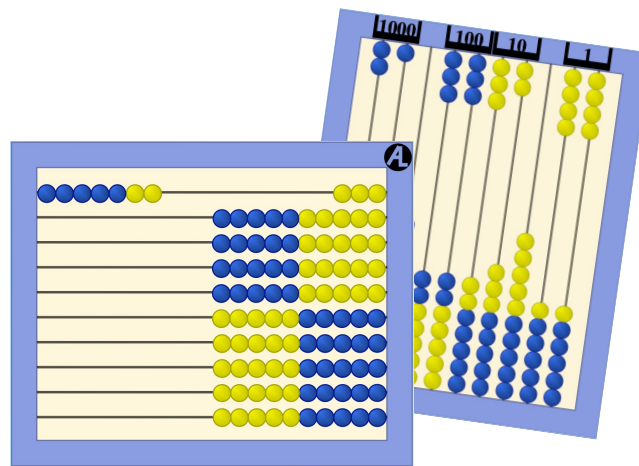



Unlock the Puzzle with RightStart™ Math



based on the work of Dr. Joan A. Cotter

Unlock the Puzzle

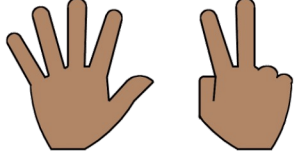


Subitizing

2


Subitizing

- Subitizing is quick recognition of quantity without counting.



3


Grouping



Try to visualize 8 apples without grouping.

4


Grouping



Now try to visualize 5 as red and 3 as green.

5


Grouping by 5s



Need grouping to visualize quantities.

6

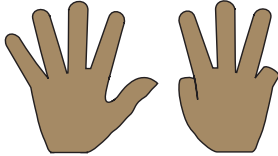
Grouping by 5s



Need grouping to visualize quantities.

7

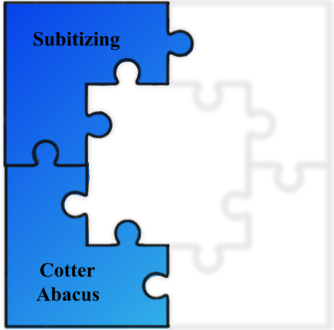
Grouping by 5s



Need grouping to visualize quantities.

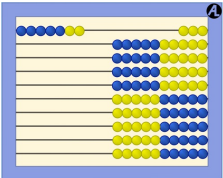
8

Unlock the Puzzle



9

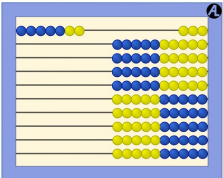
Cotter Abacus



- Visual and tactile manipulative
- Develops mental images of
 - Quantities
 - Strategies
 - Mathematical Operations

10

Cotter Abacus

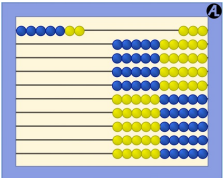


“The role of physical manipulatives is to help the child form those visual images and thus to eliminate the need for the physical manipulatives.”

—Ginsberg and others

11

Cotter Abacus

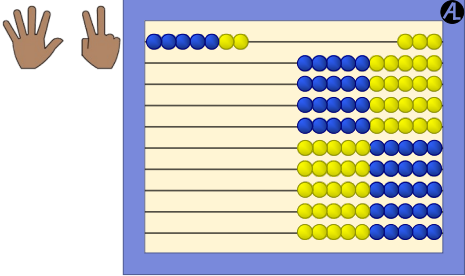


“Think in pictures, because the brain remembers images better than it does anything else.”

*—Ben Pridmore
World Memory Champion, 2009*

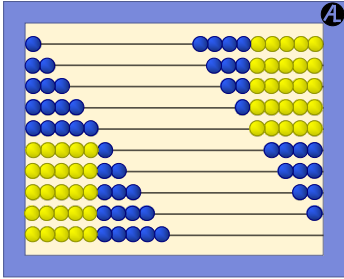
12

Entering Quantities



13

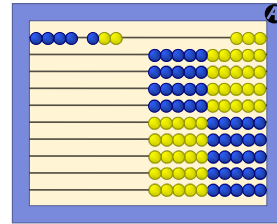
Entering Quantities



14

Adding

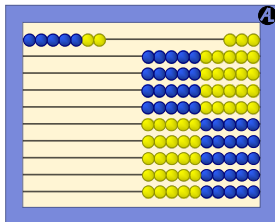
$$4 + 3 = \underline{\quad}$$



15

Adding

$$4 + 3 = \underline{7}$$



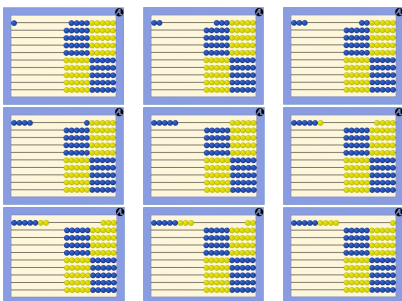
16

Typical Worksheet



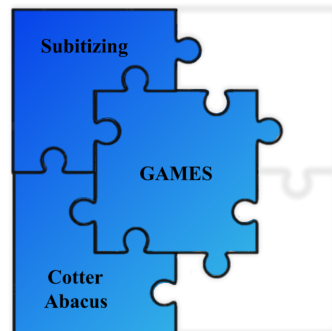
17

Facts Equaling 10



18

Unlock the Puzzle

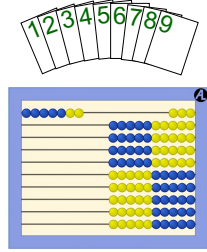


19

Go to the Dump Game

A “Go Fish” type of game where the pairs are:

