RightStart™ Mathematics

by Joan A. Cotter, Ph.D.
with Kathleen Cotter Lawler

LEVEL F LESSONS

Second Edition

Activities for Learning, Inc.

A special thank you to Maren Ehley, Rebecca Walsh, and Kelsie Burza for their work in the final preparation of this manual.

Note: Levels are used rather than grades. For example, Level A is kindergarten and Level B is first grade and so forth.

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RightStart™ Mathematics Objectives for Level F

	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Numeration				
Finds squares and square roots				
Reads, writes, rounds, and compares numbers				
Multiplication and Division				
Applies commutative, associative, and distributive properties				
Multiplies multiples of 10 and exponents				
Does division using factors				
Does long division by a two-digit divisor	N/A			
Problem Solving				
Solves two-step problems involving fractions and decimals				
Uses dimensional analysis to solve problems	N/A	N/A	N/A	
Decimals and Percents				
Rounds and compares decimals to the thousandths	N/A			
Adds and subtracts decimals to three decimal places	N/A			
Divides decimals by whole numbers and decimals	N/A			
Understands and uses simple percentages	N/A	N/A		
Solves percentage problems with a calculator	N/A	N/A		
Fractions				
Adds and subtracts mixed fractions with unlike denominators	N/A	N/A		
Converts between mixed numbers and improper fractions	N/A	N/A		
Finds equivalent fractions on the multiplication table	N/A	N/A		
Multiplies and divides various fractions	N/A	N/A		
Measurement				
Understands cubic units: cm^3, dm^3, in^3, ft^3, and yd^3	N/A	N/A		
Uses dimensional analysis to convert measurements	N/A	N/A	N/A	
Converts measurements between SI and US customary (e.g., m to ft)	N/A	N/A	N/A	
Probability and Combinations				
Calculates the probability of an event	N/A	N/A		
Calculates probabilities	N/A	N/A		
Finds probabilities using combinations	N/A	N/A		
Coordinate Systems				
Finds locations using a coordinate system	N/A	N/A	N/A	
Makes line plots and interprets data	N/A	N/A	N/A	
Finds points on a Cartesian coordinate system using ordered pairs	N/A	N/A	N/A	
Places negative points on a Cartesian coordinate system	N/A	N/A	N/A	
Plots equations on a Cartesian coordinate system	N/A	N/A	N/A	
Geometry				
Classifies shapes by attributes	N/A	N/A	N/A	
Scales figures	N/A	N/A	N/A	
Constructs regular polygons incribed in a circle	N/A	N/A	N/A	
Constructs inscribed circles in polygons	N/A	N/A	N/A	
Constructs inscribed squares in triangles	N/A	N/A	N/A	

How This Program Was Developed

We have been hearing for years that students in Japan do better than U.S. students in math. The Asian students are ahead by the middle of first grade. And the gap widens every year thereafter.

Many explanations have been given, including less diversity and a longer school year. Japanese students attend school 240 days a year.

A third explanation given is that the Asian public values and supports education more than we do. A first grade teacher has the same status as a university professor. If a student falls behind, the family, not the school, helps the child or hires a tutor. Students often attend after-school classes.

A fourth explanation involves the philosophy of learning. Asians and Europeans believe anyone can learn mathematics or even play the violin. It is not a matter of talent, but of good teaching and hard work.

Although these explanations are valid, I decided to take a careful look at how mathematics is taught in Japanese first grades. Japan has a national curriculum, so there is little variation among teachers.

I found some important differences. One of these is the way the Asians name their numbers. In English we count ten, eleven, twelve, thirteen, and so on, which doesn't give the child a clue about tens and ones. But in Asian languages, one counts by saying ten-1, ten-2, ten-3 for the teens, and 2-ten 1, 2-ten 2, and 2-ten 3 for the twenties.

Still another difference is their criteria for manipulatives. Americans think the more the better. Asians prefer very few, but insist that they be imaginable, that is, visualizable. That is one reason they do not use colored rods. You can imagine the one and the three, but try imagining a brown eight–the quantity eight, not the color. It cannot be done without grouping.

Another important difference is the emphasis on non-counting strategies for computation. Japanese children are discouraged from counting; rather they are taught to see quantities in groups of fives and tens.

For example, when an American child wants to know 9 + 4, most likely the child will start with 9 and count up 4. In contrast, the Asian child will think that if he takes 1 from the 4 and puts it with the 9, then he will have 10 and 3, or 13. Unfortunately, very few American first-graders at the end of the year even know that 10 + 3 is 13.

I decided to conduct research using some of these ideas in two similar first grade classrooms. The control group studied math in the traditional workbook-based manner. The other class used the lesson plans I developed. The children used that special number naming for three months.

They also used a special abacus I designed, based on fives and tens. I asked 5-year-old Stan how much is 11 + 6. Then I asked him how he knew. He replied, "I have the abacus in my mind."

The children were working with thousands by the sixth week. They figured out how to add 4-digit numbers on paper after learning how on the abacus.

Every child in the experimental class, including those enrolled in special education classes, could add numbers like 9 + 4, by changing it to 10 + 3.

I asked the children to explain what the 6 and 2 mean in the number 26. Ninety-three percent of the children in the experimental group explained it correctly while only 50% of third graders did so in another study.

