

Math Card Games

Over 300 Games for Learning and Enjoying Math

fifth edition

Joan A. Cotter, Ph.D.

ABOUT THE AUTHOR

Joan A. Cotter Ph.D holds a bachelor's degree in electrical engineering from the University of Wisconsin-Madison, an AMI Montessori diploma for ages 3-6, a master's degree in curriculum and instruction from the University of St. Thomas (formerly College of St. Thomas), and a Ph.D. in mathematics education from the University of Minnesota. Her research was on primary children learning mathematics, especially place value.

She has taught ages 3-6 as a Montessori teacher, taught grades 6-8 as a mathematics teacher, and tutored students with special needs.

Dr. Cotter designed the double-sided Cotter Abacus and wrote the pre-school through middle-school *RightStart™ Mathematics* and *RightStart™ Mathematics, second edition* program along with the *RightStart™ Tutoring* series. She continues to write and present at national and international conferences.

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Activities for Learning, Inc.
321 Hill Street
Hazelton, ND 58544-4416
U.S.A.

888-272-3291 or 701-782-2000

order@RightStartMath.com
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How to use the *Math Card Games* manual

Wow! Over three hundred games! Where do you begin? Here are some pointers to help you maximize the amazing learning tool you have in your hands.

1. This manual is broken into eight chapters: Number Sense, Addition, Clocks, Multiplication, Money, Subtraction, Division, and Fractions.
2. Each chapter begins with easier games and progresses to more advanced games.
3. Read the game's objectives to help you choose a game that best meets the student's needs. You will find various games with similar objectives. For example, Go To the Dump (game A3), Handshaking Game (A3.1), Old Main (A4), and Fish at the Dump (A5) focus on the facts equaling ten. Different games provide variety while the children continue to master a skill or concept.
4. When learning how to play a game, go through the instructions using the materials as directed. For some games, you can find additional instructions and graphics on our blog: RightStartMath.com/blog. Search by the game name or game number.

If you prefer instructional videos, you can find all the games used in the RightStart™ Mathematics curriculum on Vimeo for a small subscription fee. Most of the games in this *Math Card Games* manual have video instructions.



RightStart Math blog
for some of the games



Instructional videos
for most of the games

If you have questions on how to play a game, please call our office at 888-272-3291, and one of our helpful customer service people will be delighted to assist you.

5. Make notes on which games were played, when they were played, and if it was enjoyed. We suggest writing general notes next to the game, right in this manual. This will help you in two ways:
First, fun and easy games will be quickly identified and are a good review on more challenging days.
Second, the notes will help identify games that the child found challenging. It could be they were not ready for that game, they were having a tough day, or they just did not like that particular game. At a later date, try the game again to see if the student's knowledge, ability, or perspective has changed.
6. Keep a log of the games played. Ten to 15 minutes of game time is the same as a math worksheet. So, when you look at the log, you can see the time spent practicing and working on math skills. A blank log is available on the next page and in the student's RightStart™ Mathematics worksheets.

Enjoy these games. Have a great day, and go play a math card game!

*Joan A. Cotter, Ph.D., Kathleen Cotter Clayton,
and the RightStart™ Mathematics team*

INTRODUCTION

Mathematics is a fundamental force in this technological and information age. The field has doubled in the last thirty years. It is critically important that every student master mathematics. It is unfortunate when a person cannot pursue a desired career because of a lack of the necessary background in mathematics.

Mathematics education has alternated between teaching by rote and teaching for understanding. Rote learning is high maintenance and has led to one-third of the school year being spent reviewing. Many people with learning disabilities, about one out of seven, find rote memorization extremely difficult and cannot rote memorize the 390 facts.

Even though teaching for understanding takes more time initially, much less time needs to be spent in review. Students who understand math retain it longer, can apply it to new situations, and have a greater chance to succeed in advanced math courses.

This manual has games for learning all the major concepts of arithmetic with special emphasis on memorizing the facts. These games are a fun and interesting way to learn basic math.

Counting for the facts is slow, tedious, and often inaccurate. Recent research shows that songs or rhymes learned to help memorize multiplication tables are stored in the language part of the brain, making them less useful for math. Visual strategies based on fives and tens are quicker and more permanent.

Mindy Holte said, “In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic.” The Cotter Abacus, based on fives and tens, is an ideal tool for visual learning. I asked five-year-old Stan how much 11 plus 6 is. When he said 17, I asked him how he knew. He explained by saying, “I have the abacus in my mind.”

WHY PLAY GAMES

When a child learns to read, they can practice the skill by reading for pleasure. In the same way, these card games combine practice with pleasure. Although learning math requires hard work, it can be enjoyable. When a person is interested in and loves their work, they can more easily tackle the challenging segments found in any activity.

These games allow adults and students of various ages and abilities to play together. It does away with anxiety-producing flash cards, which cast the parent or teacher in the role of judge. The only person who enjoys flash cards is the person who doesn't need them.

Games create a stress-free atmosphere that allows all to learn at their own pace. Then the parent or teacher becomes a partner in the learning process.

There is another reason to make mathematics enjoyable for children. Along with the information recorded in our memories are the feelings we experienced when we learned it. These feelings are often recalled along with the fact or experience. For this reason, information stored with negative feelings tends to be avoided, then forgotten. Children who associate math with feelings of failure and inadequacy will find learning difficult, and worse yet, they will ignore applications of math to daily life.

On the other hand, when students recall feelings of discovery and success, they will want to continue learning and will apply that knowledge to other areas. Therefore, it is important that learning be a pleasant experience.

Finally, remember that 10 to 15 minutes of a card game are equivalent to a worksheet. As we know, practice is needed for the facts to become automatic. These games provide much better review and practice than flash cards, timed tests, or traditional worksheets.

THE MATH GAMES

The games teach math while the players enjoy the games. The players need not know their facts before playing.

The principles that influenced the design of these games are as follows:

1. A logical and organized approach using visual strategies.
2. Concrete and mental work before paper work.
3. Minimal counting.
4. Manipulatives that are visualizable as well as visual.
5. Concept or strategy explained.
6. Interesting repetition.
7. Some method for error detection.

Rote memorizing is a low-level thinking skill. Strategies, on the other hand, give confidence and independence. An example of a strategy is finding $9 + 6$ by taking 1 from the 6 and giving it to the 9 to make 10 and 5, which is 15. Those who learn strategies have better number sense and are less likely to resort to finger counting.

Manipulatives, such as the Cotter Abacus or multiplication tables, are not to be regarded as crutches. They enable the students to build a mental model, necessary for concept formation. In practice, students will refer to them less and less and finally not at all. Let each child decide when they no longer need them. Sometimes just the security of having them nearby helps, even if they are not looked at. At the right time, a child may respond to the challenge of playing without them.

What looks like a simple step to us is often several steps for the student. That explains the variety of games. The games progressively get harder, building on previous concepts. The background section found in some of the games offers suggestions for presenting new concepts. Often a concept can be learned in more than one way, resulting in several games for the same concept.

We know how useless it is to have a corrected math paper returned a week later. To be helpful, errors must be corrected immediately. The games allow the players to discover errors themselves or with the help of the other players. Most of the solitaires cannot be won if an error is made.

PLAYING THE GAMES

This manual can be used with any mathematics program. The games provide a way to help both students needing remediation and those needing enrichment. Playing the games over the summer months keeps the players ready to learn more mathematics.

Who can teach

Anyone can play these games with children. Once the children know how to play, they can play by themselves. To help two groups simultaneously, form two circles like a figure eight and sit where the circles join.

You need not be a math expert to teach these games. Any person knowing the most basic arithmetic can do it. Algebra is not a prerequisite for this book. If you have anxiety when faced with math, you will enjoy this approach. It is my hope that after playing a few of the games you will start to enjoy math too.

What is taught

There are eight chapters: Number Sense, Addition, Clocks, Multiplication, Money, Subtraction, Division, and Fractions. Within each chapter the beginning games are easy and gradually become harder. The final games in each chapter teach more advanced concepts topics, for example, binary numbers.

It is not necessary or advisable to complete each chapter before starting another chapter. The clock games can be played while still playing the addition and the multiplication games. However, to prevent confusion, do not teach the subtraction facts until the child knows all of the addition facts. Likewise, teach the division facts only after the child knows the multiplication facts.

NUMBER SENSE

The term *number sense* refers to understanding numbers: a sense of their relative values, a sense of their structure through place value, and a sense of the changes following arithmetic operations. Some considerations to keep in mind when helping young children learn about numbers are recognizing and visualizing quantities, naming quantities, and counting.

Mere counting does not help children understand quantity. Numbers are different from other lists. In counting, seven includes quantities one to seven, but when we say the alphabet, G does not include the letters A to G.