- 1 & 9
- 2 & 8
- 3 & 7
- 4 & 6
- 5 & 5



20

Go to the Dump Game

- App for your device



Go to Ten

21

Why Games

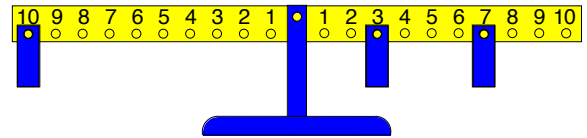
$$\frac{\text{Games}}{\text{Math}} = \frac{\text{Books}}{\text{Reading}}$$

Games provide interesting repetition needed for automatic responses in a social setting.

More importantly, games provide an application for the new information!

22

Math Balance



23

Unlock the Puzzle



24

Place Value

- The author of *Treviso Arithmetic of 1478*, written over 500 years ago, considered place value so important that it was listed first among the “five” operations of arithmetic.
- Place value organizes numbers into neat packets.
- Without place value, computational algorithms make little sense.

25

Place Value

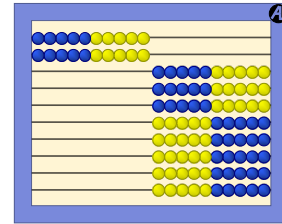
Yet, people often think
of 14 as 14 ones,
not ten and 4 ones.

The pattern that is needed to make
sense of tens and ones is hidden
because of the English language!

26

Transparent Place Value

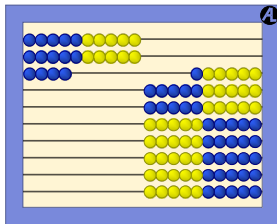
2-ten



27

Transparent Place Value

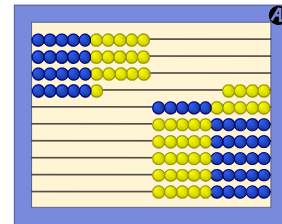
2-ten 4



28

Transparent Place Value

3-ten 6



29

Transparent Number Naming

10 = ten	20 = 2-ten
11 = ten 1	21 = 2-ten 1
12 = ten 2	22 = 2-ten 2
13 = ten 3	23 = 2-ten 3
14 = ten 4
....
19 = ten 9	99 = 9-ten 9

