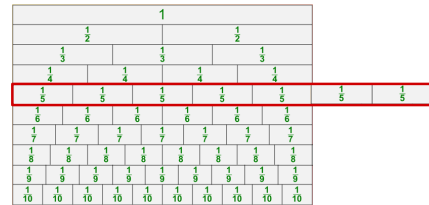
Proper and Improper Fractions



$$\frac{4}{5} + \frac{3}{5} = \frac{7}{5}$$

© Activities for Learning, Inc. 2023

Proper and Improper Fractions

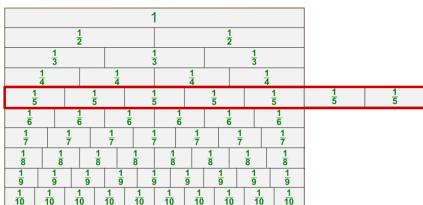


$$\frac{4}{5} + \frac{3}{5} = \frac{7}{5}$$

Rewrite the improper fraction using a whole number.

© Activities for Learning, Inc. 2023

Proper and Improper Fractions

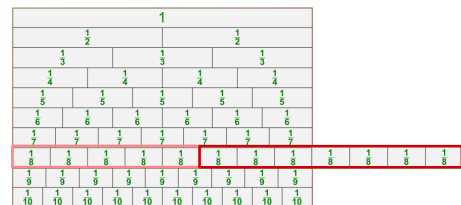


$$\frac{4}{5} + \frac{3}{5} = \frac{7}{5} = 1\frac{2}{5}$$

Rewrite the improper fraction using a whole number.

© Activities for Learning, Inc. 2023

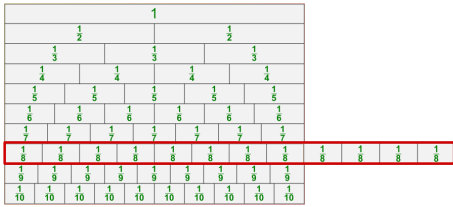
Proper and Improper Fractions



$$\frac{5}{8} + \frac{7}{8} =$$

© Activities for Learning, Inc. 2023

Proper and Improper Fractions



$$\frac{5}{8} + \frac{7}{8} = \frac{12}{8} = 1\frac{4}{8}$$

Rewrite the improper fraction using a whole number.

© Activities for Learning, Inc. 2023

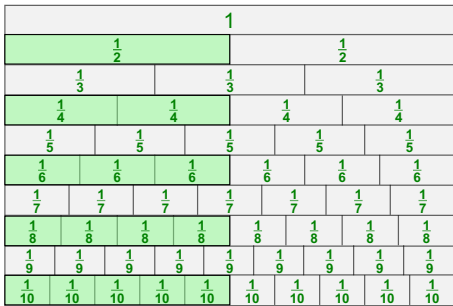
Help your child
discover the algorithm.

*“What you have been obliged to discover by yourself
leaves a path in your mind which you can use again
when the need arises.”*

– G.C. Lichtenberg
18th century physicist

© Activities for Learning, Inc. 2023

Simplifying Fractions



© Activities for Learning, Inc. 2023

Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

© Activities for Learning, Inc. 2023

Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

© Activities for Learning, Inc. 2023

Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

21
28

© Activities for Learning, Inc. 2023

Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$\frac{45}{72}$

© Activities for Learning, Inc. 2023

Simplifying Fractions

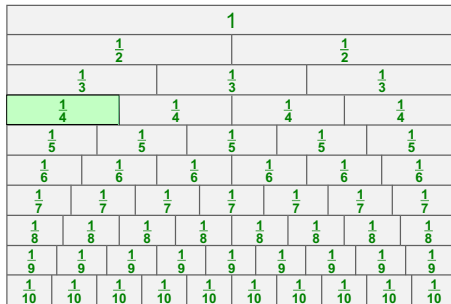
1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$\frac{45}{72}$

Why does this work?

© Activities for Learning, Inc. 2023

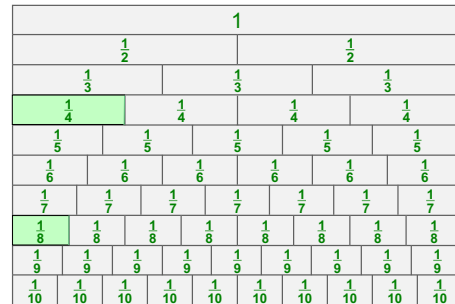
Fraction Chart



What is $\frac{1}{4} + \frac{1}{8}$?

© Activities for Learning, Inc. 2023

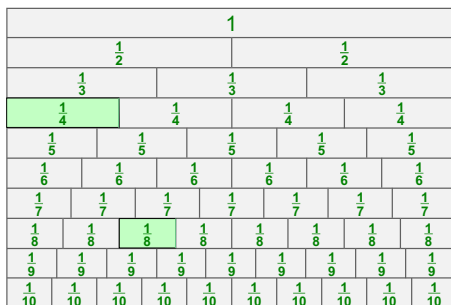
Fraction Chart



What is $\frac{1}{4} + \frac{1}{8}$?

