Learning Challenges in Math?
Challenge NO MORE!

based on work of Joan A. Cotter, Ph.D.
info@RightStartMath.com

Learning Challenges

- Approximately one in ten children has learning challenges.
- Traditional methods of teaching math are a source of frustration and failure.
- Rote memorization is nearly impossible.
- When memorization does occur, it doesn't stick very long.

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Rote Memorization

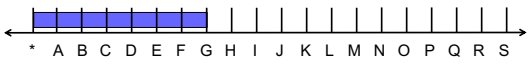
Because we're so familiar with 1, 2, 3, we'll use letters.

A = 1
B = 2
C = 3
D = 4
E = 5, and so forth

3

Rote Memorization

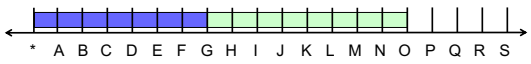
G + H = ____



4

Rote Memorization

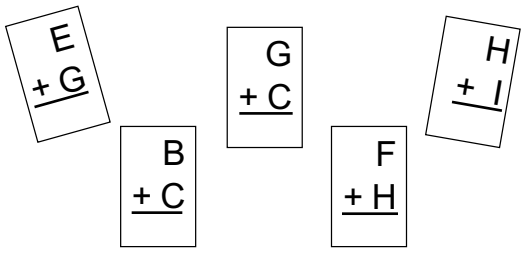
G + H = _O_



5

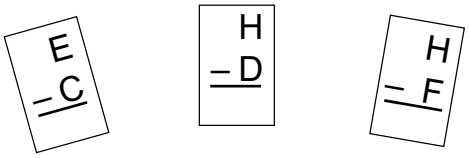
Rote Memorization

Now Memorize the Facts!!



6

Rote Memorization



Now try subtraction by "taking away."

7

Learning Challenges

- Often tempting to delay teaching math and concentrate on reading.
- Research shows that young children competent in math, excluding counting skills, will be more competent in reading in elementary grades.
- Math actually provides a foundation for reading!

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Learning Challenges

- Children learn better when they are active.
- They need to physically manipulate objects, not watch someone do it for them.
- Need to explore and learn.
- Manipulatives need to be visualizable.

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Learning Challenges

- Visual: seeing with our eyes.
- Visualizable: seeing in our mind's eye.
- Research shows that we learn best with visualizable images.
- For quantities to be visualizable, they must be grouped in fives and tens.

10

Subitizing

- Subitizing is the instant recognition of quantity.

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Subitizing

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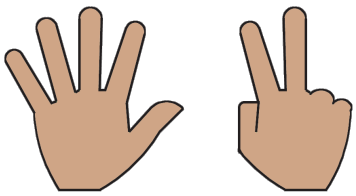

12

Subitizing

- Subitizing is the instant recognition of quantity.
- Subitizing can be extended when quantities are grouped in fives and tens.

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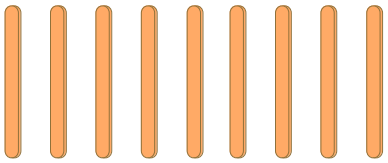
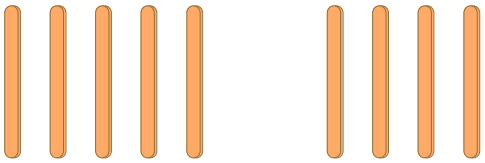
Grouping in 5s



How many?

14

Grouping in 5s



5 has a middle; 4 does not.

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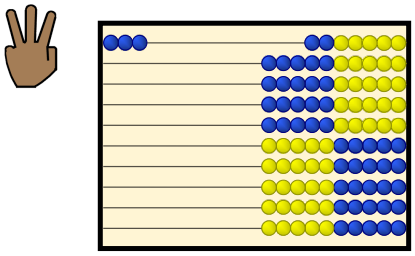
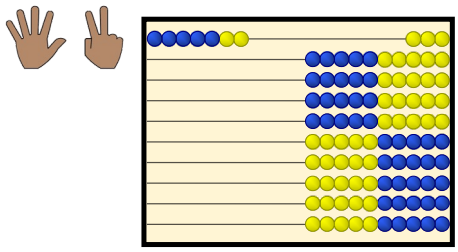
Visualizable Quantities