30

Transparent Number Naming

- Use this for two reasons:
 - Patterning

3 million
3 thousand
3 hundred
3 ten

31

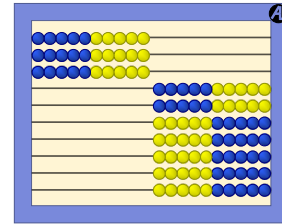
Transparent Number Naming

- Use this for two reasons:
 1. Patterning
 2. Place value

32

Transparent Number Naming

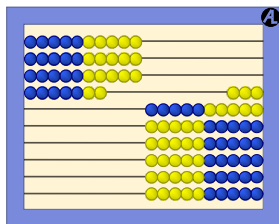
3-ten 30



33

Transparent Number Naming

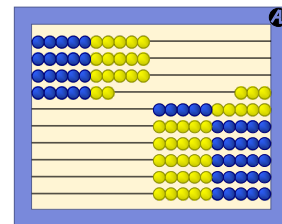
3-ten 7 30 7



34

Transparent Number Naming

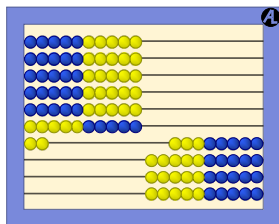
3-ten 7 37



35

Transparent Number Naming

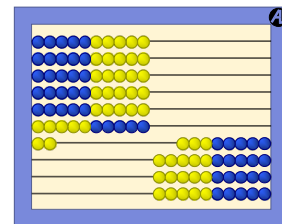
6-ten 2 60 2



36

Transparent Number Naming

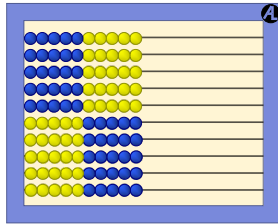
6-ten 2 62



37

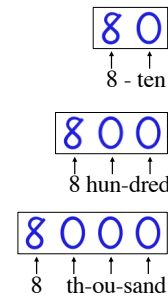
Transparent Number Naming

1 hundred 100



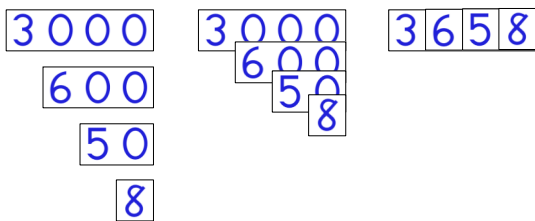
38

Transparent Number Naming



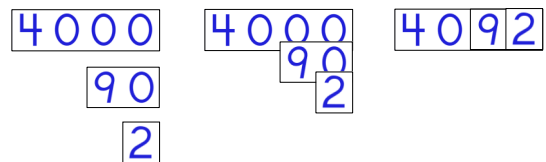
39

Transparent Number Naming



40

Transparent Number Naming



41

Transparent Number Naming

- Just as reciting the alphabet doesn't teach reading, counting doesn't teach arithmetic.
- Just as we first teach the *sound* of the letters, we first teach the *name* of the quantity (math way).

42

Transparent Number Naming

- Asian languages use the math way of number naming.
- The children understand place value in first grade; only half of U.S. children understand place value at the end of fourth grade.
- Mathematics is the science of patterns. The patterned math way of number naming greatly helps children learn number sense.

43

Unlock the Puzzle



44

Strategies

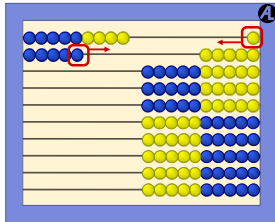
A strategy is a way to learn a new fact or recall a forgotten fact.

A visual representation is a powerful strategy.