Researchers have found that 5-month-old babies can distinguish between 1, 2, and 3 objects and half of 12-month-old babies can distinguish up to 4 objects. When you point to objects in counting, the young child loses the concept of the whole and assumes you are naming the objects. That is why if you ask a young child to count four objects and then ask her to give you four, she will frequently give you only the fourth object. Therefore, never count (by pointing to each object) fewer than 5 objects. Instead, refer to the collection by the number, for example, 4 apples.

To recognize quantities 6 to 10, they must be grouped. To understand the necessity of grouping, try to see mentally a group of eight apples in a line without any grouping—impossible. Next try to see five of those apples as red and three as green; most likely you can visualize them. Grouping by fives corresponds, of course, to our fingers. Such grouping is not a new idea; the Romans grouped in fives with their numerals: V for 5 and L for 50. Another example is piano music, which is written with two groups of five lines. Who could read it if the staves were not separated?

The words a child uses for naming numbers plays an important role in understanding place value. English is inconsistent in naming numbers 11 to 99, particularly from 11 to 19.

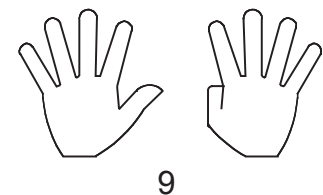
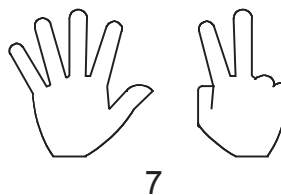
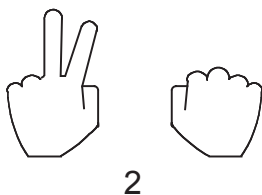
On the other hand, Asian languages follow a simple pattern. Their number words are as follows: 1 to 10, then ten 1 (11), ten 2 (12), ten 3 (13). The twenties are 2-ten, 2-ten 1, 2-ten 2, and so forth. In Asian languages, only 11 words are needed to count to 100, while 28 words are needed in English. Research shows that children learning the “math” way of number naming understand place value about three years earlier than those who do not. Therefore, teach children to count with ten 1, ten 2, . . . , 9-ten 9. Do this for several months.

Activities in this chapter focus on naming and recognizing quantities, evens and odds, and place value with tens and ones.

N1 ACTIVITIES USING FINGERS TO SHOW QUANTITIES

Most children are taught to raise a certain number of fingers when asked their age. One of the joys of having a birthday is being able to hold up one more finger.

Because we read from left to right, ask the child to use his left hand to represent the quantities 1 to 5. It does not matter which fingers on the left hand the child uses. Do not count. To teach three, do not say, “This is one, two, three”; say, “This is three.” Then teach 6 to 10 with 5 on the left hand and the amount over 5 on the right hand. See the figures below.



Then do the inverse, show quantities on your hands and ask the children to name them. Teach *Yellow is the Sun*. (The song and sheet music may be found on RightStartMath.com.)

Be sure the children use their fingers at the appropriate times. Also teach them the rhyme, *One, Two, Buckle my Shoe*, which will be used later for even and odd numbers.

Yellow is the Sun

*Yellow is the sun.
Six is five and one.
Why is the sky so blue?
Seven is five and two.
Salty is the sea.
Eight is five and three.
Hear the thunder roar.
Nine is five and four.
Ducks will swim and dive.
Ten is five and five.*

One, Two, Buckle my Shoe

*One, two, buckle my shoe.
Three, four, shut the door.
Five, six, pick up sticks.
Seven, eight, lay them straight.
Nine, ten, a big, fat hen.*

N2 FINGER CARDS IN ORDER

This is simple sorting.

Objective: To recognize the finger representations from 1 to 10.

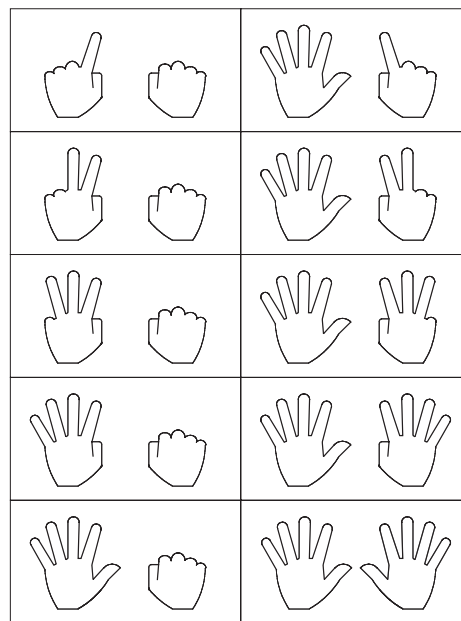
Number of players: One.

Cards: One set of Finger Cards from 1 to 10, found on Appendix page 1. (Two sets will be needed for Finger Card Memory (N4).)

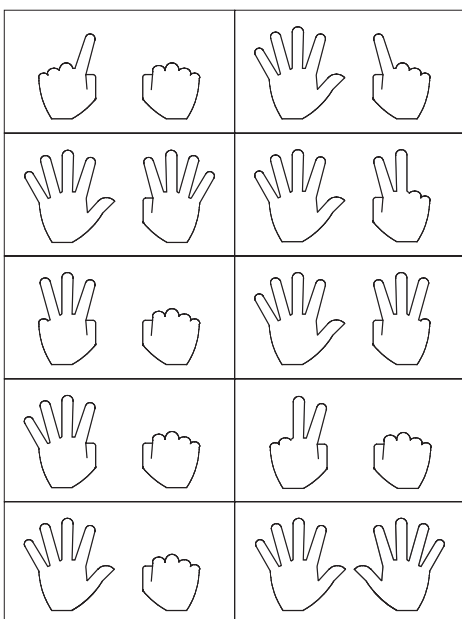
Layout: Lay the cards face up in random order.

Object of the game: To put the cards in some kind of order.

Play: Let the player decide what order to use. Shown here are the first column with 1 to 5 and the second column with 6 to 10.



The finger cards in order.



Cards with 2 and 9 are mixed-up.

N3 MIXED-UP FINGER CARDS

This game is fun for young players.

Objective: To find errors in the pattern made in the previous game (N2).

Number of players: Two.

Cards: A set of Finger Cards from 1 to 10, found on Appendix page 1.

Layout: The layout from the previous game.

Object of the game: To find the mixed-up cards.

Play: While the first player turns his back, the second player switches any two cards in the layout. The first player then looks and corrects the mixed-up cards. The players then exchange roles.

ADDITION

Research has shown that children learn the facts easier if they use strategies. Visual strategies are more powerful than other types of strategies because our visual memories are quick and permanent. A summary of visual addition strategies using the Cotter Abacus is given in the Appendix on page 10.

The terms needed to discuss addition are addend and sum, or total.

$$\begin{array}{r} 8 \text{ addend} \\ + 4 \text{ addend} \\ \hline 12 \text{ sum (total)} \end{array}$$

About half of the games in this chapter concentrate on small groups of facts using these strategies. The strategies used are (a) the facts that make 10, (b) adding 1 or 2, (c) the doubles and near doubles, (d) adding 9s and 8s, (e) facts that make 11, and (f) facts that make 9. The Corners™ games and Chain games reinforce larger groups of facts.

In some of the games, players will be collecting many pairs of cards. Ask the players to form two piles face up, one for each card of the pair. This allows everyone to verify the last pair and it prevents a difficult shuffling problem. At the end of the game combine the two piles. By pressing down on two piles, it is possible to determine which pile has the most cards to within one card.

A1 FIND THE PAIRS

The first few games deal with learning the facts that make 10. This game for one or two prepares the players for finding pairs equaling 10.

Objective: To find the pairs that total 10.

Background: To determine a pair using the Cotter Abacus, ask a player to enter the number on the card onto the abacus, for example, 2. Next slide over the remaining beads in the row (8) leaving a finger's width. Ask, "How many beads are needed with 2 to make 10?" [8] Ask the player to find the 8-card.

Manipulatives: Cotter Abacus for each player.

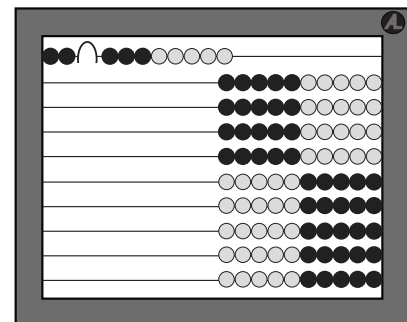
Number of players: One or two.

Cards: Two of each basic number card from 1 to 9.

Layout: Scatter the cards face up.

Object of the game: To pair all the cards.