16

Visualizable Quantities



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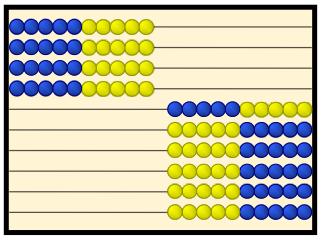
Visualizable Quantities

4-ten



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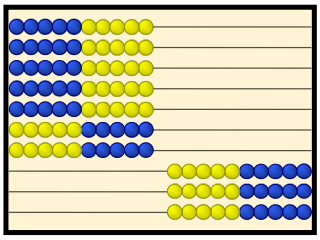
Visualizable Quantities

7-ten

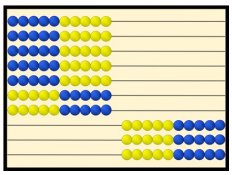


19

Learning Challenges

- Reduce the memory load.
  - No recital of the numbers 10 to 100.
  - Rather, use transparent number names.

7-ten



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Transparent Number Names

11 = ten 1	20 = 2-ten
12 = ten 2	21 = 2-ten 1
13 = ten 3	22 = 2-ten 2
14 = ten 4	23 = 2-ten 3
. . . .	. . . .
19 = ten 9	. . . .
	99 = 9-ten 9

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Transparent Number Names

- Just as reciting the alphabet doesn't teach reading, counting doesn't teach arithmetic.
- Just as we first teach the *sound* of the letters, we first teach the *name* of the quantity (math way).

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Transparent Number Names

- Only 11 words are needed to count to 100 the transparent number way; 28 in English.
- Asian languages use the math way of number naming.
- The children understand place value in first grade; only half of U.S. children understand place value at the end of fourth grade.

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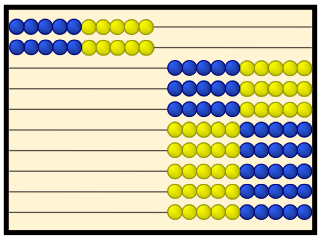
Transparent Place Value

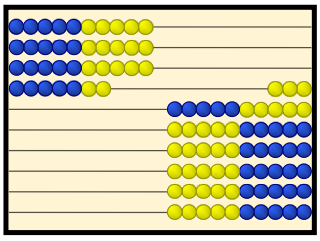
2-ten 2 0



24

Transparent Place Value

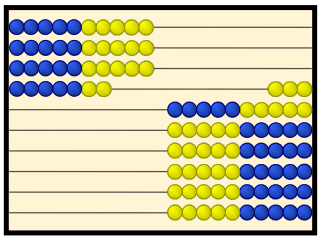
3-ten 7 3 0 7



25

Transparent Place Value

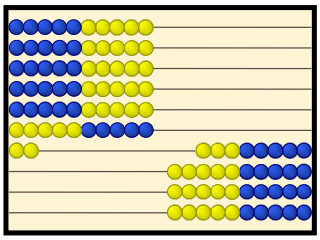
3-ten 7 3 7



26

Transparent Place Value

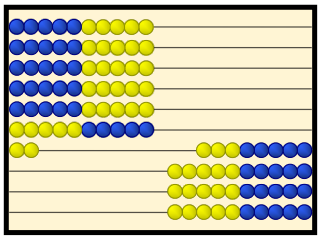
6-ten 2 6 0 2



27

Transparent Place Value

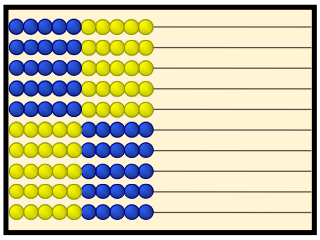
6-ten 2 6 2



28

Transparent Place Value

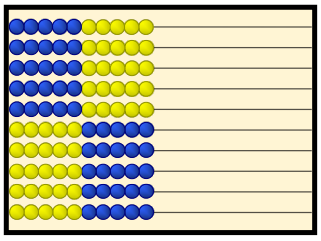
10-ten 1 0 0



29

Transparent Place Value

1 hundred 1 0 0



30

Learning Challenges

- Reduce the memory load.
  - No recital of the numbers 10 to 100.
  - Rather, use transparent number names.
    twenty-three is 2-ten 3
    seventy-four is 7-ten 4
  - Gives order and clarity to numbers.
  - Makes place value a natural part of numbers.