45

Strategy: Complete the Ten

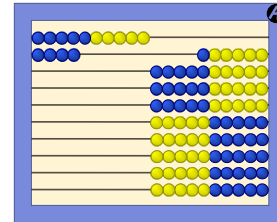
$$9 + 5 = \underline{\quad}$$



46

Strategy: Complete the Ten

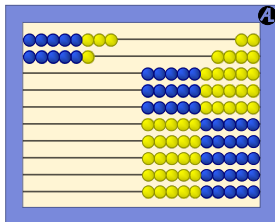
$$9 + 5 = \underline{14}$$



47

Strategy: Two Fives

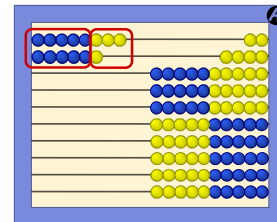
$$8 + 6 = \underline{\quad}$$



48

Strategy: Two Fives

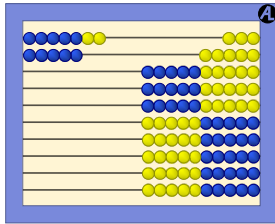
$$8 + 6 = \underline{14}$$



49

Strategy: Two Fives

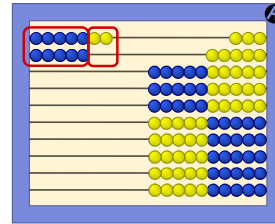
$$7 + 5 = \underline{\quad}$$



50

Strategy: Two Fives

$$7 + 5 = \underline{12}$$



51

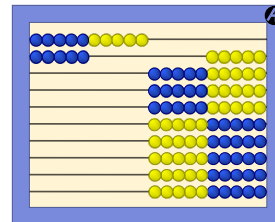
Subtraction Strategies

- Part from Ten
- All from Ten
- Going Up

52

Strategy: Part from Ten

$$15 - 9 = \underline{\quad}$$

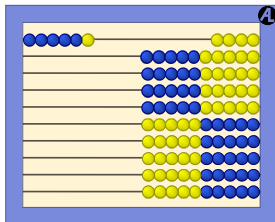


Subtract 5,
then 4

53

Strategy: Part from Ten

$$15 - 9 = \underline{6}$$

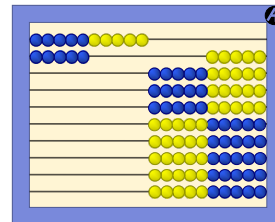


Subtract 5,
then 4

54

Strategy: All from Ten

$$15 - 9 = \underline{\quad}$$

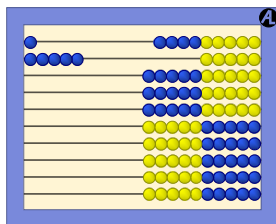


Subtract 9
from the 10

55

Strategy: All from Ten

$$15 - 9 = \underline{6}$$

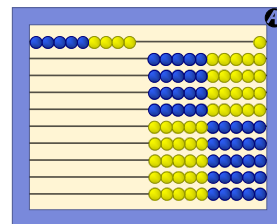


Subtract 9 from the 10

56

Strategy: Going Up

$$15 - 9 = \underline{\quad}$$

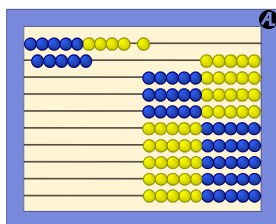


Start at 9; go up to 15

57

Strategy: Going Up

$$15 - 9 = \underline{6}$$

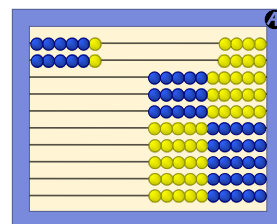


Start at 9; go up to 15

58

Multiplication

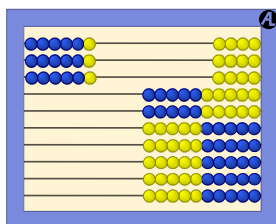
6 taken 2 times; 6×2



59

Multiplication

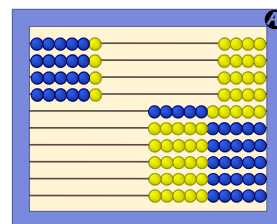