Play: Without regard for turn, the players each pick up any card, determine the number needed to make 10, and search for that card. The pairs are placed on two stacks. Repeat with another card. Continue until all the cards are paired.



$$\begin{array}{|c|} \hline 2 \\ \hline 2 \\ \hline \end{array} \quad \begin{array}{|c|} \hline 8 \\ \hline 8 \\ \hline \end{array} \quad 2 + 8 = 10$$

A2 FIND THE TENS MEMORY

This memory game collects pairs totaling 10.

Objective: To provide practice in finding pairs with sums equaling 10.

Manipulatives: Cotter Abacus. See Find the Pairs (A1) for instructions on using the abacus.

Number of players: Two.

Cards: Two sets of basic number cards from 1 to 9.

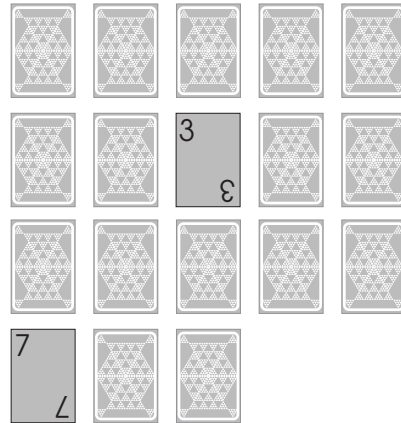
Layout: Lay out all the cards face down in rows.

Object of the game: To collect the most pairs equaling 10.

Play: The first player turns over a card. She determines the pair and says it aloud. She then decides where the pair could be and turns over that card. If they equal 10, the player collects them by putting them face up on two stacks and takes another turn. If they do not equal 10, the cards are returned face down and the next player takes his turn.

At the end of the game, the players combine their two stacks and see whose stack is higher.

Variation: For a player who needs a more concrete representation, or is not sure yet of the numbers, play this game with the Bead Cards (Appendix page 3).



Finding pairs that equal 10 for Find the Tens Memory.

A3 GO TO THE DUMP

This game is similar to the popular Go Fish, but here the pairs must equal 10. Players enjoy playing it, often for hours. As they begin to memorize the facts, they will use the Cotter Abacus less and less. The abacus provides a mental model for mastering these facts.

Objective: To provide repetition for learning the facts that equal 10.

Manipulatives: Cotter Abacus for each player.

Number of players: Two to four.

Cards: Six of each basic number card from 1 to 9.

Object of the game: To collect the most cards through pairing.

Deal: Each player takes five cards with the remaining cards placed in a stack face down in the center forming the dump.

Play: The players check over the cards in their hand for pairs that total 10. To do this, they look at each card, decide what is needed to make 10 and look for that number among their cards. Any pairs found are placed face up, one card on each of two stacks in front of the player.

The first player asks the player to his left for a number that he needs to complete a pair. If the player asked has it, she must give it to him and he receives another turn. If she does not have it, she says, “Go to the dump,” and the first player picks up the top card from the dump and his turn is complete, even if he picked up a card that makes a new pair. The next player takes her turn by asking the person on her left for a card.

For example, if the first player has a 4, he could ask, “Do you have a 6?” If the player on his left has the 6, she must give it to him, and he receives another turn. Otherwise, she says, “Go to the dump,” and then takes her turn.

When a player runs out of cards, he takes five more cards from the dump and his turn is ended. When the dump is exhausted, players can then ask any player for a card.

A3.1 HANDSHAKING GAME

Children love to shake hands, and in this game, they shake hands with their friends.

Objective: To find pairs that total 10.

Background: Before playing this game, demonstrate the proper way to shake hands. The right hand with the thumb up and the fingers together is offered to the other person, who grasps the hand firmly, but not hard, and moves a short distance up and down two times.

Manipulative: Cotter Abacus.

Number of players: Ten. If ten players are not available, have the player place each of the numbers beside a stuffed animal or doll. At the proper time, the objects “shake hands.”

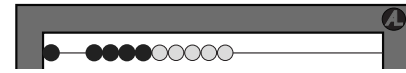
Cards: Basic number cards 1 to 9 and an extra 5.

Object of the game: To shake hands at least once.

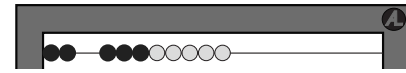
Deal: Each player takes a card and holds it in the left hand.

Play: Enter 10 on the abacus. Move 9 beads to the right; the player holding the 1-card and the 9-card shake hands. Move one bead to the left. Players with the 2-card and 8-card shake hands. Continue until the abacus shows 9 and 1.

After the game, ask who shook hands twice. [all except those with the 5-cards] Ask who shook hands only once. [those with the 5s] Ask why.



Players with 1 and 9 shake hands.



Players with 2 and 8 shake hands.

A4 OLD MAID

This game is similar to the familiar Old Maid. But now, the unwanted card is called the “dead end.” Again the pairs are the facts totaling 10.

Objective: To learn the facts equaling 10.

Number of players: From two to six.

Cards: Use one of each basic number card from 1 to 9 for each player, but be sure to have an even number of 5s.

Object of the game: To avoid having the single card, the dead end, at the end of the game.

Deal: Without looking, remove one card from the deck and set it aside. (In subsequent games, the loser may choose the card.) Deal all of the remaining cards to the players. It does not matter if some players receive an extra card.

Play: The players look for pairs totaling 10 and set them aside.

The first player takes a card from the player on her left. The player giving up a card may turn his cards over or hold his cards upright. The first player then checks her hand for a possible match with her new card. Meanwhile, the player who gave up a card takes his turn by drawing a card from the player on his left.

Play continues until one player is left with the dead end card. Verify that this card is the pair of the card set aside earlier.

A5 FISH AT THE DUMP

This game is similar to, but a little more complicated than Go to the Dump (A3).

Objective: To provide more practice for the players to use the tens facts.

Number of players: From three and five.

Cards: Six of each basic number cards from 1 to 9.

Deal: Each player receives five cards.

Object of the game: To collect the most cards by pairing cards that total 10.

Play: The first player asks any other player for a card he needs to make a pair. If he receives the card asked for, his turn continues. He may ask the same or another player. If the player asked does not have it, she says, “Go to the dump.” If he picks up the last card he asked for, he shows it and his turn continues. If not, the next player on his left takes a turn.

The game ends when all the cards are played.

CLOCKS

Children are aware that the numbers on a clock dictate many events in our lives. In some ways the preponderance of digital clocks makes telling time more difficult. Learning to read an analog clock is still a very necessary skill. Analog clocks provide a mental model, for example, to tell how much time there is between 10:42 and 11:00.

Children can play the clock games without knowing any addition or multiplication facts. The first seven games only require reading numbers to 12. After that the child must be able to read numbers to 60.

Actually time-telling is based on a simple principle. The hour (shorter) hand uses low numbers while the minute (longer) hand uses higher numbers, which are invisible. These games provide children with the necessary practice.

The games approach telling time in the following six steps:

1. First, the children work exclusively with numbers representing the hour hand. After a while, she will know the positions even on an unmarked clock.
2. Next the children work with the minute hand. She learns the placement of the “invisible” numbers. You can prepare children with this phase in advance by teaching them to count by 5s. Play the Multiples Memory Game (P2). They can also learn it by entering 5s on the Cotter Abacus and naming the quantity.
3. The next few games help the children to put the two hands together to tell time.
4. Then there is series of games that teach the children to recognize a half hour later or earlier and other common times.
5. Next the children become familiar with the minutes between the numbers.
6. Finally, the children read any number of minutes before the hour. This step is delayed until the children have thoroughly mastered formal time telling to prevent confusion with before and after the hour.

The clock cards needed for playing these games consist of two sets of 24 cards, in different colors. Each set has 12 hour cards, with numbers from 1 to 12, and 12 minute cards, with numbers :00, :05, :10, ... :55. Two clock representations are included in the appendix and may be reproduced.

Finally, use time in everyday conversations to the children. Remember to keep the games fun. Most of them should be played more than once, some many times. Soon the children will be able to read analog clocks.

C1 HOUR CARDS AROUND THE CLOCK

Although we realize the numbers around the clock are consecutive, children need to experience this.

