# RightStart™ Mathematics

by Joan A. Cotter, Ph.D.
with Kathleen Cotter Lawler

## LEVEL E LESSONS

Second Edition

Activities for Learning, Inc.

A special thank you to Maren Ehley and Rebecca Walsh for their work in the final preparation of this manual.

Note: Levels are used rather than grades. For example, Level A is kindergarten and Level B is first grade and so forth.



Printed in the United States of America

**www.RightStartMath.com**

For more information: info@RightStartMath.com
Supplies may be ordered from: www.RightStartMath.com

Activities for Learning, Inc.
321 Hill Street
Hazelton, ND 58544-0468
United States of America
888-775-6284 or 701-782-2000
701-782-2007 fax

ISBN 978-1-942943-11-2

May 2024

# RightStart™ Mathematics Objectives for Level E

| | Quarter 1 | Quarter 2 | Quarter 3 | Quarter 4 |
|---|---|---|---|---|
| **Numeration** | | | | |
| Understands and finds prime numbers | N/A | | | |
| Factors numbers | N/A | | | |
| Reads, writes, rounds, and compares numbers to the billions | | | | |
| **Addition and Subtraction** | | | | |
| Adds and subtracts multi-digit numbers in multiple ways | | | | |
| **Multiplication and Division** | | | | |
| Knows multiplication facts to 10 × 10 | N/A | | | |
| Knows division facts, including remainders | N/A | N/A | | |
| Applies commutative, associative, and distributive properties | N/A | | | |
| Multiplies multiples of 10, e.g. 80 × 7 | N/A | | | |
| Multiplies multi-digit numbers by a 2-digit number | N/A | N/A | | |
| Does short division to divide multi-digit number by a single digit | N/A | N/A | | |
| **Problem Solving** | | | | |
| Solves two-step problems involving four operations | | | | |
| Writes equations to represent story problems | | | | |
| Solves division story problems with remainders | | | | |
| Solves elapsed time, distance, money, and capacity problems | N/A | N/A | N/A | |
| **Measurement** | | | | |
| Understands square units: $cm^2$, $dm^2$, sq ft, and sq yd | N/A | N/A | | |
| Finds perimeter and area in customary and metric units | N/A | N/A | | |
| Converts measurements in same system (e.g., g to kg) | N/A | N/A | N/A | |
| **Fractions** | | | | |
| Adds and subtracts simple fractions and mixed numbers | N/A | | | |
| Understands $a/b$ as $1/b$ multiplied by $a$ | N/A | | | |
| Understands $n\frac{a}{b}$ as a whole number plus a fraction | N/A | | | |
| Compares and finds equivalences on the fraction chart | N/A | | | |
| Multiplies fractions times a whole number | N/A | | | |
| **Decimals and Percents** | | | | |
| Understands decimals as fractions of tenths or hundredths | N/A | N/A | | |
| Converts decimal fractions from tenths to hundredths and back | N/A | N/A | | |
| Adds, subtracts, and compares decimals to two decimal places | N/A | N/A | | |
| Understands and uses simple percents | N/A | N/A | | |
| **Patterns** | | | | |
| Recognizes and continues numeric and geometric patterns | N/A | N/A | N/A | |
| Uses algebraic thinking to write a pattern symbolically | | | | |
| Solves simple equations | N/A | | | |
| **Data** | | | | |
| Makes line plots and interprets data | N/A | N/A | N/A | |
| **Geometry** | | | | |
| Locates lines of symmetry and draws reflections | N/A | N/A | N/A | |
| Knows angles 30°, 45°, 60°, 90°, 180°, and 360° | N/A | N/A | N/A | |
| Classifies shapes by attributes | N/A | N/A | N/A | |
| Constructs equilateral triangle and other shapes | N/A | N/A | N/A | |

# How This Program Was Developed

We have been hearing for years that students in Japan do better than U.S. students in math. The Asian students are ahead by the middle of first grade. And the gap widens every year thereafter.

Many explanations have been given, including less diversity and a longer school year. Japanese students attend school 240 days a year.

A third explanation given is that the Asian public values and supports education more than we do. A first grade teacher has the same status as a university professor. If a student falls behind, the family, not the school, helps the child or hires a tutor. Students often attend after-school classes.

A fourth explanation involves the philosophy of learning. Asians and Europeans believe anyone can learn mathematics or even play the violin. It is not a matter of talent, but of good teaching and hard work.

Although these explanations are valid, I decided to take a careful look at how mathematics is taught in Japanese first grades. Japan has a national curriculum, so there is little variation among teachers.

I found some important differences. One of these is the way the Asians name their numbers. In English we count ten, eleven, twelve, thirteen, and so on, which doesn't give the child a clue about tens and ones. But in Asian languages, one counts by saying ten-1, ten-2, ten-3 for the teens, and 2-ten 1, 2-ten 2, and 2-ten 3 for the twenties.

Still another difference is their criteria for manipulatives. Americans think the more the better. Asians prefer very few, but insist that they be imaginable, that is, visualizable. That is one reason they do not use colored rods. You can imagine the one and the three, but try imagining a brown eight–the quantity eight, not the color. It cannot be done without grouping.

Another important difference is the emphasis on non-counting strategies for computation. Japanese children are discouraged from counting; rather they are taught to see quantities in groups of fives and tens.

For example, when an American child wants to know 9 + 4, most likely the child will start with 9 and count up 4. In contrast, the Asian child will think that if he takes 1 from the 4 and puts it with the 9, then he will have 10 and 3, or 13. Unfortunately, very few American first-graders at the end of the year even know that 10 + 3 is 13.

I decided to conduct research using some of these ideas in two similar first grade classrooms. The control group studied math in the traditional workbook-based manner. The other class used the lesson plans I developed. The children used that special number naming for three months.

They also used a special abacus I designed, based on fives and tens. I asked 5-year-old Stan how much is 11 + 6. Then I asked him how he knew. He replied, "I have the abacus in my mind."

The children were working with thousands by the sixth week. They figured out how to add 4-digit numbers on paper after learning how on the abacus.

Every child in the experimental class, including those enrolled in special education classes, could add numbers like 9 + 4, by changing it to 10 + 3.

I asked the children to explain what the 6 and 2 mean in the number 26. Ninety-three percent of the children in the experimental group explained it correctly while only 50% of third graders did so in another study.

I gave the children some base ten rods (none of them had seen them before) that looked like ones and tens and asked them to make 48. Then I asked them to subtract 14. The children in the control group counted 14 ones, while the experimental class removed 1 ten and 4 ones. This indicated that they saw 14 as 1 ten and 4 ones and not as 14 ones. This view of numbers is vital to understanding algorithms, or procedures, for doing arithmetic.

I asked the experimental class to mentally add 64 + 20, which only 52% of nine-year-olds on the 1986 National test did correctly; 56% of those in the experimental class could do it.

Since children often confuse columns when taught traditionally, I wrote 2304 + 86 = horizontally and asked them to find the sum any way they liked. Fifty-six percent did so correctly, including one child who did it in his head.

The following year I revised the lesson plans and both first grade classes used these methods. I am delighted to report that on a national standardized test, both classes scored at the 98th percentile.