6 taken 3 times; 6×3



60

Multiplication

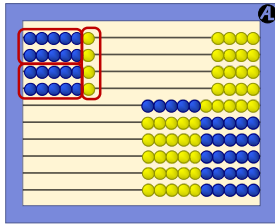
6 taken 4 times; 6×4



61

Multiplication

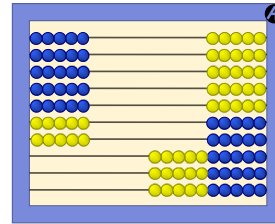
6 taken 4 times; 6×4



62

Multiplication

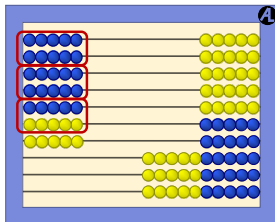
5×7



63

Multiplication

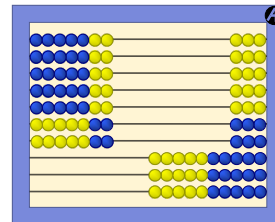
$5 \times 7 = 35$



64

Multiplication

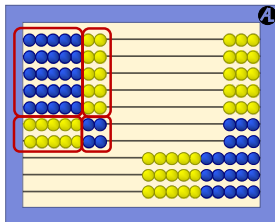
7×7



65

Multiplication

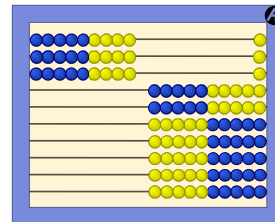
$7 \times 7 = 49$



66

Multiplication

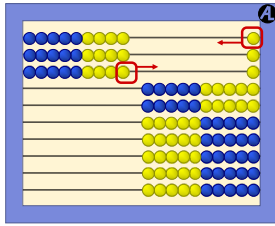
9×3



67

Multiplication

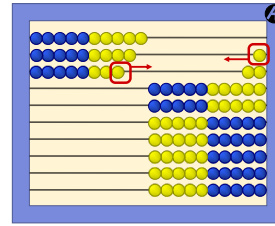
$$9 \times 3$$



68

Multiplication

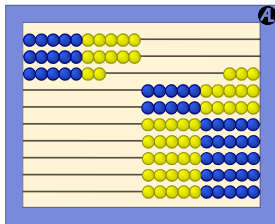
$$9 \times 3$$



69

Multiplication

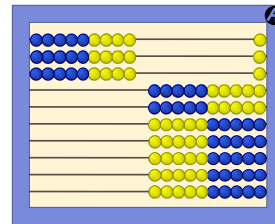
$$9 \times 3 = 27$$



70

Multiplication

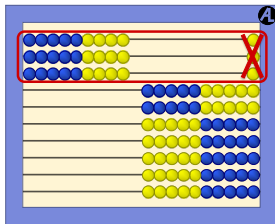
$$9 \times 3$$



71

Multiplication

$$9 \times 3 = 30 - 3 = 27$$



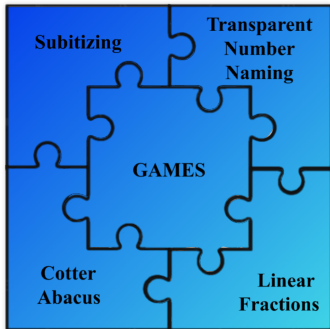
72

Ring Around the Product Game

2	6	3	5	1
8	54	12	42	7
5	50	15	9	8
9	1	6	8	3

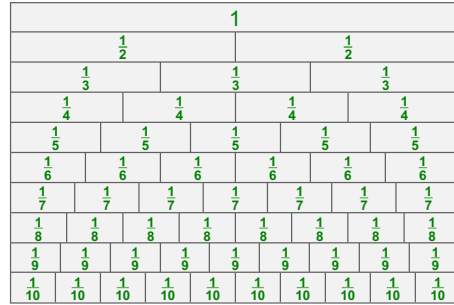
73

Unlock the Puzzle



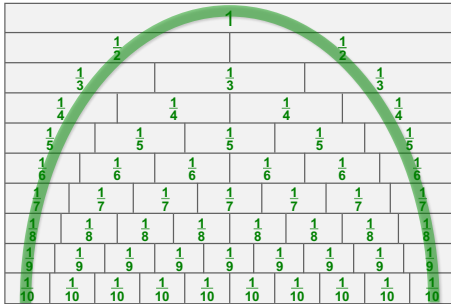
74

Fractions



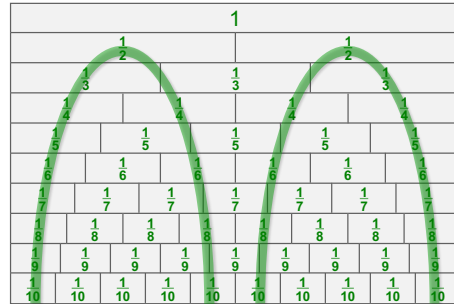
75

Fractions



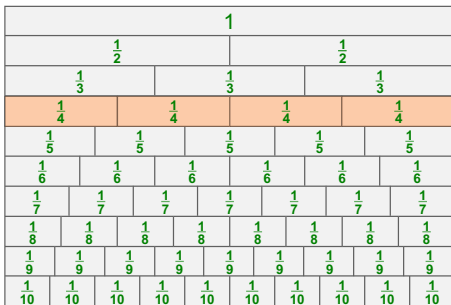
76

Fractions



77

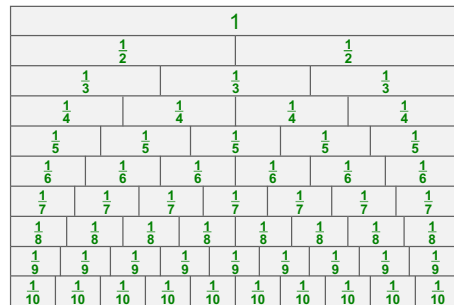
Fractions



How many fourths in a whole?

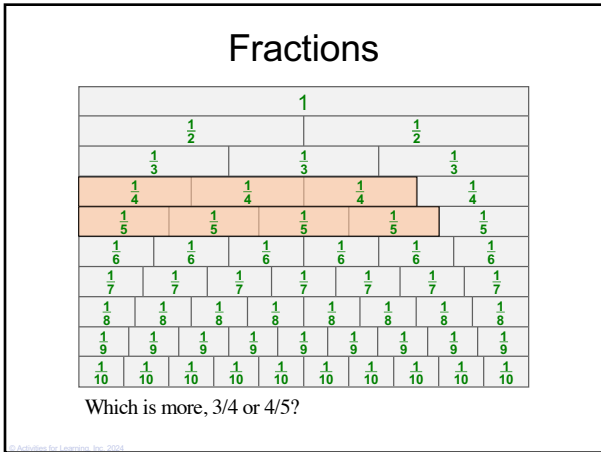
78

Fractions

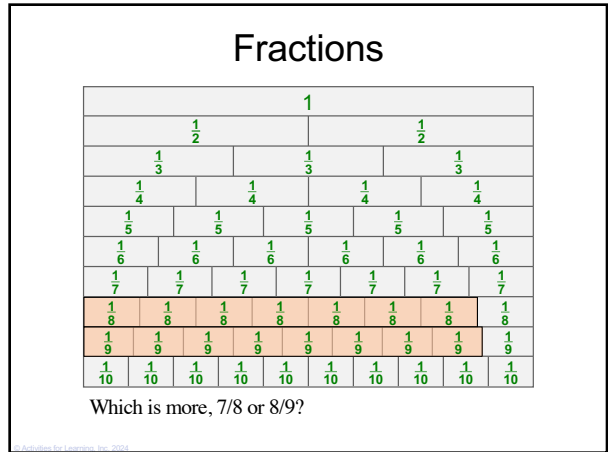


How many fourths in a whole? How many fifths? Eighths?