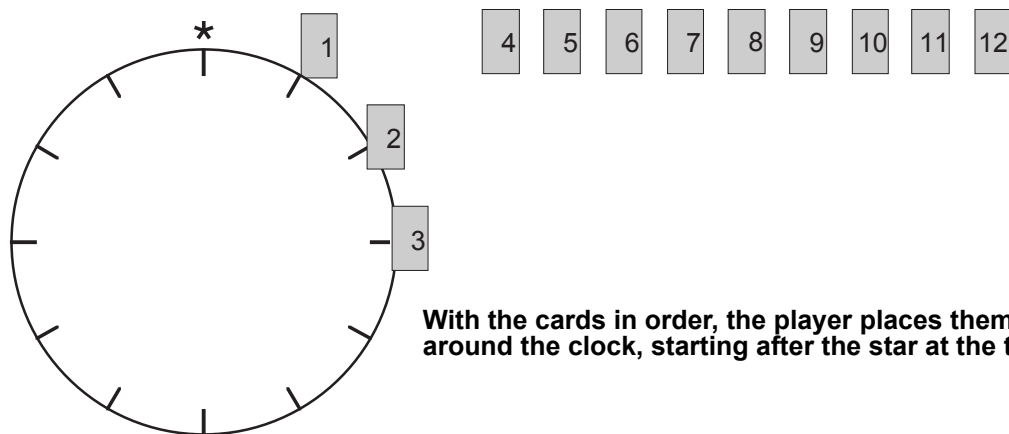
Objective: To become aware of the order of the numbers on a clock.

Number of players: One.

Cards: Use either set of 12 hour cards and Clock A (Appendix page 13).

Layout: The cards are scattered face up on the right side of the clock.

Object of the game: To place the cards around the clock in the proper positions.



With the cards in order, the player places them around the clock, starting after the star at the top.

Play: The player first puts the cards in a row in order from 1 to 12. If the player needs help, ask him to find 1. Set it aside. “What comes after 1? Can you find 2? Put it next to 1.”

When the cards are in the order, show the player the mark at the top of the clock. Explain that the 1 is placed next to that mark. The 2 is placed after the 1 and so on. The last number belongs on top.

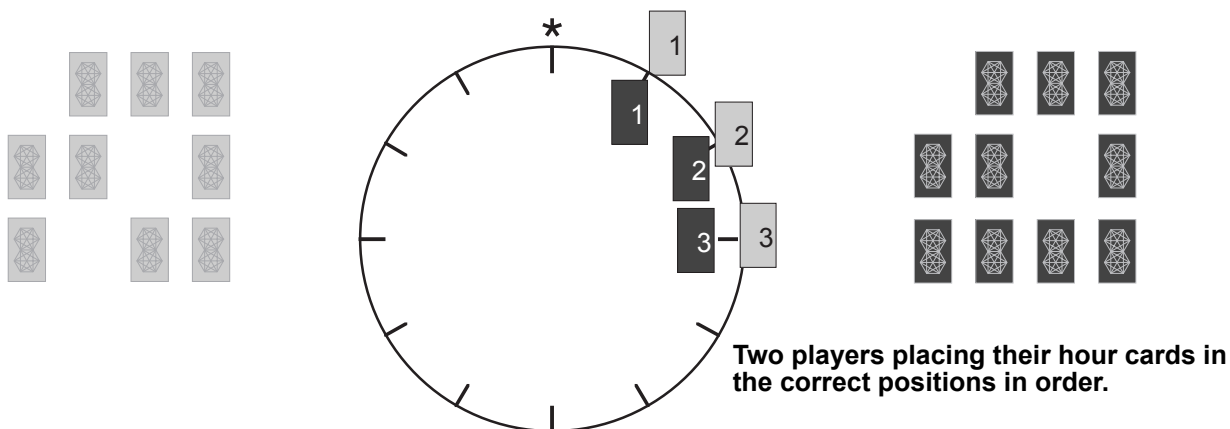
C2 HOUR MEMORY

Two players build clocks for this memory game.

Objective: To provide more practice in placing the hour cards.

Number of players: Two.

Cards: Use two sets of hour cards and Clock A (Appendix page 13).



Two players placing their hour cards in the correct positions in order.

Layout: Each player lays out a set of the hour cards as for memory.

Object of the game: To be the first player to place all his hour cards around the clock in order.

Play: The first player turns over a card. If it is the 1, he places it on the clock and receives another turn. If it is not the number wanted, he returns the card face down in the same place. Then the next player takes her turn.

Variation: Each player has his own Clock A; the hour cards are shuffled together and players may choose cards of either color.

MULTIPLICATION

The first several multiplication games can and should be played while the children are enjoying the addition and other games. Cards needed for the multiplication games are the basic number cards and the multiplication cards. Each multiplication card represents one number of the multiplication table, up to 10×10 . The 11s and 12s are not basic facts and are not included.

Memorizing the multiplication facts is approached through visualizing the multiples (also called skip counting) patterns, as shown below, and through visualizing arrays. The children should not recite the multiples in order to find a multiplication fact.

The terms needed to discuss multiplication are: *multiplicand* \times *multiplier* = *product*. That is, 8×4 is interpreted as 8 repeated 4 times, not 8 groups of 4. This is consistent with the other operations of arithmetic:

$8 + 4$ means start with 8 and transform it by adding 4.

$8 - 4$ means start with 8 and transform it by removing 4.

$8 \div 4$ means start with 8 and transform it by breaking it into either 4 groups or groups of 4.

Therefore, 8×4 should mean start with 8 and transform it by adding it a total of 4 times.

Multiples

Multiples are learned more quickly and easily when children see patterns (Appendix page 18). Arrange the 2s in two rows that show the ones repeating. The second row is 10 plus the first row. Notice also that all the multiples for the 2s are even.

Arrange the 3s in groups of 3. Look in each column and notice the ones decrease by one. Also note the 0-9 sequence starting at the lower left, moving up, and continuing at the bottom of the next column. In each column the tens have the pattern, 0, 1, 2. Also add the digits: in the first column they are 3 (1 + 2 & 2 + 1); in the second column, 6; and the third, 9.

With the 4s grouped in two rows, the ones are aligned. Observe that the first row of 4s is every other number of the 2s. The second row is 20 more than the first row.

The 5s are easily learned. Children like to recite them in sing-song fashion.

The 6s can be thought of as the even 3s for the first row. In the second row the ones are the same and the tens are 3 more.

Even the 7s have a certain pattern. Each number from 1 to 9 is used once in the ones. In each column, the ones increase by 1 and the tens increase by 2. In the rows, the tens increase by 1.

The 8s, of course, are every other 4 for the first row. Also notice that in each row, the ones are the even numbers in descending order. In each row the tens increase by 1 and the second row is 40 more than the first row.

The 9s are the most interesting of all; the sums of the digits in each case equal 9. The digits can also be reversed to obtain another multiple as

2	4	6	8	10
12	14	16	18	20

3	6	9
12	15	18
21	24	27
30		

4	8	12	16	20
24	28	32	36	40

5	10
15	20
25	30
35	40
45	50

6	12	18	24	30
36	42	48	54	60

7	14	21
28	35	42
49	56	63
70		

8	16	24	32	40
48	56	64	72	80

9	18	27	36	45
90	81	72	63	54

10	20	30	40	50
60	70	80	90	100

shown in the second row, which are written backward. Also note in the rows that the ones decrease by 1 as the tens increase by 1.

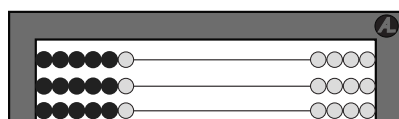
Following the games involving multiples, the next group of games emphasizes working with the “sets” individually. In this chapter the word set (also called table) is reserved for a series of the 10 numbers forming either a row or column of the Multiplication Table (Appendix page 19). The name of the set is the first, or lowest, number. For example, the 3s set is 3, 6, 9, 12, 15, 18, 21, 24, 27, and 30.

Recognizing the set or sets to which a number belongs is important for more advanced concepts, such as factoring, simplifying fractions, and finding the lowest common denominator. Other games help the players make use of the facts, learn the squares, and recognize the even-odd arrangements.

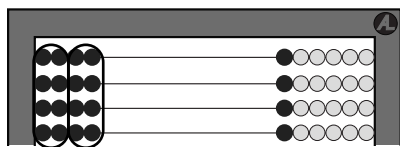
Knowing the multiples or skip counting patterns greatly facilitates learning the multiplication facts. For example, 8×3 is the third number in the top row of the 8s pattern, and 8×7 is the second number in the second row.

Arrays

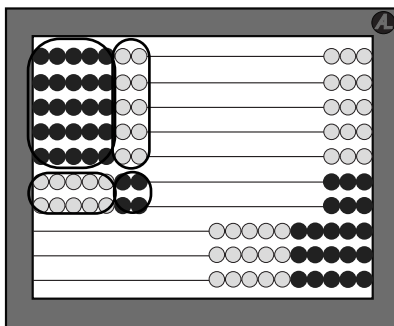
The Cotter Abacus gives players wonderful visual images for remembering the multiplication facts. Seeing groups of 2s, 5s, and 10s is the key. See the figures below. For example, 6×3 can be seen as three groups of 5 plus 3 more, or 18. The 8s and 9s facts are easy to determine when based around 10s; for example, 9×8 can be calculated by thinking of 10×8 and subtracting 8. To know a multiplication fact is to have a mental picture of the array in one’s head.