*Joan A. Cotter, Ph.D.*

# Some General Thoughts on Teaching Mathematics

1. Only five percent of mathematics should be learned by rote; 95 percent should be understood.
2. Real learning builds on what the child already knows. Rote teaching ignores it.
3. Contrary to the common myth, "young children can think both concretely and abstractly. Development is not a kind of inevitable unfolding in which one simply waits until a child is cognitively 'ready.'" —*Foundations for Success* NMAP
4. What is developmentally appropriate is not a simple function of age or grade, but rather is largely contingent on prior opportunities to learn." —Duschl & others
5. Understanding a new model is easier if you have made one yourself. So, a child needs to construct a graph before attempting to read a ready-made graph.
6. Good manipulatives cause confusion at first. If a new manipulative makes perfect sense at first sight, it is not needed. Trying to understand and relate it to previous knowledge is what leads to greater learning. —Richard Behr & others.
7. According to Arthur Baroody, "Teaching mathematics is essentially a process of translating mathematics into a form children can comprehend, providing experiences that enable children to discover relationships and construct meanings, and creating opportunities to develop and exercise mathematical reasoning."
8. Lauren Resnick says, "Good mathematics learners expect to be able to make sense out of rules they are taught, and they apply some energy and time to the task of making sense. By contrast, those less adept in mathematics try to memorize and apply the rules that are taught, but do not attempt to relate these rules to what they know about mathematics at a more intuitive level."
9. Mindy Holte puts learning the facts in proper perspective when she says, "In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic." She also emphasizes the ability to imagine or visualize, an important skill in mathematics and other areas.
10. The only students who like flash cards are those who do not need them.
11. Mathematics is not a solitary pursuit. According to Richard Skemp, solitary math on paper is like reading music, rather than listening to it: "Mathematics, like music, needs to be expressed in physical actions and human interactions before its symbols can evoke the silent patterns of mathematical ideas (like musical notes), simultaneous relationships (like harmonies) and expositions or proofs (like melodies)."
12. "More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority. This is a very important lesson to learn, an essential step in the emergence of independent thinking." —*Everybody Counts*

13. The role of the teacher is to encourage thinking by asking questions, not giving answers. Once you give an answer, thinking usually stops.
14. Putting thoughts into words helps the learning process.
15. Help the children realize that it is their responsibility to ask questions when they do not understand. Do not settle for “I don’t get it.”
16. The difference between a novice and an expert is that an expert catches errors much more quickly. A violinist adjusts pitch so quickly that the audience does not hear it.
17. Europeans and Asians believe learning occurs not because of ability, but primarily because of effort. In the ability model of learning, errors are a sign of failure. In the effort model, errors are natural. In Japanese classrooms, the teachers discuss errors with the whole class.
18. For teaching vocabulary, be sure either the word or the concept is known. For example, if a child is familiar with six-sided figures, we can give him the word, hexagon. Or, if he has heard the word, multiply, we can tell him what it means. It is difficult to learn a new concept and the term simultaneously.
19. Introduce new concepts globally before details. This lets the children know where they are headed.
20. Informal mathematics should precede paper and pencil work. Long before a child learns how to add fractions with unlike denominators, she should be able to add one half and one fourth mentally.
21. Some pairs of concepts are easier to remember if one of them is thought of as dominant. Then the non-dominant concept is simply the other one. For example, if even is dominant over odd, an odd number is one that is not even.
22. Worksheets should also make the child think. Therefore, they should not be a large collection of similar exercises, but should present a variety. In RightStart™ Mathematics, they are designed to be done independently.
23. Keep math time enjoyable. Our emotional state at the time we learn something is attached to that information. A person who dislikes math will avoid it and a child under stress stops learning. If a lesson is too hard, stop and play a game. Try the lesson again later.
24. In Japan students spend more time on fewer problems. Teachers do not concern themselves with attention spans as is done in the U.S.
25. In Japan the goal of the math lesson is that the student has understood a concept, not necessarily has done something (a worksheet).
26. The calendar must show the entire month, so the children can plan ahead. The days passed can be crossed out or the current day circled.
27. A real mathematical problem is one in which the procedures to find the answer are not obvious. It is like a puzzle, needing trial and error. Emphasize the satisfaction of solving problems and like puzzles, of not giving away the solution to others.

## RightStart™ Mathematics

Ten major characteristics make this research-based program effective:

1. Refers to quantities of up to 5 as a group; discourages counting individually. Uses fingers and tally sticks to show quantities up to 10; teaches quantities 6 to 10 as 5 plus a quantity, for example 6 = 5 + 1.
2. Avoids counting procedures for finding sums and differences. Teaches five- and ten-based strategies for the facts that are both visual and visualizable.
3. Employs games, not flash cards, for practice.
4. Once quantities 1 to 10 are known, proceeds to 10 as a unit. Temporarily uses the "math way" of naming numbers; for example, "1 ten-1" (or "ten-1") for eleven, "1-ten 2" for twelve, "2-ten" for twenty, and "2-ten 5" for twenty-five.
5. Uses expanded notation (overlapping) place-value cards for recording tens and ones; the ones card is placed on the zero of the tens card. Encourages a child to read numbers starting at the left and not backward by starting at the ones.
6. Proceeds rapidly to hundreds and thousands using manipulatives and place-value cards. Provides opportunities for trading between ones and tens, tens and hundreds, and hundreds and thousands with manipulatives.
7. Teaches mental computation. Investigates informal solutions, often through story problems, before learning procedures.
8. Teaches four-digit addition on the abacus, letting the child discover the paper and pencil algorithm.
9. Introduces fractions with a linear visual model, including all fractions from 1⁄2 to 1⁄10. "Pies" are not used initially because they cannot show fractions greater than 1. Later, the tenths will become the basis for decimals.
10. Teaches short division (where only the answer is written down) for single-digit divisors, before long division.

## Second Edition

Many changes have occurred since the first RightStart™ lessons were begun in 1994. First, mathematics is used more widely in many fields, for example, architecture, science, technology, and medicine. Today, many careers require math beyond basic arithmetic. Second, research has given us new insights into how children learn mathematics. Third, kindergarten has become much more academic, and fourth, most children are tested to ensure their preparedness for the next step.

This second edition is updated to reflect new research and applications. Topics within each level are always taught with the most appropriate method using the best approach with the child and teacher in mind.

# Daily Lessons

### Objectives

The objectives outline the purpose and goal of the lesson. Consider the words; "to introduce" is not the same as "to review." When a topic is introduced, it is not expected to be mastered during that lesson. When a topic is reviewed, proficiency should be close.

### Materials

The manipulatives needed for the lessons are specially chosen items needed to teach the lessons. Occasionally, common objects, such as scissors, will be needed and will be listed in bold type.

### Warm-up

The warm-up provides review, memory work, or an introduction of the day's topics. It can be reduced, modified, or expanded to meet a child's needs.

### Activities

Activities are the heart of the lesson. These are the instructions for teaching the lesson. When guided to ask a question, the expected answer from the child is given in square brackets.

### Explanations

Special background notes and supporting information for the teacher are provided here.

There are Overview Videos to guide and support you weekly. The provided QR code will direct you to the appropriate video.

### Games

Games, not worksheets or flash cards, provide practice. The games, found in the *Math Card Games* book, should be played as many times as necessary until proficiency or memorization takes place. Games are important to learning math, just as books are important to learning reading.

The *Math Card Games* book includes extra games for the child needing more help and more challenging games for the advanced child.