© Activities for Learning, Inc. 2023

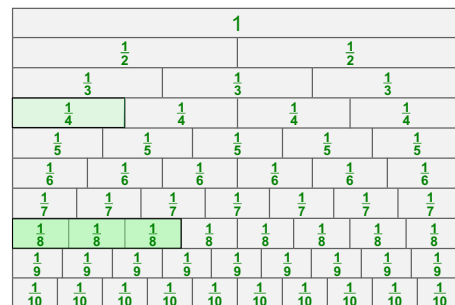
Fraction Chart



What is $\frac{1}{4} + \frac{1}{8}$?

© Activities for Learning, Inc. 2023

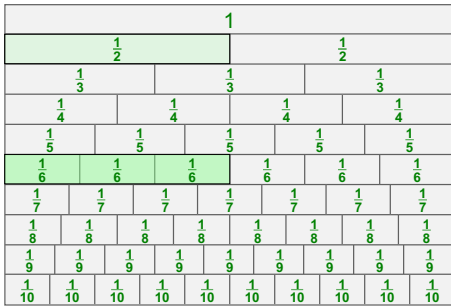
Fraction Chart



What is $\frac{1}{4} + \frac{1}{8}$?

© Activities for Learning, Inc. 2023

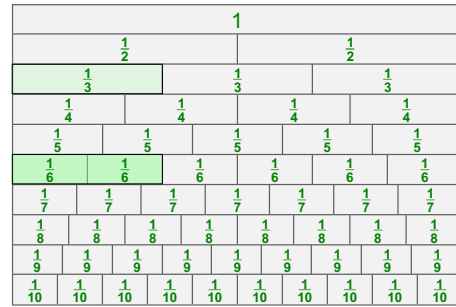
Fraction Chart



What is $\frac{1}{2} + \frac{1}{3}$?

© Activities for Learning, Inc. 2023

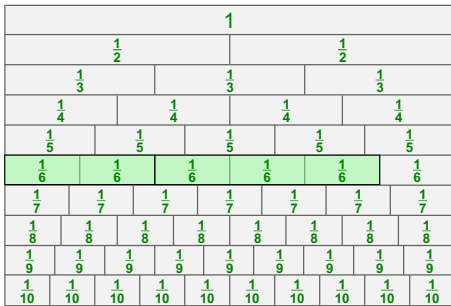
Fraction Chart



What is $\frac{1}{2} + \frac{1}{3}$?

© Activities for Learning, Inc. 2023

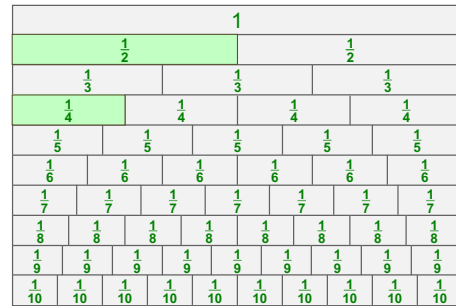
Fraction Chart



What is $\frac{1}{2} + \frac{1}{3}$? **$\frac{5}{6}$**

© Activities for Learning, Inc. 2023

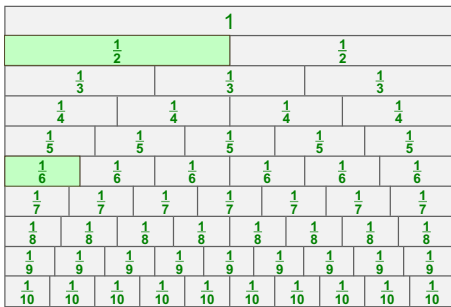
Fraction Chart



What is $\frac{1}{2}$ of $\frac{1}{2}$?

© Activities for Learning, Inc. 2023

Fraction Chart



What is $\frac{1}{3}$ of $\frac{1}{2}$? That's multiplying fractions!

© Activities for Learning, Inc. 2023

Multiplying Fractions

Multiplying is not exclusively repeated addition.

$$4 \times 4 = 4 + 4 + 4 + 4$$

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{2} + ?$$

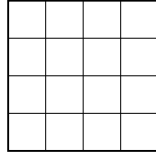
© Activities for Learning, Inc. 2023

Multiplying Fractions

Multiplying is not exclusively repeated addition.