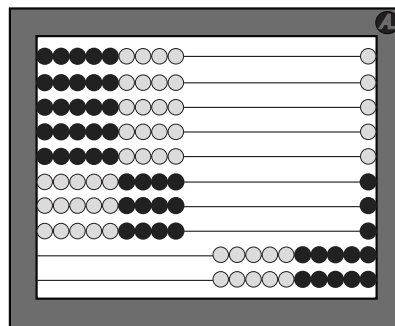
$$6 \times 3 = 5 \times 3 + 3 = 18.$$



$$4 \times 4 = 8 + 8 = 16. \text{ (See the two dot patterns for 8.)}$$



$$7 \times 7 = 25 + 10 + 10 + 4 = 49.$$



$$9 \times 8: 10 \times 8 - 8 = 72.$$

Arrays are another reason for 6×3 to be “6 taken 3 times.” In area or the common coordinate system, we do the horizontal before the vertical.

P1 SKIPPING ON THE HUNDRED CHART

This game leaves interesting patterns on the hundred chart.

Objective: To understand the relationship between multiples and skip counting while observing the patterns made on the hundred chart.

Number of players: One or two.

Cards: Choose a number between 2 and 10. Use 10 basic number cards with that number and four cards with 0. Also needed are 10 game markers and the Hundred Chart (Appendix page 12).

Object of the game: To be the player to land on 10 times the chosen number.

Play: Shuffle the 14 cards to form the stock. The first player turns over the top card and moves that many spaces on the chart. The second player turns over the next card and moves another marker (unless it is 0) that many spaces after the first marker. Players continue to alternate turns until all the markers are in place.

P2 MULTIPLES MEMORY

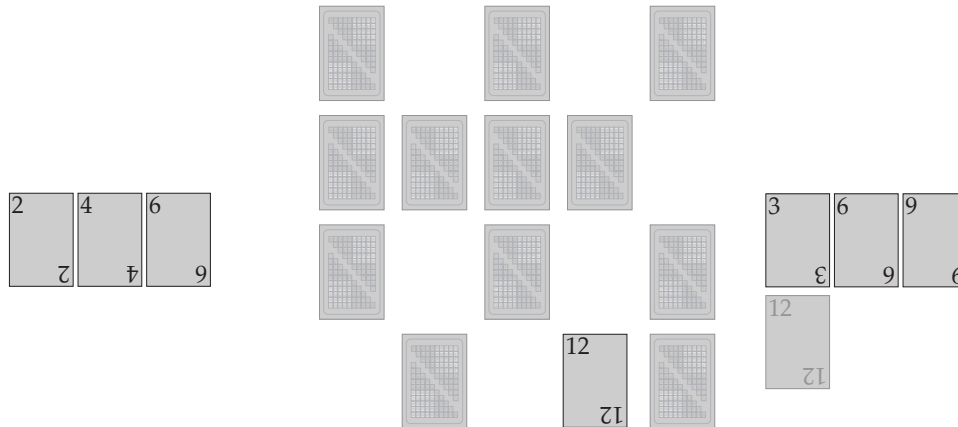
Players greatly enjoy playing this simple game, which teaches the skip counting patterns visually. These patterns help in learning the multiplication facts and for simplifying fractions and algebra. Along with the cards are envelopes with the skip counting patterns. Insert the corresponding cards into the envelopes. Some players call this game the “envelope game.”

Objective: To help the players memorize the multiples or skip counting patterns.

Number of players: Two.

Cards: An envelope containing a set of multiplication cards for each player, 20 cards in total.

Layout: Remove the cards and shuffle the two sets together slightly. Then lay them face down in four rows of five cards each.



Multiples Memory game, where one player is collecting the 2s and the other, the 3s.

Manipulatives: The Skip Counting Patterns (Appendix page 18), or on the envelopes.

Object of the game: To be the first player to pick up, *in order*, her set of multiplication cards.

Play: The first player turns over a card so both can see it. If it is the card she needs, she picks it up and takes another turn. If it is not the card she needs, it is returned face down in the original place. The other player then takes a turn. Players continue to take turns until one of them has picked up all her cards.

One player may have seven or eight cards before the second player finds his first card, but by remembering the locations, the second player can pick up most of his cards in one turn. Both players will at times need the same number, which adds interest and excitement to the game. Stress the importance of returning the cards to the correct envelopes following play.

Variation: After the players know the patterns very well, either or both players may collect the cards in reverse order.

P3 SUM RUMMY

This was Andy’s favorite game. The scoring performed at the conclusion of the game provides the players with an insight to the purpose of multiplication and its relation to addition. It is especially interesting to observe how a player finds the sum during his first game.

Objective: To help the players understand multiplication as a shortcut version of addition.

Background: Younger players can play this game without any reference to multiplication. However, players familiar with multiples or multiplication will appreciate how it simplifies the arithmetic.

Number of players: From two to six. Players may form teams.

Cards: Use about 50 to 70 assorted basic number cards for two players, 75 to 100 cards for three players, and all the basic number cards for four or more players. Omit the 0s.

Deal: Deal seven cards to each player. The remaining cards form the stock. Turn over the top card and place it beside the stock to start the discard pile.

Object of the game: To form groups of three or more identical cards. After the game, each player or team calculates the total value of all the groups he has put down. The winner is the one with the highest score.

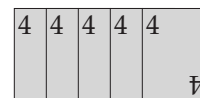
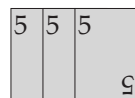
Play: Players take turns, starting with the player to the left of the dealer. During a turn, a player picks up a card or cards, lays down cards if possible, and discards one card.

A player may either take the top card from the stock or pick up, in order, as many cards as desired from the discards. If the third card in the discard pile is desired, the player must also pick up the first and second card. Then he checks his hand for groups of three or more identical cards, which are placed face up on the table. He may also add an identical card to any existing group that he already has on the table. A player may not have more than 10 cards in his hand following his turn.

A player concludes his turn by laying down a discard. The discards are laid down overlapping each other so that all the numbers can be seen. If a player runs out of cards, he may take five more from the stock. The game is over when the stock is exhausted and all cards possible have been played.

After the game, each player determines his scores by adding the value of all the groups of cards. After the players understand the role of multiplication, show them how to organize their work as shown.

Following the game, pick up the cards at random in order to simplify the shuffling for future games.



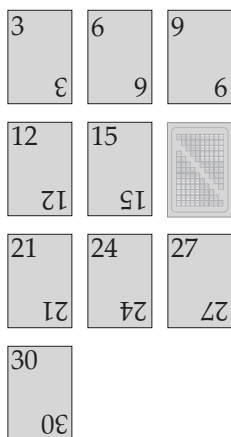
$$6 \times 4 = 24$$

$$1 \times 6 = 6$$

$$5 \times 3 = 15$$

$$4 \times 5 = \underline{20}$$

$$65$$



P4 MYSTERY MULTIPLICATION CARD

This and the next two games concentrate on one set at a time.

Objective: To help the players memorize the multiples or skip counting patterns.

Cards: One set of multiplication cards.

Number of players: Two.

Layout: The cards are arranged in the skip counting pattern (envelopes or Appendix page 18).

Object of the game: To determine the overturned card.

Play: While the first player closes her eyes or turns her back, the other player turns over a card. The first player is asked to look and name the overturned card. She then checks by turning the card over. If she names it correctly, she takes a turn at turning a card over. If not, she loses her turn.

P5 MISSING MULTIPLICATION CARDS

This game is similar to Mystery Multiplication Card (P4), but the cards are laid out in a row in order. Here the mystery card is removed and the gap closed by sliding the cards together.

P6 TREASURE HUNT

This is another fun way to reinforce learning the skip counting patterns. It involves hiding the cards around the room. Each player looks for cards from only one set.

Objective: To help the players memorize the multiples or skip counting patterns.

Number of players: From 1 to 10, depending upon the available space. Two to four or five make the best game.

SUBTRACTION

Before starting on this chapter, be sure the players thoroughly know their addition facts. When a person is trying to memorize a set of facts based on another set of facts, less confusion results if one set is thoroughly mastered first. A driver who has thoroughly learned a complicated route in one direction will find the return trip easy. But if he attempts the return trip before mastering the original trip, he is likely to become confused.

Subtraction facts are best learned through strategies. See the Appendix page 11 for various subtraction strategies.

The games in this chapter teach the subtraction facts and how to apply them in subtracting one number from another. The beginning games focus on a small group of facts; later games use more facts. A few games teach the properties of subtraction. Other games, such as Converting to Binary (S35) or Converting to Base Three (S36), require application of subtraction. Negative numbers are introduced and used in several games.