Instructional videos for all the games used in the RightStart™ Mathematics curriculum are available on Vimeo for a small subscription fee.

**Instructional game videos**

### Worksheets

The worksheets are designed to be completed independently in order to demonstrate understanding of the day's lesson. Some lessons, especially in the early levels, have no worksheet.

### In conclusion

Each lesson ends with a short summary based on the day's learning.

### Timeline

Each RightStart Math level is designed for one school year. This level should be completed in full before beginning the next level.

*Joan A. Cotter, Ph.D.*
info@RightStartMath.com

# Level E Table of Contents

# Level E Table of Contents

# Level E Table of Contents

# Level E Table of Contents

# Lesson 56: Adding Mixed Numbers with Eighths

**OBJECTIVES:**

1. To learn the terms *proper fraction* and *improper fraction*
2. To practice adding fractions with eighths
3. To convert improper eighths to proper eighths

**MATERIALS:**

1. Fraction chart and fraction pieces
2. *Math Card Games* book, F22.1
3. Math journal

**ACTIVITIES FOR TEACHING:**

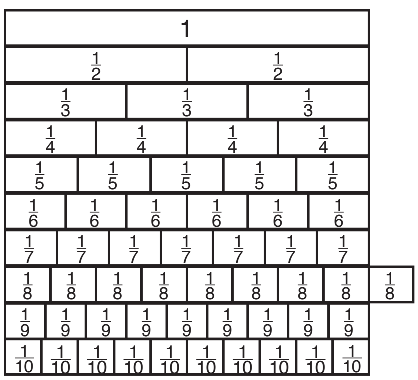
**EXPLANATIONS:**

***Warm-up.*** Ask: In the fraction one fifth, what is the denominator? [5] In the fraction one fifth, what number is the numerator? [1] If the denominator and numerator are the same, what does the fraction equal? [1]

***Improper fractions.*** Give the child the fraction chart and fraction pieces.

Write: $\frac{9}{8}$

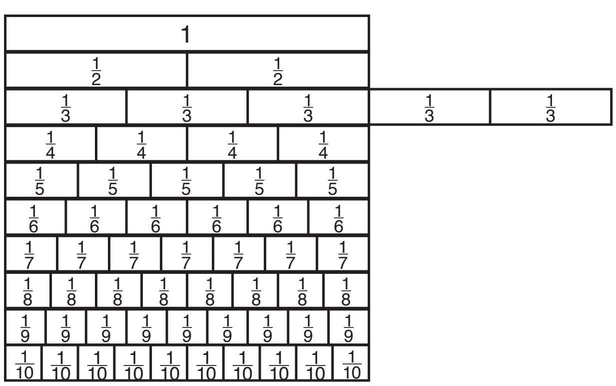
and ask the child to show it with her fraction chart and fraction pieces. [8 eighths plus 1 more eighth] See figure below.




**Showing $\frac{9}{8}$ with the fraction chart and pieces.**

Write: $\frac{5}{3}$

and tell her to show it with the fraction materials. See figure below.




**Showing $\frac{5}{3}$ with the fraction chart and pieces.**

## ACTIVITIES FOR TEACHING CONTINUED:

Write: $\frac{9}{8}$ $\frac{5}{3}$ $\frac{3}{4}$

Ask: Which of these three fractions is less than one? [$\frac{3}{4}$] How can you tell by looking only at the numerators and denominators? [The numerator is less than the denominator.]

Say: When the numerator is less than the denominator, the fraction is called a *proper fraction*. This name results from hundreds of years ago when people thought a "real" fraction had to be less than one. The word "fraction" comes from the Latin word "frangere" meaning "to break." Two other words from this root word are fracture and fragment. Mathematicians realized fractions were division and often were not less than one. They called fractions equal to or greater than one *improper fractions*.

Write: $\frac{4}{8}$ $\frac{7}{4}$ $\frac{4}{3}$ $\frac{8}{8}$ $\frac{12}{8}$ $\frac{1}{6}$

Ask: Which of these are proper fractions? [only the first and last fractions, $\frac{4}{8}$ and $\frac{1}{6}$]

Ask: How can we rewrite the improper fractions using a whole number plus a fraction? [$1\frac{3}{4}$, $1\frac{1}{3}$, 1, $1\frac{4}{8}$]

**EXPLANATIONS CONTINUED:** This can be done by referring to the fraction chart. No algorithm is necessary.

***Preparation for Corners™ with Eighths game.*** Explain that the game for the day will be a Corners™ Three game variation. Now each number on the cards will be *eighths*. For example, 3 is $\frac{3}{8}$ and 9 is $\frac{9}{8}$.

Write: $1\frac{3}{8} + \frac{9}{8} =$

and ask the child to add it. Ask the child to explain her work.

One way is: $1\frac{3}{8} + \frac{9}{8} = 1\frac{12}{8} = 2\frac{4}{8}$

Another way is:

$$1\frac{3}{8} + \overset{1\frac{1}{8}}{\cancel{\frac{9}{8}}} = 2\frac{4}{8}$$

The answers need not be in lowest terms.

Give her another example:

$$2\frac{5}{8} + \frac{18}{8} = [2\frac{23}{8} = 4\frac{7}{8} \text{ or } 2\frac{5}{8} + \overset{2\frac{2}{8}}{\cancel{\frac{18}{8}}} = 4\frac{7}{8}]$$

***Corners™ with Eighths game.*** Play Corners™ with Eighths game, found in *Math Card Games* book, F22.1. Stress that the fractions in the scoring sums must be proper fractions. Tell her to write the scoring in her math journal.

***In conclusion.*** Ask: What do we call a fraction when the numerator is greater than the denominator? [improper] What is a fraction called when the denominator is greater than the numerator? [proper]

# Lesson 59: Multiplying by Two Digits

**OBJECTIVE:**

1. To develop a procedure for multiplying by two digits

**MATERIALS:**

1. Worksheet 37, Multiplying by Two Digits

**ACTIVITIES FOR TEACHING:**

***Warm-up.*** Ask: What is 31 × 2? [62] What is 31 × 20? [620] What is 31 × 200? [6200]

Ask: What is 23 × 3? [69] What is 23 × 30? [690] What is 23 × 300? [6900]

***Multiplying by two digits.*** Write these three problems:

$$\begin{array}{r}312\\ \underline{\times\ 2}\\ 624\end{array}\qquad\begin{array}{r}312\\ \underline{\times\ 30}\\ 9360\end{array}\qquad\begin{array}{r}312\\ \underline{\times\ 32}\end{array}$$

Say: You have been multiplying problems like the first one for several months now. In yesterday's lesson you multiplied numbers with tens like the second problem. Today you will multiply numbers with two digits like the third problem.

Ask: How do you think you could do it? Tell the child to share her thoughts. Two solutions are below.

$$\begin{array}{r}312\\ \underline{\times\ 32}\\ 624\\ \underline{9360}\\ 9984\end{array}\qquad\begin{array}{r}312\\ \underline{\times\ 32}\\ 9360\\ \underline{624}\\ 9984\end{array}$$

**EXPLANATIONS:**

It is acceptable to multiply the leftmost digit first.

***Worksheet 37.*** Give the child the worksheet and tell her to do the first two rows in the left box. The solutions are below.