Area is a better model.

$$4 \times 4 =$$

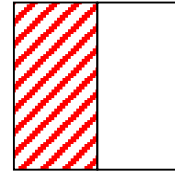


© Activities for Learning, Inc. 2023

Multiplying Fractions

$$\frac{1}{2} \times \frac{1}{2} =$$

One half
of one half

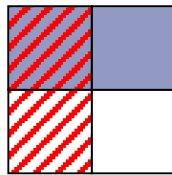


© Activities for Learning, Inc. 2023

Multiplying Fractions

$$\frac{1}{2} \times \frac{1}{2} =$$

One half
of one half

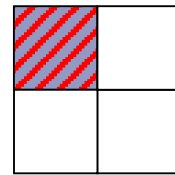


© Activities for Learning, Inc. 2023

Multiplying Fractions

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

One half
of one half

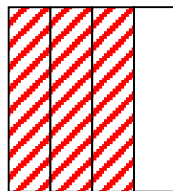


© Activities for Learning, Inc. 2023

Multiplying Fractions

$$\frac{2}{3} \times \frac{3}{4} =$$

Three fourths
of two thirds

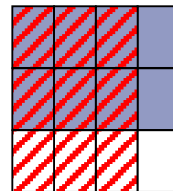


© Activities for Learning, Inc. 2023

Multiplying Fractions

$$\frac{2}{3} \times \frac{3}{4} =$$

Three fourths
of two thirds

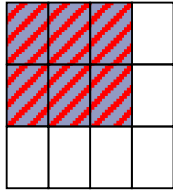


© Activities for Learning, Inc. 2023

Multiplying Fractions

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

Three fourths
of two thirds

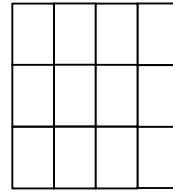


© Activities for Learning, Inc. 2023

Multiplying Fractions

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

The total number of rectangles is 3×4 .



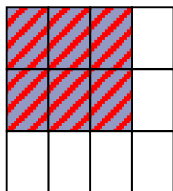
© Activities for Learning, Inc. 2023

Multiplying Fractions

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

The total number of rectangles is 3×4 .

The number of colored crosshatched
rectangles is 2×3 .



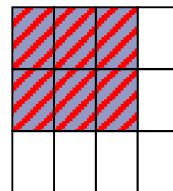
© Activities for Learning, Inc. 2023

Multiplying Fractions

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

The total number of rectangles is 3×4 .

The number of colored crosshatched
rectangles is 2×3 .

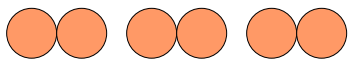


© Activities for Learning, Inc. 2023

What is Division?

$$6 \div 2 = \underline{\quad}$$

How many 2s in 6?



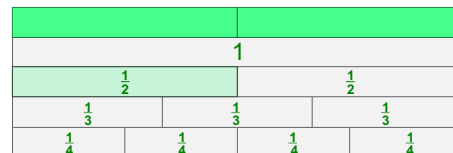
Number of groups of 2s; 3 groups

© Activities for Learning, Inc. 2023

Dividing Fractions

$$1 \div \frac{1}{2} = \underline{\quad}$$

How many $\frac{1}{2}$ s in 1?



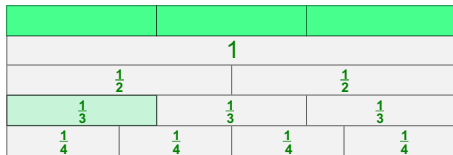
© Activities for Learning, Inc. 2023

Dividing Fractions

$$1 \div \frac{1}{2} = 2$$

$$1 \div \frac{1}{3} = 3$$

How many $\frac{1}{3}$ s in 1?



© Activities for Learning, Inc. 2023

Dividing Fractions

$$1 \div \frac{1}{2} = 2$$

$$1 \div \frac{1}{3} = 3$$

$$1 \div \frac{1}{4} = 4$$

$$1 \div \frac{1}{5} = 5$$

$$1 \div \frac{1}{6} = 6$$

© Activities for Learning, Inc. 2023

Dividing Fractions

$$1 \div \frac{1}{2} = 2$$

$$1 \div \frac{2}{3} = 1\frac{1}{2} = 1\frac{3}{2}$$

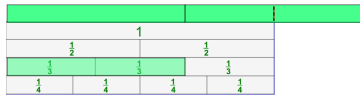
$$1 \div \frac{1}{3} = 3$$

How many $\frac{2}{3}$ s in 1?

$$1 \div \frac{1}{4} = 4$$

$$1 \div \frac{1}{5} = 5$$

$$1 \div \frac{1}{6} = 6$$



© Activities for Learning, Inc. 2023

Dividing Fractions

$$1 \div \frac{1}{2} = 2$$

$$1 \div \frac{2}{3} = \frac{3}{2}$$

$$1 \div \frac{1}{3} = 3$$

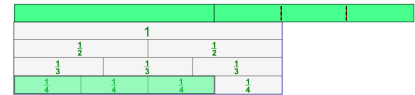
$$1 \div \frac{3}{4} = 1\frac{1}{3} = 1\frac{4}{3}$$

$$1 \div \frac{1}{4} = 4$$

How many $\frac{3}{4}$ s in 1?