Subtraction can be thought of in three ways:

1. It answers the question, how many more are needed. This is the inverse of addition, and the result is often called the *missing addend*. It is written as $7 + \underline{\quad} = 15$.
2. It answers the question, how much more is one quantity than another. The results of the comparison is called the *difference*. The smaller number is subtracted from the larger.
3. It answers the question, how much is left, or remains, when a portion is taken away from a larger quantity. The result is also referred to as the *difference*. It used to be called “remainder,” but that can be confused with remainders in division.

The terms used to discuss subtraction are *minuend*, *subtrahend*, and *difference*.

$$\begin{array}{r} 12 \text{ minuend} \\ -4 \text{ subtrahend} \\ \hline 8 \text{ difference} \end{array}$$

For young children the best term for “borrowing” is *trading*. They understand when a trade is fair. They do not understand that terms like *exchanging*, *renaming*, and *regrouping* mean equality.

In some of the games, the 1 in the tens place is assumed. The fact $8 - 9$ is to be thought of as $18 - 9$. Children have no problem with this notation and readily accept it as eighteen minus nine. We do the same thing whenever the need to “borrow” arises. For example, in the problem $43 - 27$, we think $13 - 7$ even though the 1 is absent.

Simplified Subtraction

The subtraction algorithm, or procedure, generally taught in the United States is not the only one in general use; students in Latin America use another algorithm. In fact, there are at least seven methods.

Described below is the Simplified Subtraction method. In this simpler method, which is easy to learn and easier to use, the work proceeds from left to right like division, rather than right to left like addition. The method is explained below.

According to research, it is easier for most children to complete the work for trading, or borrowing, before performing the actual subtracting.

In the following example two trades are necessary.

$$\begin{array}{r} 8572 \\ - 6913 \\ \hline \end{array}$$

First consider the thousands; is a thousand going to be needed for a trade to get more hundreds? Yes, because 913 is more than 572. One thousand will be traded for 10 hundreds. Indicate it by underlining the 8. Initially, some children may want to write a 1 before the 5 to indicate 15 hundreds.

$$\begin{array}{r} \underline{8}572 \\ -6913 \\ \hline \end{array} \quad \begin{array}{r} \underline{8}572 \\ -6913 \\ \hline \end{array}$$

Next, look at the hundreds; is a hundred needed to make more tens? No, because 13 is less than 72. A trade is not necessary.

Finally, consider the tens; will 1 ten need to be traded to get 10 more ones? Yes, a ten must be traded because 3 is more than 2. Underline the 7 as shown above.

Now the actual subtraction can take place; 8 thousand – 6 thousand = 2 thousand, but write 1 in the thousand place. See below. Note that the line under the number indicates subtracting an extra 1. Next 15 hundreds – 9 hundreds is 6 hundreds; write 6. Continue with 7 tens – 1 ten = 6 tens, but 7 is underlined so write 5. For the ones, 12 – 3 = 9.

$$\begin{array}{r} \underline{8}572 \\ -6913 \\ \hline 1 \end{array} \quad \begin{array}{r} \underline{8}572 \\ -6913 \\ \hline 16 \end{array} \quad \begin{array}{r} \underline{8}572 \\ -6913 \\ \hline 165 \end{array} \quad \begin{array}{r} \underline{8}572 \\ -6913 \\ \hline 1659 \end{array}$$

This method simplifies problems with zeroes, such as the following:

$$\begin{array}{r} \underline{4}808 \\ -3457 \\ \hline 1 \end{array} \quad \begin{array}{r} \underline{4}808 \\ -3457 \\ \hline 1351 \end{array} \quad \begin{array}{r} \underline{8}002 \\ -4567 \\ \hline 3 \end{array} \quad \begin{array}{r} \underline{8}002 \\ -4567 \\ \hline 3435 \end{array}$$

With Simplified Subtraction it is possible to calculate any digit of the difference without going through the entire procedure. When subtracting mentally, the results can be said aloud as the subtraction is performed.

Manipulatives

The Cotter Abacus helps the players develop their powers of visualization. Players will use the abacus less and less as they form mental images.

S1 TEN MINUS

This game actually is Go to the Dump (A3) but approached from the viewpoint of subtraction.

Objective: To realize the relationship between addition and subtraction for the facts with 10 as the minuend.

Manipulatives: Cotter Abacus for each player.

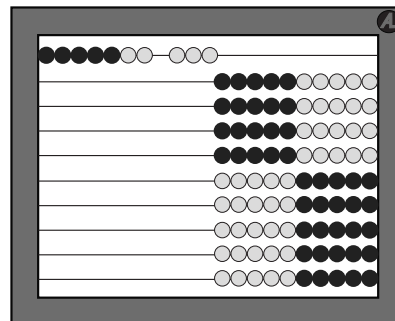
Background: Spread out a set of basic number cards 1 to 9.

Explain that we want to find 10 minus a number. Enter 10 on the abacus; choose a card, for example, a 3. Move three beads about an inch (2 cm) to the right. Ask a player how many are left. [7] Have a player find the 7 and pair it with the 3. Repeat for the other pairs.

Play: The game is identical to Go to the Dump (A3).

Variation 1: Play Eleven Minus, the subtraction version of Go to the Dump with Elevens (A24).

Variation 2: Play Nine Minus, the subtraction version of Go to the Dump with Nines (A34).



Showing $10 - 3 = 7$.

S2 TEN AS THE MINUEND

This subtraction game is another game where pairs make 10.

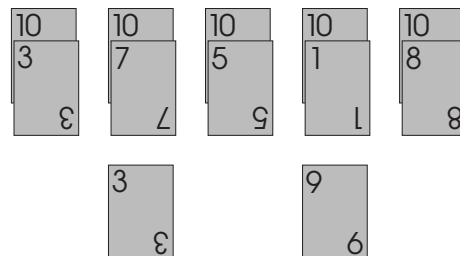
Objective: To learn the subtraction facts with 10 as the minuend.

Manipulatives: Cotter Abacus.

Number of players: Two or three, sitting on the same side of the cards.

Cards: The deck of basic number cards, but only seven 0s.

Layout: Find five 10s in the deck and lay them in a row face up. Take five cards from the stock and place one below each 10, as shown on the right.



Deal: Each player receives four cards. Following a turn, a player draws from the stock until she has four cards.

Object of the game: To collect the most cards by playing the difference cards. Only the numbers being subtracted and the difference cards are collected, leaving the 10-cards in place.

Play: The first player checks her hand for cards to form the differences. She does this by deciding what difference is needed for the first column and checking her hand to see if she has it. She checks the other four columns the same way. The differences are placed below the subtrahend cards. Then she recites the equations while collecting the cards, leaving the 10s in place.

The player then fills in the missing subtrahends with cards from the stock, then draws from the stock again until she has four cards. The next player takes his turn.

In the event that a player is unable to play, she may bury any number of cards from her hand in the stock and replace them with new cards from the stock.

The game is over when the stock is exhausted and all the cards are played.

S3 NINE AS THE MINUEND

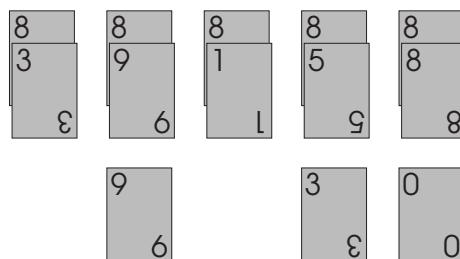
This is the same game as Ten as the Minuend (S2), but five 9s serve as the minuends.

Background: Subtracting from 9 can be thought as one less than subtracting from 10.

Cards: Use the basic number card deck, but only seven 0s and no 10s.

S4 EIGHT OR EIGHTEEN AS THE MINUEND

Here the minuend is 8. However, when the subtrahend is greater than 8, the minuend is thought of as 18. Therefore, the second column in the figure is read as $18 - 9 = 9$. No 10s are used.



Cards: Use the basic number card deck, but only seven 0s and no 10s.

S5 OTHER CONSTANT MINUENDS

Seven more games, each concentrating on a different minuend can be played, Seven or Seventeen, Six or Sixteen . . . , One or Eleven. Use the same cards and play as Eight or Eighteen as the Minuend (S4).

DIVISION

Division has often been a difficult topic in arithmetic, but it need not be. The division facts are not a separate collection of facts to learn; they are the inverse of the multiplication. However, wait until the children have mastered the multiplication facts before starting division.

Division is the process of separating into equal groups. It answers one of two questions, either the size of the groups or the number of groups. For example, a problem that asks, how many bags are needed if you put three apples into each bag, is a number of groups question. A problem that asks, how would you divide 12 cookies equally among 3 friends, is a size of groups question.