Then tell her to discuss her answer.

$$\begin{array}{r}63\\ \underline{\times\ 5}\\ \mathbf{315}\end{array}\quad\begin{array}{r}63\\ \underline{\times\ 30}\\ \mathbf{1890}\end{array}\quad\begin{array}{r}63\\ \underline{\times\ 35}\\ \mathbf{315}\\ \underline{\mathbf{1890}}\\ \mathbf{2205}\end{array}\qquad\begin{array}{r}825\\ \underline{\times\ 6}\\ \mathbf{4950}\end{array}\quad\begin{array}{r}825\\ \underline{\times\ 50}\\ \mathbf{41,250}\end{array}\quad\begin{array}{r}825\\ \underline{\times\ 56}\\ \mathbf{4950}\\ \underline{\mathbf{41250}}\\ \mathbf{46,200}\end{array}$$

## ACTIVITIES FOR TEACHING CONTINUED:

## EXPLANATIONS CONTINUED:

Repeat for the last two rows in the left box. The solutions are below.

| | | | | | |
|---|---|---|---|---|---|
| 3674 | 3674 | 3674 | 9062 | 9062 | 9062 |
| × 1 | × 80 | × 81 | × 7 | × 20 | × 27 |
| **3674** | **293,920** | **3674** | **63,434** | **181,240** | **63434** |
| | | **293920** | | | **181240** |
| | | **297,594** | | | **244,674** |

***Writing the 'carries.'*** Write:

$$\begin{array}{r} 28 \\ \underline{\times\ 43} \end{array}$$

and multiply the 28 × 3 part. See below.

$$\begin{array}{r} {\scriptstyle 2} \\ 28 \\ \underline{\times\ 43} \\ 84 \end{array}$$

Do not insist that the child write the little ones. Some can do it mentally.

Continue with multiplying the 28 × 40.

$$\begin{array}{r} {\scriptstyle 3} \\ {\scriptstyle 2} \\ 28 \\ \underline{\times\ 43} \\ 84 \\ \underline{1120} \\ 1204 \end{array}$$

Technically, it is not necessary to write the 0 in the right column of the second line. However, it helps children in their understanding that they are multiplying by 3 tens and not by 3 ones.

Unfortunately, some children have been taught to write an "x" as the placeholder. This nonstandard use of x has caused those children considerable confusion when they studied algebra.

Explain that the carries, the little numbers, can be written in rows above the problem, but many people do not write them at all; they do it mentally.

***Worksheet 37.*** Tell the child to complete the worksheet. The solutions are below.

| | | | |
|---|---|---|---|
| 81 | 143 | 572 | 2927 |
| × 52 | × 33 | × 64 | × 81 |
| **162** | **429** | **2288** | **2927** |
| **4050** | **4290** | **34320** | **234160** |
| **4212** | **4719** | **36,608** | **237,087** |

| | | | | |
|---|---|---|---|---|
| **365** | **365** | **365** | **365** | **365** |
| **× 2** | **× 9** | **× 10** | **× 55** | **× 26** |
| **730** | **3285** | **3650** | **1825** | **2190** |
| | | | **18250** | **7300** |
| | | | **20,075** | **9490** |

***In conclusion.*** Ask: If you multiply 2 by 50 and then 2 by 3 and add them together, what is the answer? [106, 2 × 53] If you multiply any number by 50 and then by 3 and add them together, what is the answer? [number × 53]

If there is additional time following this lesson, play the Multiples Solitaire game, found in *Math Card Games* book, P19.

Name: ______________________

Date: ______________________

1. Find the products by using your previous answers wherever possible.

$$\begin{array}{r} 63 \\ \times\ 5 \\ \hline \end{array} \qquad \begin{array}{r} 63 \\ \times\ 30 \\ \hline \end{array} \qquad \begin{array}{r} 63 \\ \times\ 35 \\ \hline \end{array}$$

$$\begin{array}{r} 825 \\ \times\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 825 \\ \times\ 50 \\ \hline \end{array} \qquad \begin{array}{r} 825 \\ \times\ 56 \\ \hline \end{array}$$

$$\begin{array}{r} 3674 \\ \times\ 1 \\ \hline \end{array} \qquad \begin{array}{r} 3674 \\ \times\ 80 \\ \hline \end{array} \qquad \begin{array}{r} 3674 \\ \times\ 81 \\ \hline \end{array}$$

$$\begin{array}{r} 9062 \\ \times\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 9062 \\ \times\ 20 \\ \hline \end{array} \qquad \begin{array}{r} 9062 \\ \times\ 27 \\ \hline \end{array}$$

2. Multiply.

$$\begin{array}{r} 81 \\ \times\ 52 \\ \hline \end{array} \qquad \begin{array}{r} 143 \\ \times\ 33 \\ \hline \end{array} \qquad \begin{array}{r} 572 \\ \times\ 64 \\ \hline \end{array} \qquad \begin{array}{r} 2927 \\ \times\ 81 \\ \hline \end{array}$$

3. How many days are in the following number of years? Ignore leap years.

2 yr

365

9 yr

10 yr

55 yr

26 yr

# Lesson 78: Using Decimal Points for Hundredths

**OBJECTIVES:**

1. To understand decimals as an alternate way of writing tenths and hundredths
2. To subtract tenths and hundredths in decimal format

**MATERIALS:**

1. Warm-up Practice 3
2. Cotter Abacus and about 10 centimeter cubes
3. Place-value cards
4. *Math Card Games* book, N43 and F22.2, and Math journal
5. Worksheet 51, Using Decimal Points for Hundredths

**ACTIVITIES FOR TEACHING:**

***Warm-up.*** Give the child the warm-up practice sheet. Tell him to do the second section on the page. Solutions are on the right.

| | | |
|---|---|---|
| 5678 (**8**) | 5678 (**8**) | 5678 (**8**) |
| × 2 (**2**) | × 70 (**7**) | × 72 (**0**) |
| **11 356** (**7**) | **397 460** (**2**) | **11 356** |
| | | **397 460** |
| | | **408 816** (**0**) |
| | 213 459 (**6**) | |
| | × 35 (**8**) | |
| | **1 067 295** | |
| | **6 403 770** | |
| | **7 471 065** (**3**) | |

***Writing hundredths as decimals.*** Give the child the abacus, centimeter cubes, and place-value cards.

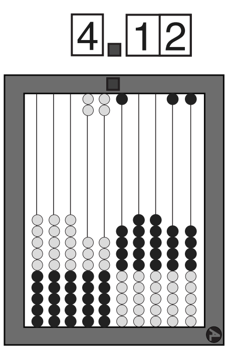
Write: $4\frac{12}{100}$

and ask: How do you think you could write this using a decimal point? Write it: 4.12

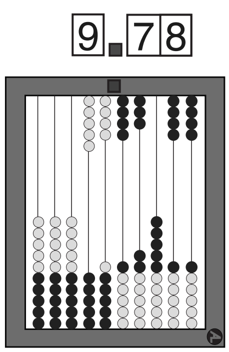
This question encourages the child to think of the big picture and to continue to think intuitively about math.

Say: We read it as 4 and 12 hundredths. Compose the number with your place-value cards and enter it on your abacus. See the left figure below.