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Learning Algorithms

- Mathematics is much more than a hodgepodge of algorithms and formulas.
- Use has changed greatly over the centuries.
- Used to add long columns of numbers and multiply large numbers together.
- Now this process is frequently handled by calculators and computers.
- Need to learn estimating skills and simple mental calculations.

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Learning Algorithms

- Teach concepts before procedures.
- Research shows what is understood is retained much longer and is more likely to be applied to other situations.

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Learning Algorithms

- A fact is considered to be known if it can be recalled in two or three seconds.
- Gives time to visualize, then produce the fact.
- Visual strategies help learn the facts.

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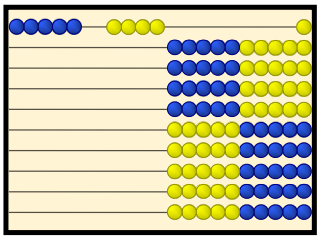
Adding

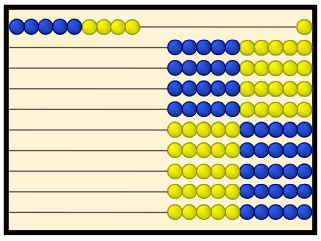
5 + 4 = ___



35

Adding

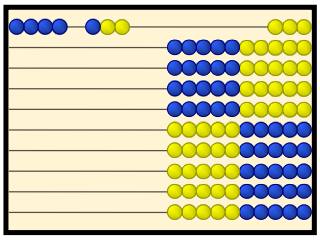
5 + 4 = 9



36

Adding

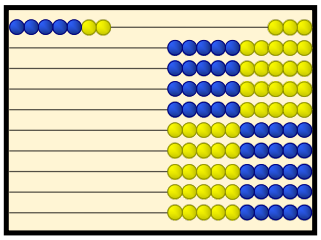
4 + 3 = ___



37

Adding

4 + 3 = _7_



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Strategies

- A strategy is a way to learn a new fact or recall a forgotten fact.
- A visual representation is a powerful strategy.

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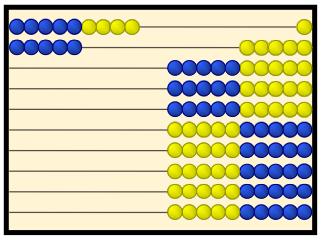
Strategy: Complete the Ten