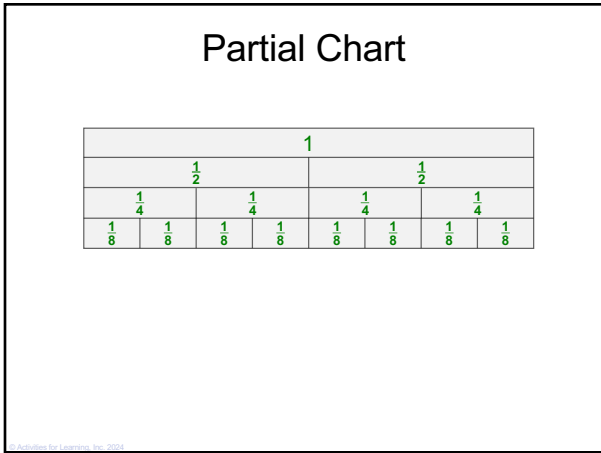
79



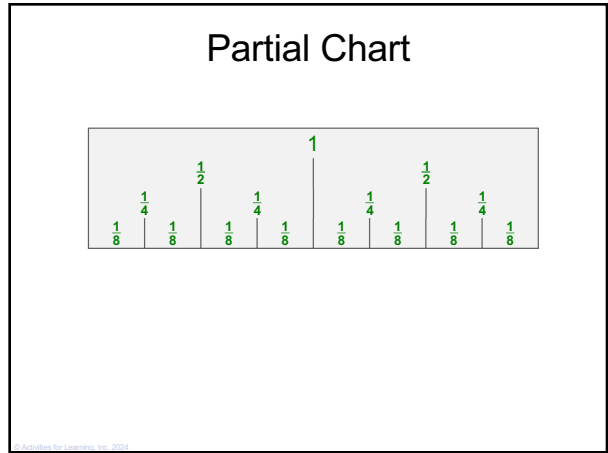
80



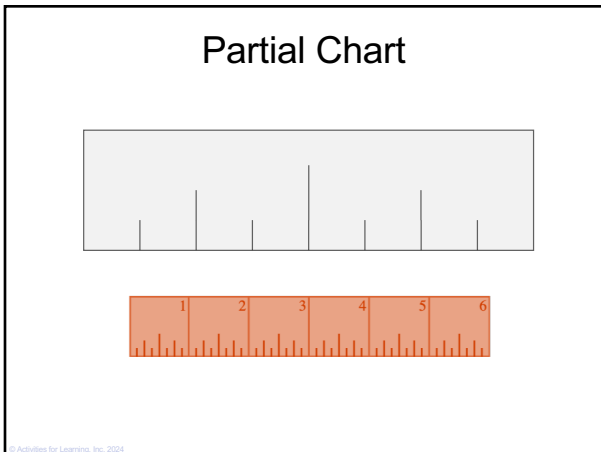
81



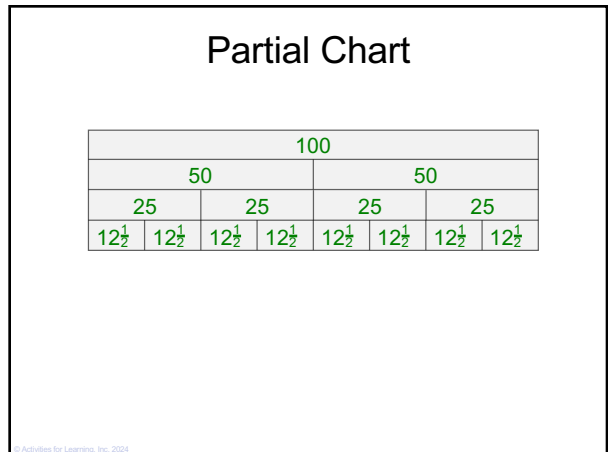
82



83



84



85

Partial Chart

100%							
50%				50%			
25%		25%		25%		25%	
$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$

86

Partial Chart

100¢							
50¢				50¢			
25¢		25¢		25¢		25¢	
$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$
10¢	10¢	10¢	10¢	10¢	10¢	10¢	10¢

87

Partial Chart

60 minutes							
30 minutes				30 minutes			
15 min		15 min		15 min		15 min	

88

Partial Chart

gallon							
half-gallon				half-gallon			
quart		quart		quart		quart	
pint	pint	pint	pint	pint	pint	pint	pint
cup	cup	cup	cup	cup	cup	cup	cup

89

Partial Chart

1							
1/2				1/2			
1/3		1/3		1/3		1/3	
1/4		1/4		1/4		1/4	
1/5		1/5		1/5		1/5	
1/6		1/6		1/6		1/6	
1/7		1/7		1/7		1/7	
1/8		1/8		1/8		1/8	
1/9		1/9		1/9		1/9	
1/10		1/10		1/10		1/10	

90

Simplifying Fractions

1							
1/2				1/2			
1/3		1/3		1/3		1/3	
1/4		1/4		1/4		1/4	
1/5		1/5		1/5		1/5	
1/6		1/6		1/6		1/6	
1/7		1/7		1/7		1/7	
1/8		1/8		1/8		1/8	
1/9		1/9		1/9		1/9	
1/10		1/10		1/10		1/10	

91

Simplifying Fractions

1											
$\frac{1}{2}$					$\frac{1}{2}$						
$\frac{1}{3}$			$\frac{1}{3}$			$\frac{1}{3}$			$\frac{1}{3}$		
$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$	
$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$	
$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$	
$\frac{1}{7}$		$\frac{1}{7}$		$\frac{1}{7}$		$\frac{1}{7}$		$\frac{1}{7}$		$\frac{1}{7}$	
$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$	
$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$	
$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$	

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Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

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Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$\frac{21}{28}$

94

Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$\frac{45}{72}$

95

Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$\frac{45}{72}$

Why does this work?