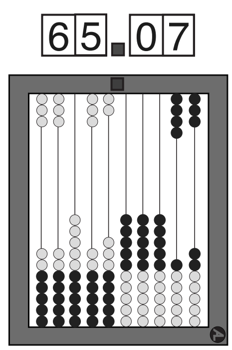
Do not at this point read 4.12 as "four point one two." This lesson is to help the child understand the relationship between fractions and decimals.



**Entering 4.12**.



**Entering 9.78**.



**Entering 65.07**.

Repeat for nine and 78 hundredths. See the middle figure above.

Repeat for 65 and 7 hundredths. (To get the zero, turn a tens card for example, the 30-card, upside down and cover the 3 with the 7.) See the right figure above. Ask: Why did you need a zero before the 7? [Without it, it would be 65 and 7 tenths.]

***Practice.*** Write and ask him to read the following:

30.72 [30 and 72 hundredths]

72.8 [72 and 8 tenths]

72.08 [72 and 8 hundredths]

9.40 [9 and 40 hundredths]

**ACTIVITIES FOR TEACHING CONTINUED:**

***Can You Find game.*** Play this variation of the Can You Find game, found in *Math Card Games* book, N43. Use place-value cards with ones and tens and seven centimeter cubes. Below are the numbers to say. Tell the child to compose the number and set it aside. All the cards will be collected at the end of the game.

1. Can you find 90 and 5 tenths?
2. Can you find 8 tenths?
3. Can you find 60 and 87 hundredths?
4. Can you find 50 and 12 hundredths?
5. Can you find 24 and 3 tenths?
6. Can you find 71 and 36 hundredths?
7. Can you find 9 hundredths? (Hint: Turn the 40-card upside down to get a zero.)

***Subtracting tenths and hundredths.*** Write:

$$\begin{array}{r} 4.1 \\ \underline{-\ .3} \end{array} \qquad \begin{array}{r} 2.37 \\ \underline{-\ 1.31} \end{array} \qquad \begin{array}{r} 3.26 \\ \underline{-\ 1.48} \end{array}$$

and ask the child to find the differences any way he can. [3.8, 1.06, 1.78] He could do it with the abacus or by thinking in terms in tenths and hundredths as fractions.

***Worksheet 51.*** Give the child the worksheet and tell him to do the problems. The solutions are below.

$1\frac{29}{100}$ **1.29** $\qquad 52\frac{52}{100}$ **52.52** $\qquad 63\frac{47}{100}$ **63.47**

$\frac{83}{100}$ **.83** $\qquad 8\frac{7}{100}$ **8.07** $\qquad 8\frac{9}{10}$ **8.9**

$$\begin{array}{r} 21.6 \\ \underline{-\ \ 3.5} \\ \mathbf{18.1} \end{array} \qquad \begin{array}{r} 9.3 \\ \underline{-\ 5.6} \\ \mathbf{3.7} \end{array} \qquad \begin{array}{r} 10.0 \\ \underline{-\ \ 8.5} \\ \mathbf{1.5} \end{array} \qquad \begin{array}{r} 9.1 \\ \underline{-\ 8.3} \\ \mathbf{.8} \end{array}$$

$$\begin{array}{r} 11.63 \\ \underline{-\ \ 2.31} \\ \mathbf{9.32} \end{array} \qquad \begin{array}{r} 9.47 \\ \underline{-\ 2.87} \\ \mathbf{6.60} \end{array} \qquad \begin{array}{r} 9.53 \\ \underline{-\ 5.28} \\ \mathbf{4.25} \end{array} \qquad \begin{array}{r} 7.41 \\ \underline{-\ 5.53} \\ \mathbf{1.88} \end{array}$$

$$\begin{array}{r} 5.68 \\ \underline{-\ 2.08} \\ \mathbf{3.60} \end{array} \qquad \begin{array}{r} 5.15 \\ \underline{-\ 2.90} \\ \mathbf{2.25} \end{array} \qquad \begin{array}{r} 8.00 \\ \underline{-\ 1.25} \\ \mathbf{6.75} \end{array} \qquad \begin{array}{r} 3.40 \\ \underline{-\ 1.25} \\ \mathbf{2.15} \end{array}$$

***Corners™ with Tenths game.*** Play this variation of Corners™ with Tenths game found in Math Card Games book, F22.2. Say: Use your math journal to write the scores using decimal points. In this game, all the numbers are considered to be hundredths. A score of 12 is now 12 hundredths, written with a decimal point (.12). Tell him to use his math journal for scoring.

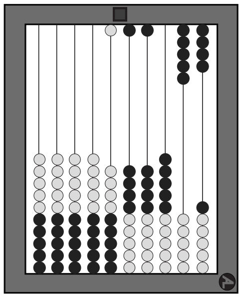
***In conclusion.*** Ask: Which is more, 7 tenths or 7 hundredths? [7 tenths] Which is more, 7 tenths or 70 hundredths? [the same] Which is more, 7 or 7 tenths? [7]

**EXPLANATIONS CONTINUED:**

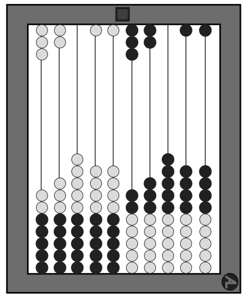
Name: ______________________________

Date: ______________________________

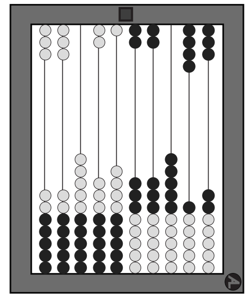
Write the quantities shown using fractions and decimal points.



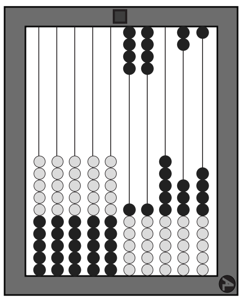
_______ _______



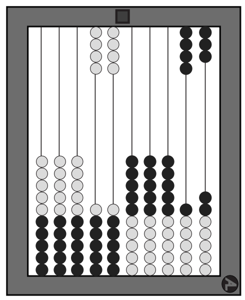
_______ _______



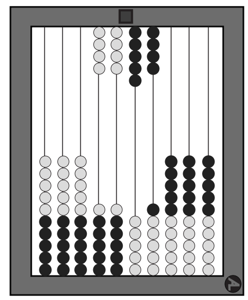
_______ _______



_______ _______



_______ _______



_______ _______

Subtract the following.

| | | | |
|---|---|---|---|
| 21.6 − 3.5 | 9.3 − 5.6 | 10.0 − 8.5 | 9.1 − 8.3 |
| 11.63 − 2.31 | 9.47 − 2.87 | 9.53 − 5.28 | 7.41 − 5.53 |
| 5.68 − 2.08 | 5.15 − 2.90 | 8.00 − 1.25 | 3.40 − 1.25 |

# Lesson 95: More Percentage Problems

**OBJECTIVES:**

1. To solve more common problems involving percentages
2. To learn about tipping and sales tax

**MATERIALS:**

1. Warm-up Practice 9
2. Worksheet 67, More Percentage Problems
3. *Math Card Games* book, F48

**ACTIVITIES FOR TEACHING:**

***Warm-up.*** Give the child the warm-up practice sheet. Tell him to do the second multivide on the page. Solutions are on the right.

***Worksheet 67.*** Give the child the worksheet and ask him to read and solve the first problem. Then tell him to explain it.