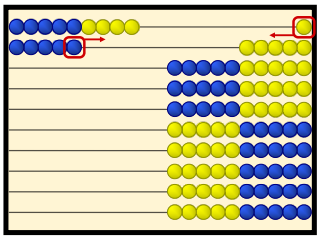
9 + 5 = ___



40

Strategy: Complete the Ten

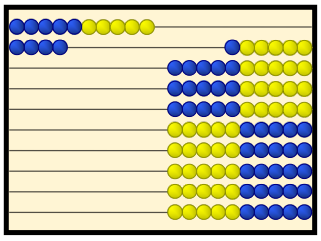
9 + 5 = ___



41

Strategy: Complete the Ten

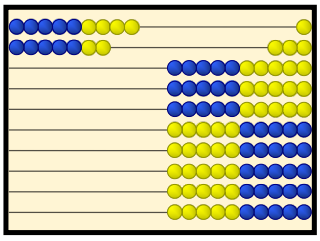
9 + 5 = _14_



42

Strategy: Complete the Ten

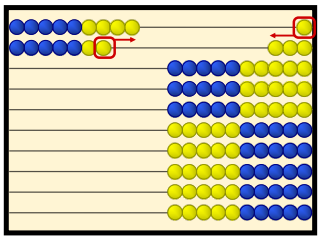
9 + 7 = ___



43

Strategy: Complete the Ten

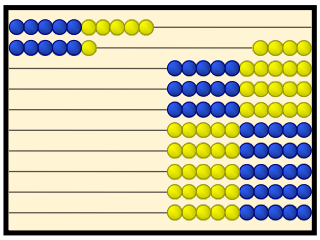
$9 + 7 =$ ___



44

Strategy: Complete the Ten

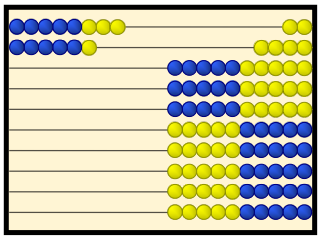
$9 + 7 =$ 16



45

Strategy: Two Fives

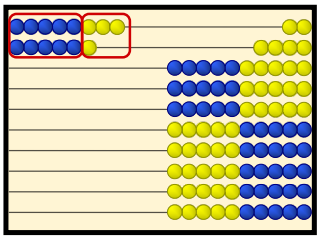
$8 + 6 =$



46

Strategy: Two Fives

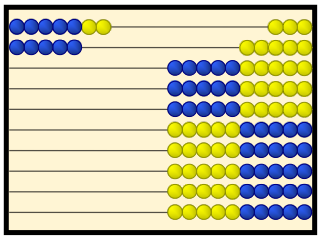
$8 + 6 = 10 + 4 = 14$



47

Strategy: Two Fives

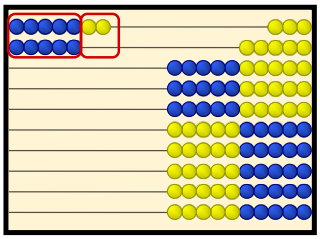
$7 + 5 =$



48

Strategy: Two Fives

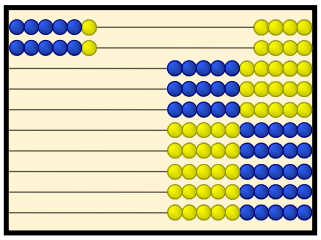
$7 + 5 = 10 + 2 = 12$



49

Multiplication

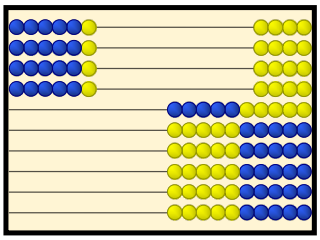
6 taken 2 times



50

Multiplication

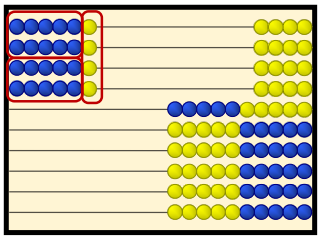
6 taken 4 times



51

Multiplication

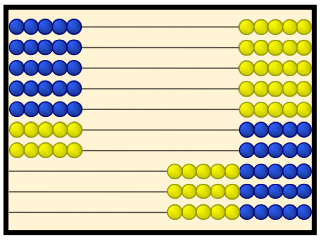
6 taken 4 times



52

Multiplication

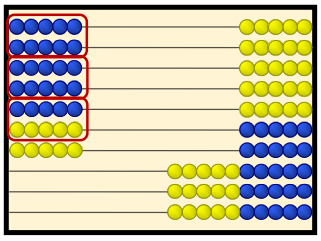
5 × 7



53

Multiplication

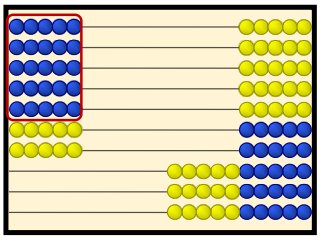
5 × 7



54

Multiplication

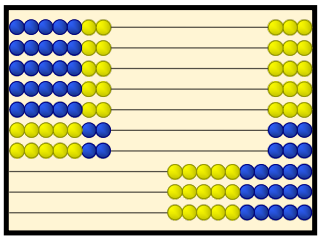
5 × 7



55

Multiplication

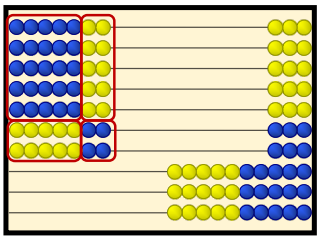
7 × 7



56

Multiplication

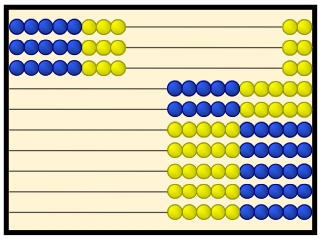
7 × 7



57

Multiplication