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Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$\frac{12}{16}$

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Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

12
16

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Fraction Chart

1											
1/2					1/2						
1/3			1/3			1/3			1/3		
1/4		1/4		1/4		1/4		1/4		1/4	
1/5		1/5		1/5		1/5		1/5		1/5	
1/6	1/6	1/6	1/6	1/6	1/6	1/6	1/6	1/6	1/6	1/6	1/6
1/7	1/7	1/7	1/7	1/7	1/7	1/7	1/7	1/7	1/7	1/7	1/7
1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8
1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9
1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10

What is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{4}$

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Fraction Chart

1											
1/2					1/2						
1/3			1/3			1/3			1/3		
1/4		1/4		1/4		1/4		1/4		1/4	
1/5		1/5		1/5		1/5		1/5		1/5	
1/6	1/6	1/6	1/6	1/6	1/6	1/6	1/6	1/6	1/6	1/6	1/6
1/7	1/7	1/7	1/7	1/7	1/7	1/7	1/7	1/7	1/7	1/7	1/7
1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8
1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9
1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10

What is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{6}$ That's multiplying fractions!

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Effective Math Users

"...The now well established fact that those who are mathematically effective in daily life seldom make use 'in their heads' of the standard written methods which are taught in the classroom."

W. H. Cockcroft, 1982
eminent mathematics educator in England

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RightStart™ Mathematics

- Uses the abacus to develop visualization.
- Teaches topics in different ways with different approaches.
- Fractions are presented in a linear format.
- Games are the practice and review.
- Uses over 20 different manipulatives.
- Arranged in levels rather than grades.

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RightStart™ Mathematics

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LESSON 119: INTRODUCING MULTIPLICATION AS ARRAYS

OBJECTIVES:
1. To introduce multiplication
2. To learn to learn product

MATERIALS:
1. Dry erase board
2. Abacus
3. Worksheet 119, Introducing Multiplication as Arrays

ACTIVITIES FOR TEACHING:
Warmup: Ask the child to write the equation for doubling 1 on his abacus board. 1 + 1 = 2. Compare with writing the equation for doubling 4. 4 + 4 = 8.

Beginning multiplication: Tell the child to enter 4 on the first row of an abacus. Now double it. Now double it again before. Ask: How much is 4? 8? 16? For 4 times 4.

These are the same: 4 + 4 = 8. 4 x 2 = 8. 2 x 4 = 8. 4 x 4 = 16. 4 x 3 = 12. 3 x 4 = 12.

Fluency 4 Times: Tell the child to enter 4 times on the abacus. Ask: How many times does 4 fit into 16? 4 times. Ask: How many times does 4 fit into 20? 5 times. Ask: How many times does 4 fit into 24? 6 times. Ask: How many times does 4 fit into 28? 7 times. Ask: How many times does 4 fit into 32? 8 times.

Fluency 4 Times: Tell the child to enter 4 times on the abacus. Ask: How many times does 4 fit into 20? 5 times. Ask: How many times does 4 fit into 24? 6 times. Ask: How many times does 4 fit into 28? 7 times. Ask: How many times does 4 fit into 32? 8 times.

EXPLANATIONS:
In the first row, an arrangement of 4 ones is shown. This is 4. In the second row, 4 more ones are added to the first row. This is 4 + 4 = 8. In the third row, 4 more ones are added to the second row. This is 8 + 4 = 12. In the fourth row, 4 more ones are added to the third row. This is 12 + 4 = 16. In the fifth row, 4 more ones are added to the fourth row. This is 16 + 4 = 20. In the sixth row, 4 more ones are added to the fifth row. This is 20 + 4 = 24. In the seventh row, 4 more ones are added to the sixth row. This is 24 + 4 = 28. In the eighth row, 4 more ones are added to the seventh row. This is 28 + 4 = 32.

ACTIVITIES FOR TEACHING:
Ask the child to enter the equation, 3 times 3 equals 9. Show how to use the equation written:

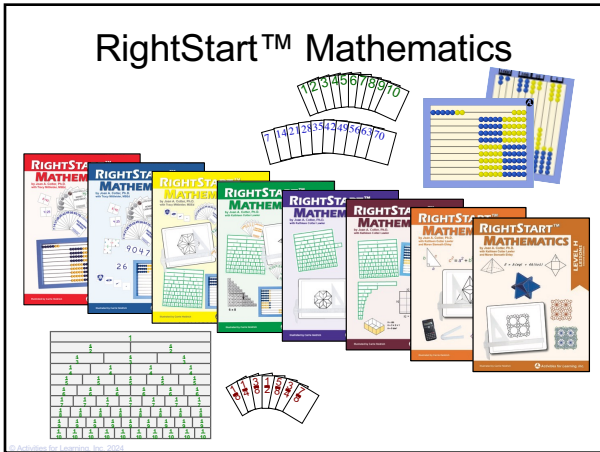
Fluency 4 Times: Tell the child to enter 4 times 4 on the abacus. Ask: How many times does 4 fit into 16? 4 times.

EXPLANATIONS:
When the child enters 4 times 4 on the abacus, he has 4 rows of 4 ones each. This is 16. When he enters 4 times 4, he has 4 rows of 4 ones each. This is 16. When he enters 4 times 4, he has 4 rows of 4 ones each. This is 16. When he enters 4 times 4, he has 4 rows of 4 ones each. This is 16.