1. In a certain class 50% of the children are girls. There are 12 girls. How many children are in the class? [24 children]

If 50% are girls, then 50% must be boys. The total number will be 12 × 2 = 24 children.

Repeat for the remaining problems.

2. The usual tip at a restaurant is 15% of the cost of the food. Many people figure it out by first finding 10%, then finding 5%, which is half of 10%, and adding them together. What is the tip if the food costs $8.00? [$1.20]

Ten percent of $8 is $0.80. Half of that is $0.40. Adding $0.80 and $0.40 is $1.20.

3. What is the 15% tip if the food bill is $12.00? What is the total cost? [$13.80]

Tip is $1.20 + $0.60 = $1.80. Total is $12 + $1.80 = $13.80.

4. In some places people pay sales tax on certain things they buy. If the sales tax is 5%, what is the total bill for a car that cost $4000? [$4200]

Ten percent of $4000 is $400. Half of that is $200. Total is $4000 + $200 = $4200 total cost.

**EXPLANATIONS:**

```
            24  (6)
          × 18  (0)
           192
           240
           432  (0)
           × 6  (6)
         2 592  (0)
          × 35  (8)
        12 960
        77 760
        90 720  (0)
          × 96  (6)
       544 320
     8 164 800
  9)8 709 120   (0)
    8)967 680   (0)
    7)120 960   (0)
     6)17 280   (0)
      5)2 880   (0)
        4)576   (0)
        3)144   (0)
         2)48   (3)
           24
```

**ACTIVITIES FOR TEACHING CONTINUED:**

5. The original price for a game is $10.00. In Store A it went on sale at 10% off and then it went on sale again with 50% off of the sale price. In Store B it went on sale at 50% off and then it went on sale again with 10% off of the sale price. Which store has the better price? [the same, $4.50]

At Store A, the price after the first reduction is $10 × 90% = $9. After the second price reduction, it is $9 × 50% = $4.50.

At Store B, the price after the first reduction is $10 × 50% = $5. After the second price reduction, it is $5 × 90% = $4.50.

***Percentage War game.*** Have him play the Percentage War game, found in *Math Card Games* book, F48.

***In conclusion.*** Ask: Which is more, one half or 60%? [60%] Which is more, three eighth or 20%? [three eighths] Which is more, two thirds or four fifths? [four fifths]

**EXPLANATIONS CONTINUED:**

Note that the final price for Store A is $10 × 50% × 90% and for Store B it is $10 × 90% × 50%, which gives the same result.

Name: ______________________________

Date: ______________________________

Solve the following problems.

1. In a certain class 50% of the children are girls. There are 12 girls. How many children are in the class?

2. The usual tip at a restaurant is 15% of the cost of the food. Many people figure it out by first finding 10%, then finding 5%, which is half of 10%, and adding them together. What is the tip if the food costs $8.00?

3. What is the 15% tip if the food bill is $12.00? What is the total cost?

4. In some places people pay sales tax on certain things they buy. If the sales tax is 5%, what is the total bill for a car that cost $4000?

5. The original price for a game is $10.00. In Store A it went on sale at 10% off and then it went on sale again with 50% off of the sale price. In Store B it went on sale at 50% off and then it went on sale again with 10% off of the sale price. Which store has the better price?

# Lesson 100: Measuring Angles

**OBJECTIVES:**

1. To introduce the term *angle*
2. To measure angles with the goniometer
3. To measure and add the angles in a triangle

**MATERIALS:**

1. Warm-up Practice 10
2. Goniometer
3. 45 triangle and 30-60 triangle
4. Worksheet 72, Measuring Angles

**ACTIVITIES FOR TEACHING:**

***Warm-up.*** Give the child the warm-up practice sheet. Tell him to do only the first multivide. Solutions are on the right.

**EXPLANATIONS:**

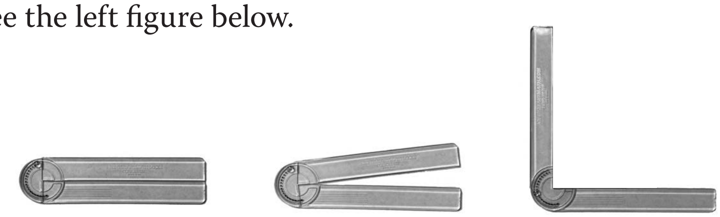
| | |
|---|---|
| 22 | (4) |
| × 27 | (0) |
| 154 | |
| 440 | |
| 594 | (0) |
| × 35 | (8) |
| 2 970 | |
| 17 820 | |
| 20 790 | (0) |
| × 24 | (6) |
| 83 160 | |
| 415 800 | |
| 498 960 | (0) |
| × 16 | (7) |
| 2 993 760 | |
| 4 989 600 | |
| 9)7 983 360 | (0) |
| 8)887 040 | (0) |
| 7)110 880 | (0) |
| 6)15 840 | (0) |
| 5)2 640 | (3) |
| 4)528 | (6) |
| 3)132 | (6) |
| 2)44 | (8) |
| 22 | |

***The goniometer.*** Give the child the goniometer and triangles. Tell him that a goniometer (GON-ee-OM-i-ter) measures the *angles*. An angle is the space between two lines at their vertex, or intersecting point.

Goniometers were briefly introduced in Level D, Lesson 116.

Lay the goniometer flat on a surface and demonstrate how to open it by holding the bottom part with your right hand and gently opening the top part with your left hand. See the left figure below.

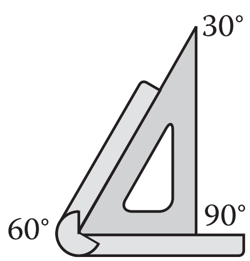
If the two parts of the goniometer come apart, they can be snapped back together. Align the part with the bump on top of the other part and press down.



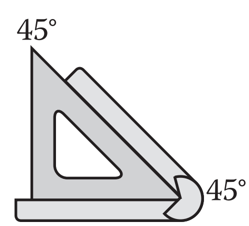
**The goniometer.**

Tell him to open his goniometer so the inside edges are perpendicular to make a right angle. See the right figure above. Tell him to look at the number inside the little magnifying bubble and ask: What number do you see? [90] Tell him to read it as 90 degrees. Tell him to continue to open the goniometer to twice 90°. Ask: What is the angle? [180°]

***Measuring angles.*** Tell him to measure the angles in the triangles with his goniometer. See the figures below.




**Measuring 60°.**




**Measuring 45°.**

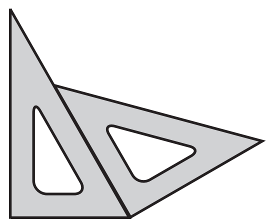
Ask: Which triangle has two angles that are congruent, or the same? [45 triangle]

Congruent is defined as fitting exactly on top.

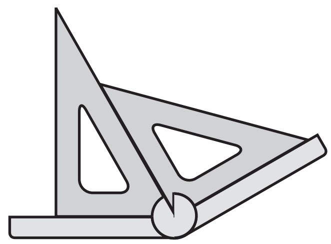
## ACTIVITIES FOR TEACHING CONTINUED:

## EXPLANATIONS CONTINUED:

***Combining angles.*** Tell him to place the 90° angle of the 45 triangle next to the 60° angle of the 30-60 triangle. See the left figure shown below. Ask: What do you think the angle is now? [60 + 90 = 150°] Tell him to use his goniometer to check. [150°] See the right figure below.