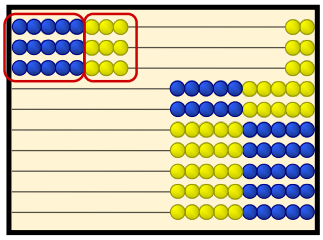
8 × 3



58

Multiplication

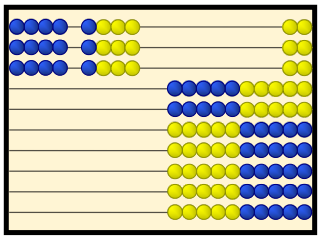
8 × 3 = 15 + 9 = 24



59

Multiplication

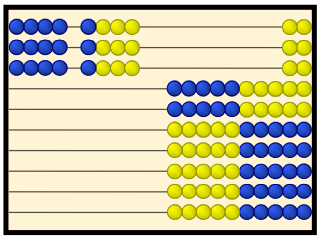
8 × 3



60

Multiplication

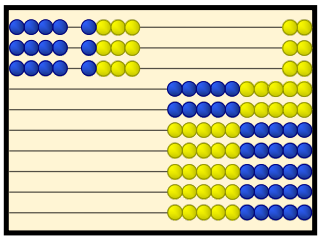
8 × 3 = 4 × 3 doubled



61

Multiplication

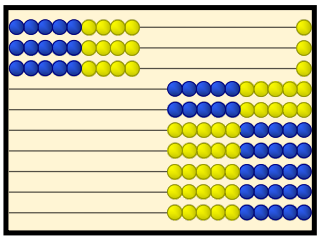
$8 \times 3 = 12$ doubled $= 24$



62

Multiplication

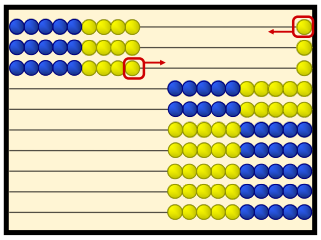
9×3



63

Multiplication

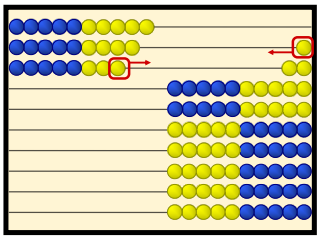
9×3



64

Multiplication

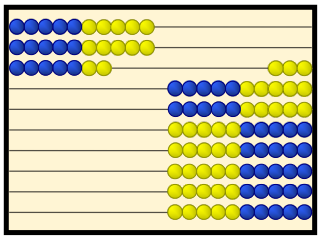
9×3



65

Multiplication

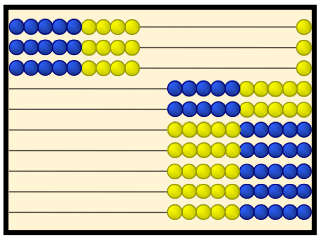
$9 \times 3 = 27$



66

Multiplication

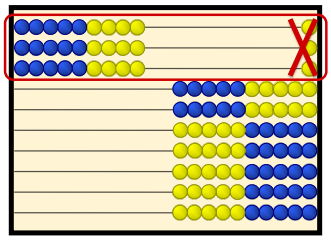
9×3



67

Multiplication

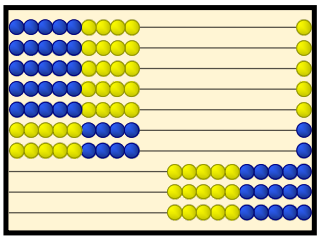
$9 \times 3 = 30 - 3 = 27$



68

Multiplication

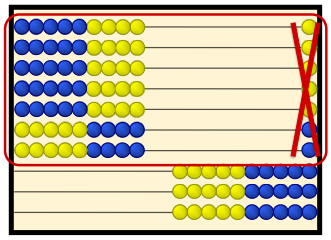
9×7



69

Multiplication

9×7



70

Learning Algorithms

- A fact is considered to be known if it can be recalled in two or three seconds.
- Gives time to think about a strategy, then produce the fact.
- Visual strategies help learn the facts.
- Use games, not worksheets, for practice.

71

Why Games

$$\frac{\text{Games}}{\text{Math}} = \frac{\text{Books}}{\text{Reading}}$$

Games provide interesting repetition needed for automatic responses.

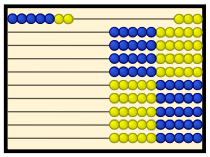
More importantly, games provide an application for the new information!