Four terms describe the various numbers of the division process.

$$\begin{array}{r} \text{quotient} \quad \text{remainder} \\ \text{divisor} \overline{) \text{dividend}} \end{array} \qquad 4 \overline{) 33} \begin{array}{l} 8 \text{ r} 1 \\ 32 \\ \hline 33 \end{array}$$

The quotient (the word is Latin for how many times) tells how many times the divisor goes into the dividend. The remainder tells what is left over. The relationship to multiplication is quotient \times divisor + remainder = dividend, for example, $(8 \times 4) + 1 = 33$.

There are four ways to write division problems as shown below. The first three are introduced in the games. The slanted form (/) is confusing for children and is rarely used in computations.

$$\begin{array}{r} 3 \\ 5 \overline{) 15} \end{array} \qquad 15 \div 5 = 3 \qquad \frac{15}{5} = 3 \qquad 15/3 = 3$$

How important is long division? Long division is unnecessary for single-digit divisors. It does not help in understanding the process, is very inefficient, interferes with learning a more efficient method, and is not a necessary preparation for double-digit division. Short division, where nothing is written below the dividend, is the preferred operation.

Unfortunately, about 25 percent of adults in the U.S. did not learn short division in school. However, it is quite simple to master and is explained on page 112.

After mastering short division, most children should learn long division. It is necessary, for example, to observe repeating decimals. On the other hand, children with learning problems should not bother to learn it. With calculators so readily available, they will never use it. Their time is better spent in learning other mathematics.

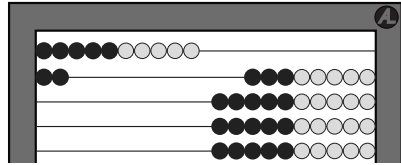
Some division concepts are included in the Fraction chapter.

D1 FIND THE QUOTIENTS

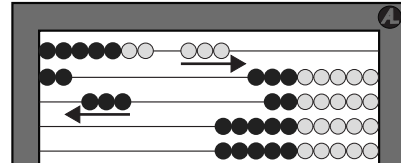
This game asks the player or players to match the dividends with the corresponding quotients for a particular divisor.

Objective: To see the relationship between multiplication and division.

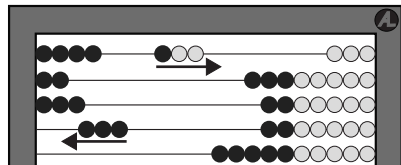
Background: Ask a player to enter 12 on the Cotter Abacus. Then ask, "How many 3s are in 12?" Since only 3 beads can remain on each wire, remove 3 extra beads from the first row (see the next page) and move them to the third wire. This is done in one smooth operation, with the right hand removing 3 beads and the left hand entering 3 beads. Remove another 3 beads and move them to the fourth wire. Finally, move the last extra bead to the second wire. Repeat the question. [4] "There are four 3s in 12 and 3×4 is 12." Repeat with other examples.



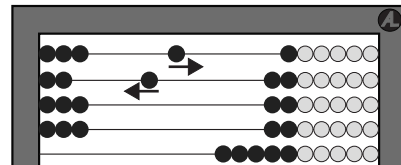
12 entered.



3 from the first row moved to the third row.



3 from the 1st row moved to the 4th row.



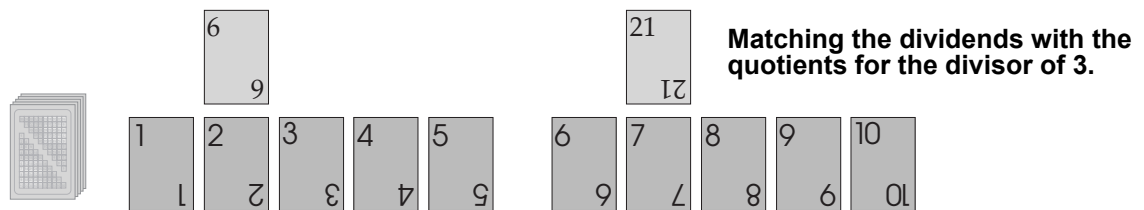
1 last bead moved to the second row.

Number of players: One or two.

Manipulatives: Cotter Abacus or cards for counters.

Cards: Use a set of basic number cards 1 to 10 and one set of multiplication cards.

Layout: Lay out in a row face up the cards 1 to 10 with a space after 5. The set of shuffled multiplication cards face down forms the stock.



Matching the dividends with the quotients for the divisor of 3.

Object of the game: To match all the cards.

Play: The player picks up the top multiplication card, determines how many times the divisor is contained in that number, and places the card above the corresponding number in the row. For example, if the divisor is 3 and 21 is picked up, the player determines the quotient to be 7 and places 21 above 7. Two players take turns.

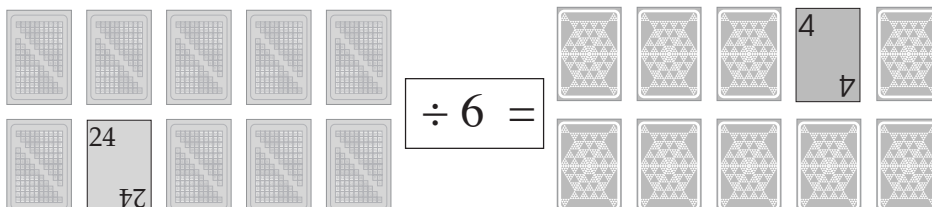
D2 DIVISION MEMORY

This game is essentially the same as Multiplication Memory (P10) but here the multiplication card is turned over first.

Objective: To see the relationship between multiplying and dividing.

Background: The Skip Counting Patterns (Appendix page 18) provides a good reference. To find how many 6s in 24, look at the 6 multiples and observe the position of 24. [4th position]

6	12	18	24	30
36	42	48	54	60



Matching the dividends with the quotients for the divisor of 6.

Number of players: Two, sitting on the same side of the cards.

Cards: A set of basic number cards from 1 to 10 and any set of multiplication cards.

Layout: Lay the multiplication cards out in rows face down. To the right lay out the basic number cards. If desired, put a piece of paper between them saying, for example, “ $\div 6 =$.”

Object of the game: To collect the most pairs.

Play: The first player turns over a multiplication card and says the quotient aloud. He then chooses the most likely basic number card. If it is the correct quotient, the player collects both cards and receives another turn. If it is not correct, both cards are returned face down and the second player takes her turn.

D3 MIXED-UP QUOTIENTS

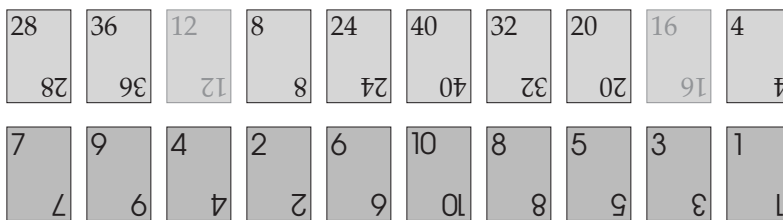
The title of this little game describes it well.

Objective: To practice recognizing dividends and quotients.

Number of players: Two or possibly three.

Cards: Use one set of multiplication cards and a set of basic number cards 1 to 10.

Layout: Lay out in random order, a row face up the set of multiplication cards. In the row below, place the corresponding quotient.



Finding and correcting the two mixed-up cards for the divisor of 4.

Play: The first player turns around while the second player interchanges two cards in one of the rows. The first player finds the mix-ups and corrects it. She then takes a turn.

D4 QUOTIENT RACE

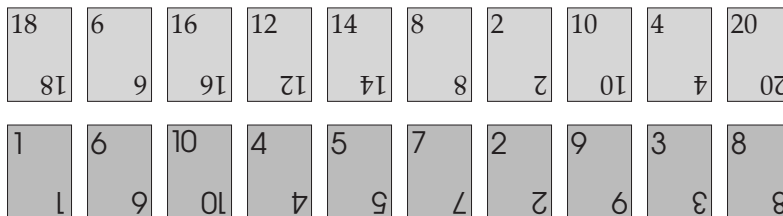
Only players who can stand the pressure should play.

Objective: To work quickly matching dividends and quotients.

Number of players: Two to six.

Cards: One set of multiplication cards and one set of basic number cards 1 to 10 for each player.

Layout: Players lay out their cards face up but in random order, the 10 multiplication cards in the top row and the basic number cards in the second row.



Finding and correcting the mixed-up rows for the divisor of 2.

Object of the game: To be the first player to match all her cards.

Play: Be sure the players know their divisors. At the starting signal, players race to match the corresponding dividend and quotient.