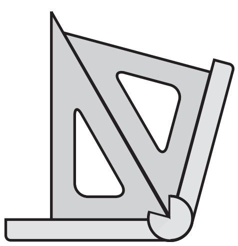


**60° + 90°**

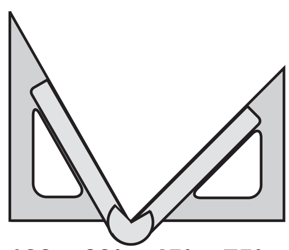


**Measuring 150°.**

Tell him to place the 45° angle next to the 60° angle. Ask: What is the combined angle? [105°] Tell him to use his goniometer to check. [105°] See the left figure below.



**60° + 45° = 105°.**



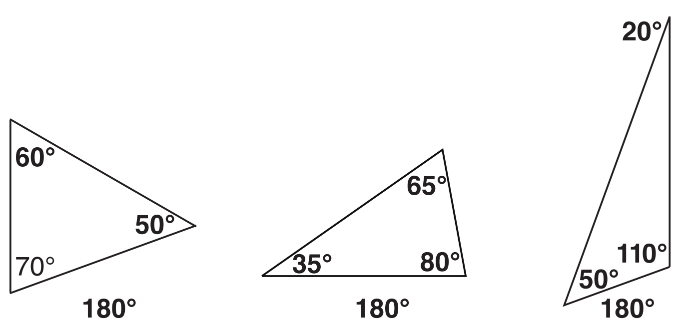
**180 – 60° – 45° = 75°**

Tell him to place the two triangles on a straight line with the right angles on the outside as in the right figure above. Ask: How could you find the angle between them? [180 – 60° – 45° = 75°] Tell him to check with his goniometer. [75°]

***Worksheet 72.*** Give the child the worksheet and tell him to complete it. He will need a goniometer. The solutions are below.

If the worksheets are coil bound, it may be easier to remove the page so that the goniometer lays flat on the page.

| | | |
|---|---|---|
| 45 + 60 = 105° | **45 + 30 = 75°** | **90 + 90 = 180°** |
| **45 + 90 = 135°** | **30 + 90 = 120°** | **90 + 60 = 150°** |
| **180 – 45 – 60 = 75°** | **45 – 30 = 15°** | **60 – 45 = 15°** |



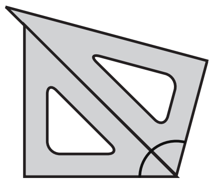

***In conclusion.*** Ask: What are the angles in the 45 triangle? [45°, 45°, 90°] What are the angles in the 30-60 triangle? [30°, 60°, 90°] How many degrees are in a right angle? [90°]

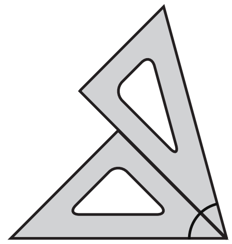
If there is additional time following this lesson, play the Subtracting from One Hundred game, found in *Math Card Games* book, S33.

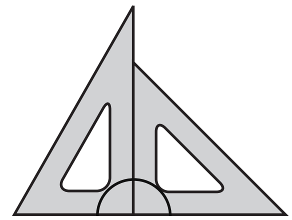
Name: ______________________________

Date: ______________________________

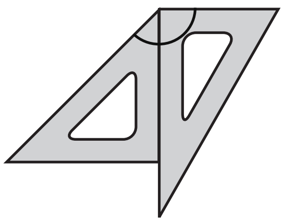
For each figure below, calculate the angle identified by the arc. Then check it with a goniometer.

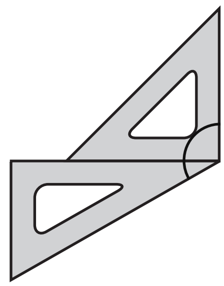


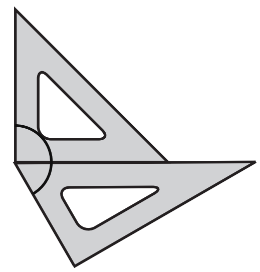


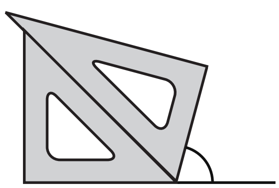


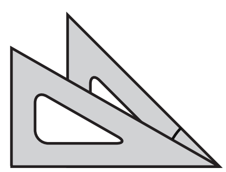
$45 + 60 = 105°$ ✓

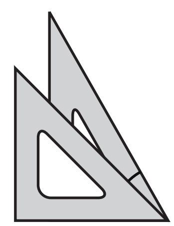




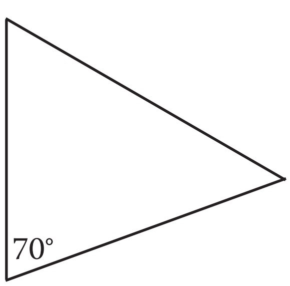


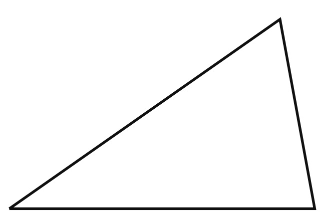


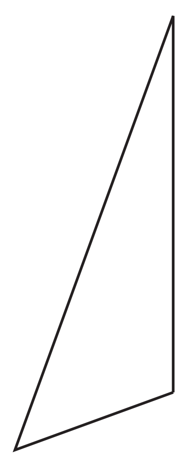




For each triangle, measure the angles and add them up.





# Lesson 122: Isometric Drawings

**OBJECTIVES:**

1. To introduce isometric drawing
2. To practice visualizing objects
3. To make some simple isometric drawings

**MATERIALS:**

1. Warm-up Practice 12
2. Worksheet 94, Isometric Drawings
3. 35 centimeter cubes
4. Drawing board
5. T-square and 30-60 triangle
6. 10 Colored 1” × 1” Tiles

**ACTIVITIES FOR TEACHING:**

***Warm-up.*** Give the child the warm-up practice sheet. Tell him to do the second multivide on the page. Solutions are on the right.

***Worksheet 94.*** Give the child the worksheet, centimeter cubes, drawing board, T-square, triangle, and tiles. Tell him to tape the worksheet to his drawing board.

***Problem 1.*** Tell the child to read the instructions on the worksheet for Problem 1 then to use his triangle to find the angles of the lines. [90° and 30°]

Explain that the word “isometric” (i-so-MET-ric) comes from two Greek words, “iso” meaning “equal” and “metric” meaning “measure.” Ask: What other mathematical word starts with “iso”? [isosceles] What does isosceles mean? [equal legs]

Ask: What small figures makes up the background for the isometric drawings? [equilateral triangles] What is special about them? [All three sides are equal.] Say: This means that the units are the same in each direction. Isometric drawings are a way to show three dimensions on a flat surface.

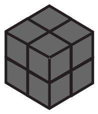
Tell the child that the terms *width*, *length*, and *height* do not have exact definitions. Sometimes *breadth* and *depth* are also used. Because of possible confusion, companies that sell boxes do not use these words to describe the dimensions of their boxes, but use drawings or just the measurements instead.