72

Go to the Dump Game

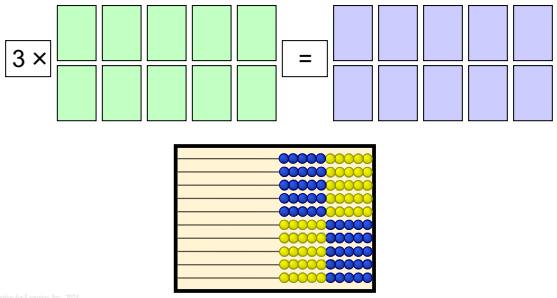
A "Go Fish" type of game where the pairs are:

1 & 9
2 & 8
3 & 7
4 & 6
5 & 5



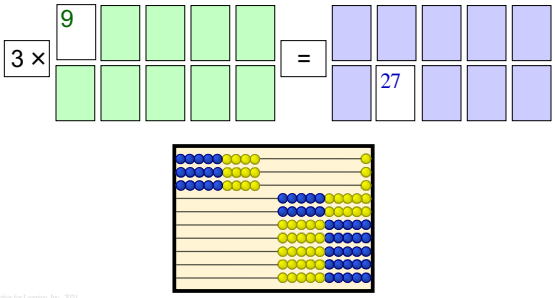
73

Multiplication Memory



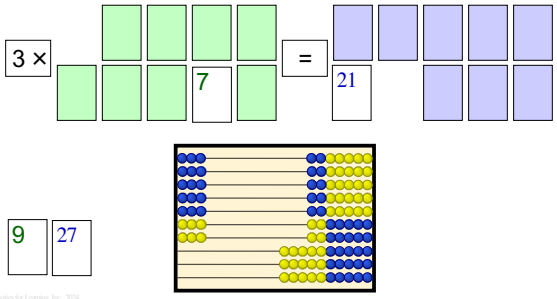
74

Multiplication Memory



75

Multiplication Memory



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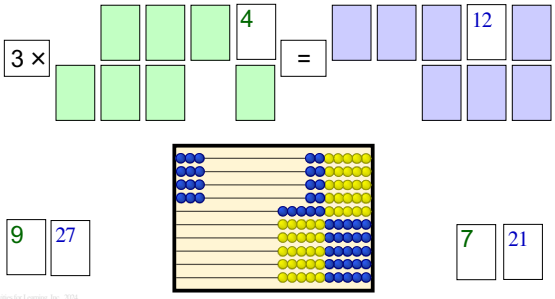
Multiplication Memory



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Multi-Fun

Building the multiplication table as a game.

- Each player draws six cards from the stock, then replaces the cards used, keeping six cards in hand.
- Lay as many cards as possible in the proper spaces in either a row or column.
- Only one row or column can be used during a turn.
- One point is awarded for each card played.

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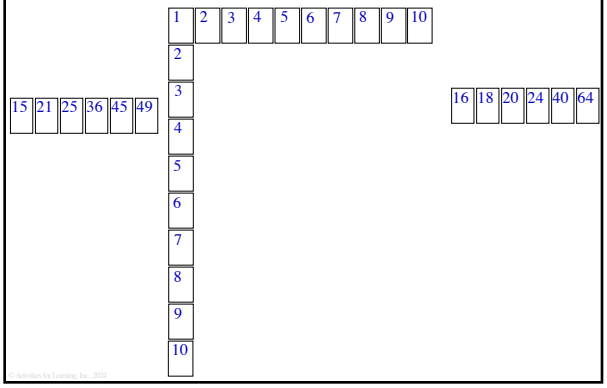
Multi-Fun



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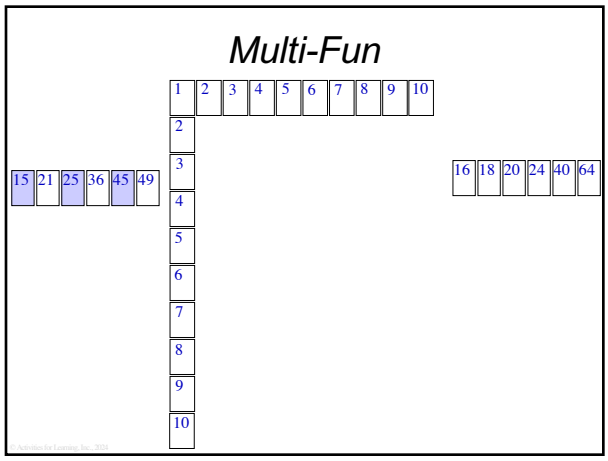