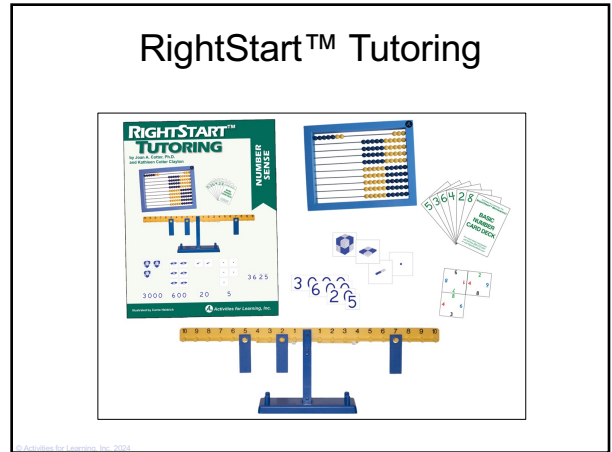
Worksheet 119: Ask the child to enter the top half of the worksheet. The worksheet contains a grid of 20 squares. The child is to enter 4 in the first row, 4 in the second row, 4 in the third row, and 4 in the fourth row. This is 4 times 4. The child is to enter 8 in the fifth row, 12 in the sixth row, 16 in the seventh row, and 20 in the eighth row. This is 4 times 5. The child is to enter 12 in the ninth row, 16 in the tenth row, 20 in the eleventh row, and 24 in the twelfth row. This is 4 times 6. The child is to enter 16 in the thirteenth row, 20 in the fourteenth row, 24 in the fifteenth row, and 28 in the sixteenth row. This is 4 times 7. The child is to enter 20 in the seventeenth row, 24 in the eighteenth row, 28 in the nineteenth row, and 32 in the twentieth row. This is 4 times 8.

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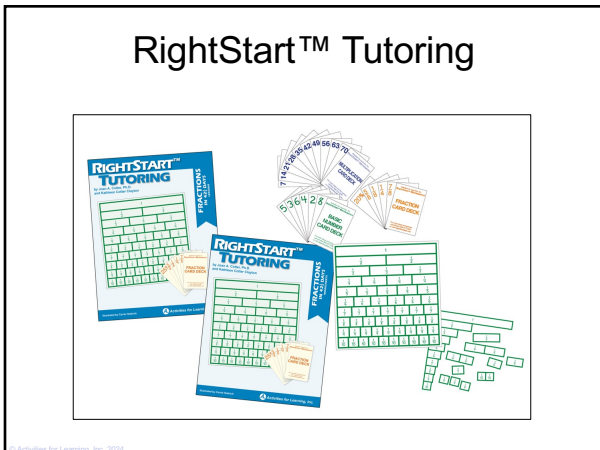
105



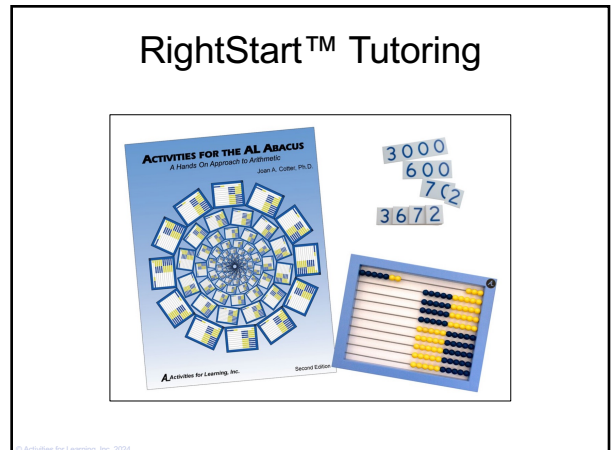
106



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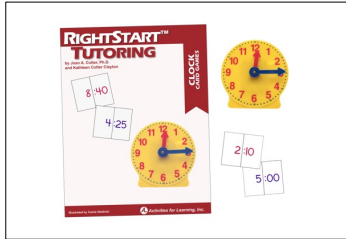


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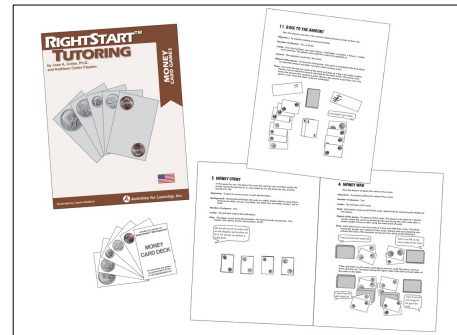
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RightStart™ Tutoring



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RightStart™ Tutoring



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Awards



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Awards



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Awards



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In Conclusion ...

Math needs to be taught so
95 percent is understood and
only 5 percent memorized.

Richard Skemp
— major pioneer in
mathematics education

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In Conclusion ...

Our goal as a teacher of mathematics is to help our children transform, expand, and refine these beginning ideas into deeper mathematical thinking.

– *Dr. Joan A. Cotter*

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