Tell him to make a cube with his centimeter cubes that measures 2 cm on a side. See the left figure on the next page. Then tell him to make another cube that measures 3 cm on a side. See the right figure. Ask: How does the length, width, and height change? [increases by 1 cm]

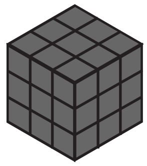
**EXPLANATIONS:**

| | |
|---:|---|
| 16 | (7) |
| × 90 | (0) |
| 1 440 | (0) |
| × 56 | (2) |
| 8 640 | |
| 72 000 | |
| 80 640 | (0) |
| × 72 | (0) |
| 161 280 | |
| 5 644 800 | |
| 9)5 806 080 | (0) |
| 8)645 120 | (0) |
| 7)80 640 | (0) |
| 6)11 520 | (0) |
| 5)1 920 | (3) |
| 4)384 | (6) |
| 3)96 | (6) |
| 2)32 | (5) |
| 16 | |

## ACTIVITIES FOR TEACHING CONTINUED:

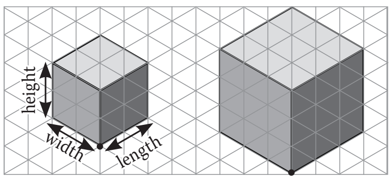


Cube with 2 cm side.

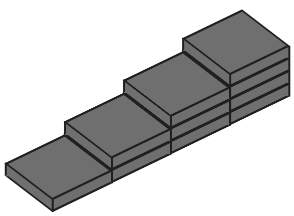


Cube with 3 cm side.

Tell him to draw the 3 cm cube for Problem 1. The solution is shown below.

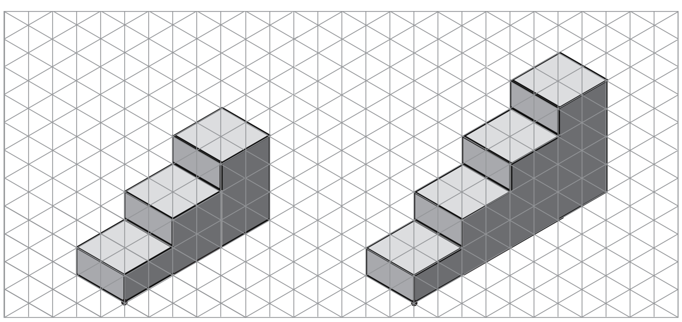



***Problem 2.*** Tell the child to read the instructions for the second problem. Tell him to make the stairs he needs with tiles first. See the figure below.

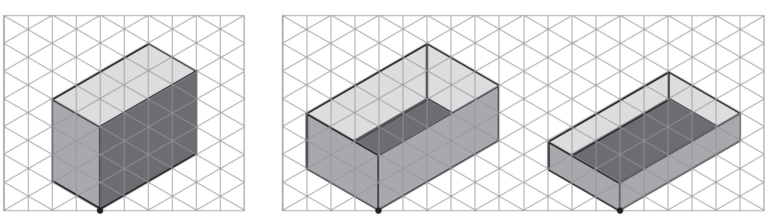


**The stairs built with tiles.**

Then tell him to draw the stairs. The solution is shown below. Tell him to explain his solution.



***Problems 3 and 4.*** Tell him to complete the worksheet. The solutions are below.



***In conclusion.*** Ask: Do you see any rectangular prisms in the room? [possibly a brick, book, picture frame, box, table top, and window glass.]

## EXPLANATIONS CONTINUED:

Shading isn't strictly necessary, but it makes the figure more realistic.

The child will need this worksheet for the next lesson.

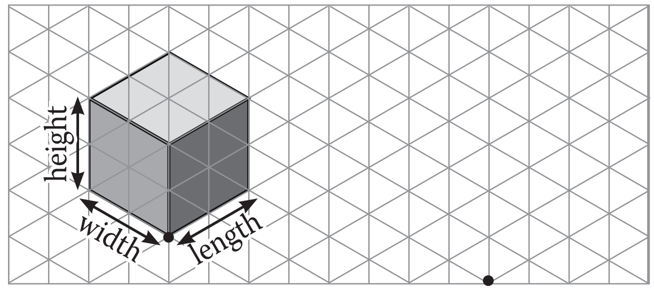
If there is additional time following this lesson, play the Card Exchange game, found in *Math Card Games* book, P27.

Name: ____________________

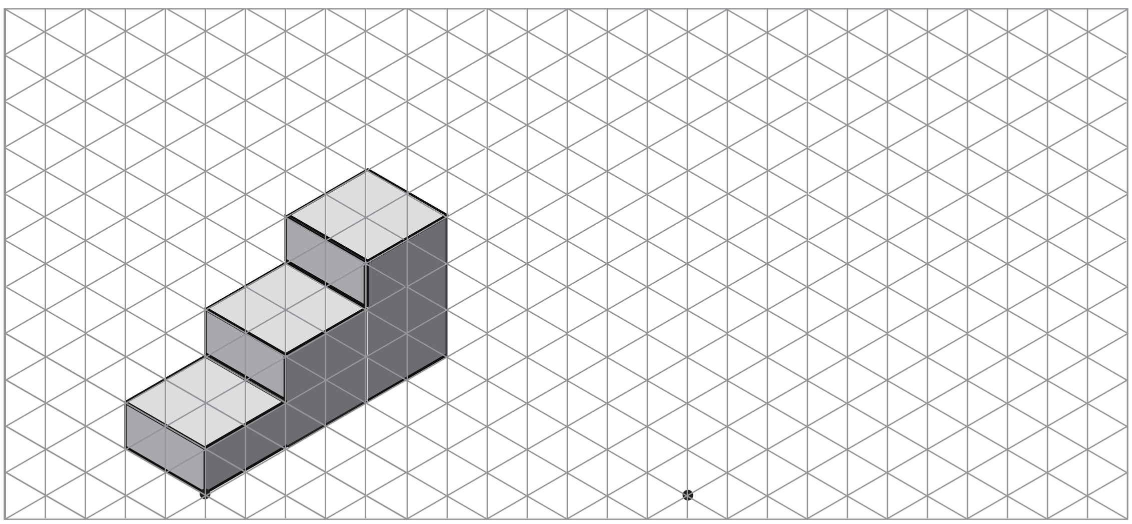
Date: ____________________

1. The drawing at the right is called an isometric drawing. Isometric drawings show equal distances along the three dimensions: width, length, and height. Engineers, architects, and designers use isometric drawings to show how a product will look.

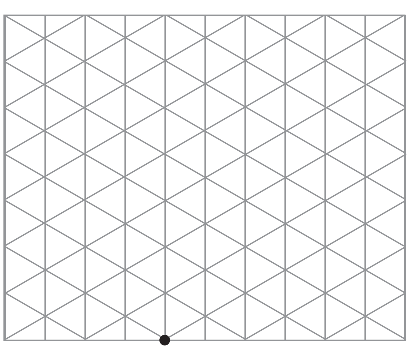
The cube shown is 2 units on each edge. Using your drawing tools, draw another cube that is 3 units on an edge. Start at the dot. Make each side different by shading or hatching.




2. Copy the stairs, but make it one step higher.



3. Draw a rectangular prism (a box) that is 2 units wide, 4 units long, and 3 units high.



4. Draw an empty box like the box shown, but make it only 1 unit high. Be sure the bottom of the box shows.