$$1 \div \frac{1}{5} = 5$$

$$1 \div \frac{1}{6} = 6$$



© Activities for Learning, Inc. 2023

Dividing Fractions

$$1 \div \frac{1}{2} = 2$$

$$1 \div \frac{2}{3} = \frac{3}{2}$$

$$1 \div \frac{1}{3} = 3$$

$$1 \div \frac{3}{4} = \frac{4}{3}$$

$$1 \div \frac{1}{4} = 4$$

$$1 \div \frac{2}{5} = \frac{5}{2}$$

$$1 \div \frac{1}{5} = 5$$

$$1 \div \frac{5}{8} = \frac{8}{5}$$

$$1 \div \frac{1}{6} = 6$$

$$1 \div \frac{4}{7} = \frac{7}{4}$$

Guide the child to making the discovery that the answers are the inverted form of the divisor.

© Activities for Learning, Inc. 2023

Fraction Division War

Purpose: Practice in dividing fractions.
To help the children realize the quotient is the inverted form of the divisor.

Goal: To collect all, or most, of the cards by having the greatest quotient.

© Activities for Learning, Inc. 2023

Fraction Division War

© Activities for Learning, Inc. 2023

Fraction Division War

© Activities for Learning, Inc. 2023

Fraction Division War

© Activities for Learning, Inc. 2023

Fraction Division War

© Activities for Learning, Inc. 2023

Fraction Division War

© Activities for Learning, Inc. 2023

More Dividing Fractions

$$1 \div \frac{1}{4} = 4$$

© Activities for Learning, Inc. 2023

More Dividing Fractions

$$1 \div \frac{1}{4} = 4$$

$$2 \div \frac{1}{4} = 2 \times (1 + \frac{1}{4})$$
$$= 2 \times 4 = 8$$

© Activities for Learning, Inc. 2023

More Dividing Fractions

$$1 \div \frac{1}{4} = 4$$

$$2 \div \frac{1}{4} = 2 \times (1 + \frac{1}{4}) = 2 \times 4 = 8$$

© Activities for Learning, Inc. 2023

More Dividing Fractions

$$1 \div \frac{1}{4} = 4$$

$$2 \div \frac{1}{4} = 2 \times (1 + \frac{1}{4}) = 2 \times 4 = 8$$

$$3 \div \frac{1}{4} = 3 \times (1 + \frac{1}{4})$$
$$= 3 \times 4 = 12$$

© Activities for Learning, Inc. 2023

More Dividing Fractions

$$1 \div \frac{1}{4} = 4$$

$$2 \div \frac{1}{4} = 2 \times (1 + \frac{1}{4}) = 2 \times 4 = 8$$

$$3 \div \frac{1}{4} = 3 \times (1 + \frac{1}{4}) = 3 \times 4 = 12$$

© Activities for Learning, Inc. 2023

More Dividing Fractions

$$1 \div \frac{1}{4} = 4$$

$$2 \div \frac{1}{4} = 2 \times (1 + \frac{1}{4}) = 2 \times 4 = 8$$

$$3 \div \frac{1}{4} = 3 \times (1 + \frac{1}{4}) = 3 \times 4 = 12$$

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times (1 + \frac{1}{4})$$
$$= \frac{1}{2} \times 4 = 2$$

© Activities for Learning, Inc. 2023

More Dividing Fractions

$$1 \div \frac{1}{4} = 4$$

$$2 \div \frac{1}{4} = 2 \times (1 + \frac{1}{4}) = 2 \times 4 = 8$$

$$3 \div \frac{1}{4} = 3 \times (1 + \frac{1}{4}) = 3 \times 4 = 12$$

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times (1 + \frac{1}{4}) = \frac{1}{2} \times 4 = 2$$

$$\frac{1}{3} \div \frac{1}{4} = \frac{1}{3} \times (1 + \frac{1}{4}) = \frac{1}{3} \times 4 = \frac{4}{3}$$

$$\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times (1 + \frac{1}{4}) = \frac{3}{4} \times 4 = \frac{12}{4} = 3$$

© Activities for Learning, Inc. 2023

Fraction Chart

Allow the child to explore the whole picture and relationships within the whole using the linear perspective.

© Activities for Learning, Inc. 2023

In Conclusion ...

Math needs to be taught so
95 percent is understood and
only 5 percent memorized.

Richard Skemp
-- major pioneer in
mathematics education

© Activities for Learning, Inc. 2023

In Conclusion ...

Our goal as a teacher of mathematics
is to help our children transform,
expand, and refine these beginning ideas
into deeper mathematical thinking.

– Dr. Joan A. Cotter

© Activities for Learning, Inc. 2023