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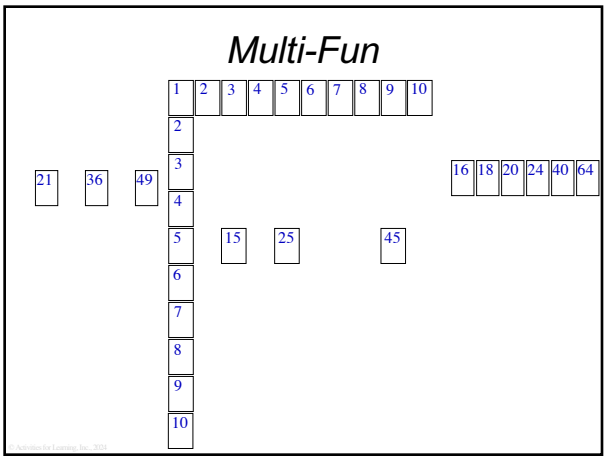

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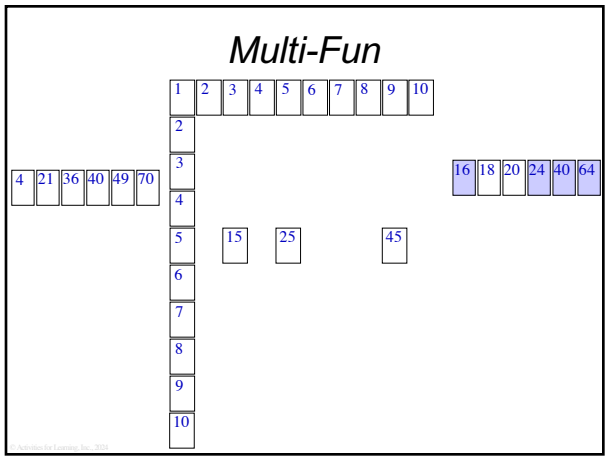

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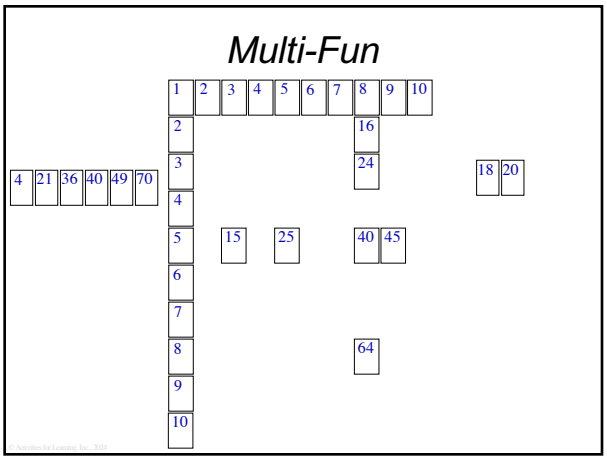

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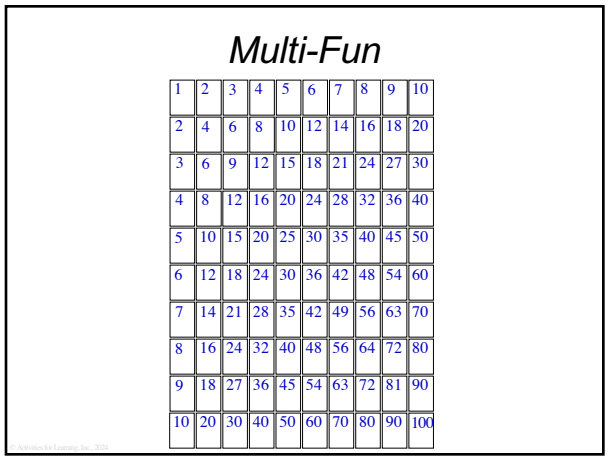


84

Learning Algorithms

- A fact is considered to be known if it can be recalled in two or three seconds.
- Gives time to think about a strategy, then produce the fact.
- Visual strategies help learn the facts.
- Use games, not worksheets, for practice.
- Short division before long division.

85

Division

quotient remainder

divisor) dividend

86

Short Division

$3\overline{)471}$

87

Short Division

$3\overline{)4^{1}71}$ → $1\_\_$

88

Short Division

$3\overline{)4^{1}7^{2}1}$ → $15\_$

89

Short Division

$3\overline{)4^{1}7^{2}1}$ → 157

90

Short Division

$9\overline{)8053}$

91

Short Division

$$9\overline{)8\,0\,5\,3}$$

92

Short Division

$$9\overline{)8\,0\,5\,3}$$

93

Short Division

$$9\overline{)8\,0\,5\,3}$$

94

Problem Solving

- Often a troublesome area for children.
- Yet problem solving is the heart of mathematics.
- Approach a math problem like a puzzle.
  - struggling is natural and necessary; develop persistence.
  - imagine the story problem with simple numbers, then draw a sketch.