FRACTIONS

Many people become uncomfortable when fractions are mentioned. I know an excellent cabinetmaker who can add fractions only with the help of a tape measure. A mathematics teacher tells this story: During sixth grade while attending a parochial school, she was introduced to dividing fractions. She followed the invert and multiply rule but believed that the children in the public school had a different rule to use. She was not bothered that the answers would be different. To her it was only a paper and pencil exercise; she was unconcerned about striving for a consistent or useful solution.

Introduce children to fractions early—in kindergarten or first grade. They are part of every-day life. When children do not learn fractions until the fourth or fifth grades, they are so convinced that 5 is always more than 4 that they have a problem with $\frac{1}{5}$ being less than $\frac{1}{4}$. In working with fractions, children should only use the horizontal form, not the slanted line.

Most work with fractions should be done with fraction charts, a copy of which is in the Appendix on page 21. Sturdier charts the are made of plastic are available from Activities for Learning, Inc. at RightStartMath.com. Although circles are often used to represent fractions, comparing fractions with them is difficult. Are we comparing areas, angles, or arcs? Showing fractions greater than one is also more difficult with circles.

To be successful with more advanced fractions, children must be thoroughly familiar with multiplication and division of whole numbers. This background is needed to simplify a fraction to lowest terms and to find a lowest common denominator.

Some might feel that fractions are becoming obsolete. The metric system does not need them and calculators use decimals. However, fractions are the culmination of arithmetic and are essential for understanding algebra and other advanced topics. In essence, fractions are division.

Generally, the games should be played in the order given. Do not be in a hurry to play the next game. Frequently go back to games already learned; the children will often play them from a new perspective.

F1 BEGINNING FRACTION ACTIVITIES

These initial activities introduce fractions concretely. They have been used with children as young as five and six and with adults.

Cut one fraction chart into its individual pieces. The players will also need an uncut fraction chart. A paper chart can be found on Appendix page 21.

Activity 1: Show the players the fraction chart and ask them to use their pieces to build it. They should not build it on top of the model.

Activity 2: Tell the players that what the word *fraction* means is to break, or divide, into pieces; it is derived from the Latin “frangere.” Until this place in arithmetic, 1 was considered the smallest unit. Now one will be fractured, although always into equal pieces.

Give the players strips of paper the same length as the 1 on the fraction chart. Demonstrate and then have them fold a strip in half and tear or cut it apart at the fold. Write the fraction $\frac{1}{2}$ and explain that it means 1 divided by 2. The line between the numbers means *divided by*. Repeat for $\frac{1}{3}$ and $\frac{1}{4}$ with other strips.

Now ask the players to find one of each of the following fraction pieces: 1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9, and 1/10. Except for 1, these are the unit fractions.

Help the players pronounce the names correctly. With the exception of one-half, the names are the same as the ordinal numbers: third, fourth, fifth, sixth, and so on. Some players may be helped by remembering that third and fifth are similar to 30 and 50, or 13 and 15.

Activity 3: Ask the players to find the 1 and set it aside. Then ask them to find the 1/2 and place it above the 1 with the left edges even, as shown. Continue with the 1/3 and 1/4. Let them complete the “stairs” on their own.

When the stairs are completed, ask if they could climb such a stairs. Ask, “What happens near the top?” This curve is known as a hyperbola.

With the pieces arranged in the stairs, ask such questions as: “Which is more, 4 or 5? But which is more 1/4 or 1/5? Which is more 2 or 3? 1/2 or 1/3? 1/2 or 1?”

Activity 4: Ask them to complete the chart. Then ask such questions as: “How many thirds do you need to make a whole? How many fourths? How many fifths?” Use the terms “whole” and “one” interchangeably.

Activity 5: To explain the meaning of 3/4, lay three 1/4s under the 1. Ask a player how many 1/4s she sees. [3] Then show that we write it as 3/4. Repeat with several other combinations. Then explain that the top number, called the numerator, tells the number of parts. Also give each player a fraction card and ask her to bring the corresponding fraction pieces. (Do not teach at this stage that 3/4 is 3 divided by 4.)

The unit fractions.

F2 FRACTION MEMORY

It takes time to adjust to the concept that the larger the denominator, the smaller the fraction.

Objective: To help the players become familiar with the relative sizes of the fraction pieces and to realize that as the denominator increases, the fraction decreases.

Cards: Use only eight fraction cards: 1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/8, and 1/10. Use also the eight corresponding fraction pieces.

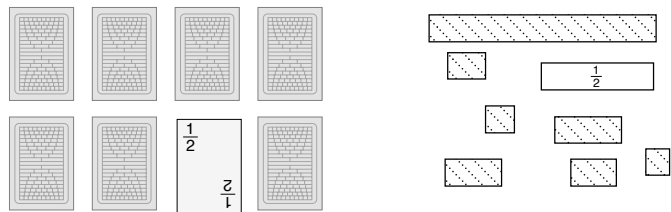
Number of players: Two only.

Layout: Lay the fraction cards out on the table in random order face down. Nearby lay out the fraction pieces from the fraction chart, also face down.

Object of the game: To collect the most cards.

Play: The first player turns over a card. Then he turns over a fraction piece. If they match, he collects them, but no extra turn is given. If they do not match, both the card and piece are returned face down.

The second player then takes her turn. Turns continue until all the cards are collected.



Matching fraction cards and pieces.

F2.1 UNIT FRACTION WAR

This is another game where players compare unit fractions.

Objective: To practice comparing unit fractions.

Cards: Use all the unit fraction cards: 1/2, 1/3, 1/4, 1/5, 1/6, 1/8, and 1/10, and four 1s.

Manipulatives: The fraction chart.

Number of players: Two or two teams.

Deal: Keep the cards face down and divide them evenly between the two players.

Object of the game: To capture all the cards.

Play: Each player takes the top card from his stack and lays it down in the middle of the table face up. The player whose card is greater takes both cards. Players alternate deciding whose card is greater.

Players continue comparing cards until they play identical cards, causing a “war.” To resolve a war, both players play one card face down and then play a third card face up to be compared. The player who has the higher card in the last comparison takes all six cards.

F3 CONCENTRATING ON ONE

Players need time to understand the concept that two halves, three thirds, or eight eighths are needed to make a whole. This memory game will help.

1				
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Seeing that five 1/5s equal 1.

Objective: To help the players realize that two halves, three thirds, and so forth, equal one.

Being told this fact does not necessarily mean understanding it.

Background: Review that $\frac{2}{3}$ means two $\frac{1}{3}$ s. Then lay down various fraction cards and ask the players to find the equivalent fraction pieces.

Now ask a player to lay the fraction pieces for $\frac{3}{5}$ under the 1. Then ask her how many more fifths are needed to make 1. [two $\frac{1}{5}$ s] Repeat this for other fractions, such as $\frac{1}{6}$ and $\frac{7}{10}$. Players sometimes have trouble with $\frac{1}{2}$. [one $\frac{1}{2}$]

Some players find the fraction chart to be very helpful. With it they can see what they have and count how many more are needed. With the left index finger, the player counts what she has. With the left finger still in place, she counts with her right index finger how many more she needs. Explain that these are the pairs needed for this game.

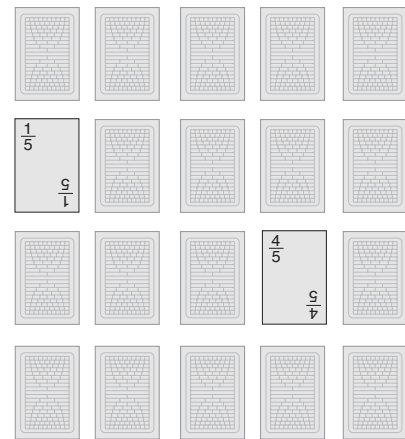
Cards: Twenty fraction cards: two $\frac{1}{2}$ s and one of each of the following: $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{10}$, $\frac{3}{10}$, $\frac{7}{10}$, and $\frac{9}{10}$.

Number of players: Two or two teams.

Layout: Lay the fraction cards out on the table face down in rows as shown.

Object of the game: To collect the most pairs of fractions that equal one.

Play: The first player turns over a card and says aloud how many more are needed to make 1. She then chooses a probable card. If she is correct, she collects both cards and takes another turn. If they do not match, both cards are returned face down. The second player then takes his turn. Turns continue until all the cards are collected.



Two fractions that equal 1.

F4 FRACTION OLD MAID

This new version of the childhood favorite Old Maid uses fractions totaling 1 as pairs. The loser is the player with the “dead end” card.

Objective: To help the player realize that two halves, three thirds, or eight eighths are needed to make a whole.

Manipulatives: The fraction pieces or the fraction chart. Generally the younger players prefer the pieces while the older players like the chart.

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