95

Problem Solving

- Avoid teaching "key words," such as *all together* or *how many left*.

  There are 40 tiles per box and we bought 15 boxes. How many tiles do we have **altogether**?

96

Remainder Problems

Problem 1.

Thirteen children are going on a field trip. If 4 children can ride in a car, how many cars are needed?

[4 cars]

97

Remainder Problems

Problem 2.

Pauline has 13 petunias to plant. She wants exactly 4 in a row. How many rows can she plant?

[3 rows]

98

Remainder Problems

Problem 3.

Four children have $13 to split evenly. How much does each one receive?

[$3.25]

99

Remainder Problems

Problem 4.

Four children divide 13 candy bars. How much does each one receive?

[$3\frac{1}{4}$ bars]

100

Remainder Problems

Problem 5.

Jack packages 13 cookies with 4 per bag and eats the leftovers. How many does he eat?

[1 cookie]

101

Remainder Problems

Five problems with same numbers, but five different answers.

[4 cars]
[3 rows]
[$3.25]
[$3\frac{1}{4}$ bars]
[1 cookie]

102

Problem Solving

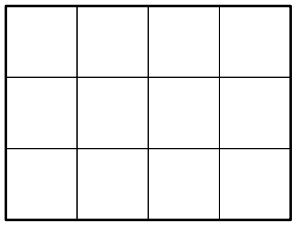
- Area is the number of square units that fit inside.

103

Problem Solving

- Area is the number of square units that fit inside.

$A = 12$



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Problem Solving

- Area is the number of square units that fit inside.

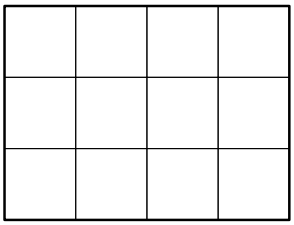
$A = 12$

$A = w \times h$

$A = 4 \times 3$

$A = 12$ square units

$A = 12$ units^2



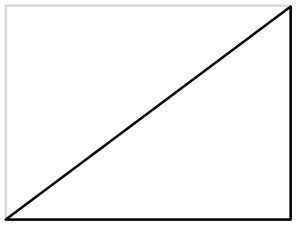
105

Problem Solving

- Area is the number of square units that fit inside.

$A = w \times h$

$A = \frac{1}{2}\ w \times h$



106

Problem Solving

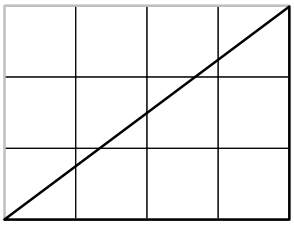
- Area is the number of square units that fit inside.

$A = w \times h$

$A = \frac{1}{2}\ w \times h$

$A = \frac{1}{2} \times 4 \times 3$

$A = 6$ units^2



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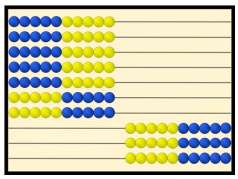
Summary

- Traditionally: Insisting all children memorize the counting words to 100 before doing any meaningful math.
  - About 20% of children have difficulty with the counting words and often fall behind.
- Solution: Teach the names of quantities to 10, use transparent number names for 11 to 99, then connect to the traditional names.

108

Summary

- Traditionally: Ignoring children's ability to visualize.
- Solution: Use visualizable manipulatives that group quantities in fives as well as tens.



109

Summary

- Traditionally: Using flash cards and timed tests.
- Solution: Teach strategies for learning facts.
- Solution: Use games the children enjoy for practice.

110

Summary

- Traditionally: Teaching math like it's a bunch of rules without rhyme or reason.
  - This makes advanced math much more difficult and applications mystifying.
- Solution: Teach for understanding.
  - Ask questions that require the child to think, not just parrot an answer.

111

In Conclusion …

Math needs to be taught so
95 percent is understood and
only 5 percent memorized.

– Richard Skemp
major pioneer in mathematics education

112

In Conclusion …

Our goal as a teacher of mathematics
is to help our children transform,
expand, and refine these beginning ideas
into deeper mathematical thinking.

– Dr. Joan A. Cotter

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