I gave the children some base ten rods (none of them had seen them before) that looked like ones and tens and asked them to make 48. Then I asked them to subtract 14. The children in the control group counted 14 ones, while the experimental class removed 1 ten and 4 ones. This indicated that they saw 14 as 1 ten and 4 ones and not as 14 ones. This view of numbers is vital to understanding algorithms, or procedures, for doing arithmetic.

I asked the experimental class to mentally add 64 + 20, which only 52% of nine-year-olds on the 1986 National test did correctly; 56% of those in the experimental class could do it.

Since children often confuse columns when taught traditionally, I wrote 2304 + 86 = horizontally and asked them to find the sum any way they liked. Fifty-six percent did so correctly, including one child who did it in his head.

The following year I revised the lesson plans and both first grade classes used these methods. I am delighted to report that on a national standardized test, both classes scored at the 98th percentile.

Joan A. Cotter, Ph.D.

Some General Thoughts on Teaching Mathematics

1. Only five percent of mathematics should be learned by rote; 95 percent should be understood.
2. Real learning builds on what the child already knows. Rote teaching ignores it.
3. Contrary to the common myth, "young children can think both concretely and abstractly. Development is not a kind of inevitable unfolding in which one simply waits until a child is cognitively 'ready.'" —*Foundations for Success* NMAP
4. What is developmentally appropriate is not a simple function of age or grade, but rather is largely contingent on prior opportunities to learn." —Duschl & others
5. Understanding a new model is easier if you have made one yourself. So, a child needs to construct a graph before attempting to read a ready-made graph.
6. Good manipulatives cause confusion at first. If a new manipulative makes perfect sense at first sight, it is not needed. Trying to understand and relate it to previous knowledge is what leads to greater learning. —Richard Behr & others.
7. According to Arthur Baroody, "Teaching mathematics is essentially a process of translating mathematics into a form children can comprehend, providing experiences that enable children to discover relationships and construct meanings, and creating opportunities to develop and exercise mathematical reasoning."
8. Lauren Resnick says, "Good mathematics learners expect to be able to make sense out of rules they are taught, and they apply some energy and time to the task of making sense. By contrast, those less adept in mathematics try to memorize and apply the rules that are taught, but do not attempt to relate these rules to what they know about mathematics at a more intuitive level."
9. Mindy Holte puts learning the facts in proper perspective when she says, "In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic." She also emphasizes the ability to imagine or visualize, an important skill in mathematics and other areas.
10. The only students who like flash cards are those who do not need them.
11. Mathematics is not a solitary pursuit. According to Richard Skemp, solitary math on paper is like reading music, rather than listening to it: "Mathematics, like music, needs to be expressed in physical actions and human interactions before its symbols can evoke the silent patterns of mathematical ideas (like musical notes), simultaneous relationships (like harmonies) and expositions or proofs (like melodies)."
12. "More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority. This is a very important lesson to learn, an essential step in the emergence of independent thinking." —*Everybody Counts*

13. The role of the teacher is to encourage thinking by asking questions, not giving answers. Once you give an answer, thinking usually stops.
14. Putting thoughts into words helps the learning process.
15. Help the children realize that it is their responsibility to ask questions when they do not understand. Do not settle for "I don't get it."
16. The difference between a novice and an expert is that an expert catches errors much more quickly. A violinist adjusts pitch so quickly that the audience does not hear it.
17. Europeans and Asians believe learning occurs not because of ability, but primarily because of effort. In the ability model of learning, errors are a sign of failure. In the effort model, errors are natural. In Japanese classrooms, the teachers discuss errors with the whole class.
18. For teaching vocabulary, be sure either the word or the concept is known. For example, if a child is familiar with six-sided figures, we can give him the word, hexagon. Or, if he has heard the word, multiply, we can tell him what it means. It is difficult to learn a new concept and the term simultaneously.
19. Introduce new concepts globally before details. This lets the children know where they are headed.
20. Informal mathematics should precede paper and pencil work. Long before a child learns how to add fractions with unlike denominators, she should be able to add one half and one fourth mentally.
21. Some pairs of concepts are easier to remember if one of them is thought of as dominant. Then the non-dominant concept is simply the other one. For example, if even is dominant over odd, an odd number is one that is not even.
22. Worksheets should also make the child think. Therefore, they should not be a large collection of similar exercises, but should present a variety. In RightStart™ Mathematics, they are designed to be done independently.
23. Keep math time enjoyable. Our emotional state at the time we learn something is attached to that information. A person who dislikes math will avoid it and a child under stress stops learning. If a lesson is too hard, stop and play a game. Try the lesson again later.
24. In Japan students spend more time on fewer problems. Teachers do not concern themselves with attention spans as is done in the U.S.
25. In Japan the goal of the math lesson is that the student has understood a concept, not necessarily has done something (a worksheet).
26. The calendar must show the entire month, so the children can plan ahead. The days passed can be crossed out or the current day circled.
27. A real mathematical problem is one in which the procedures to find the answer are not obvious. It is like a puzzle, needing trial and error. Emphasize the satisfaction of solving problems and like puzzles, of not giving away the solution to others.

RightStart™ Mathematics

Ten major characteristics make this research-based program effective:

1. Refers to quantities of up to 5 as a group; discourages counting individually. Uses fingers and tally sticks to show quantities up to 10; teaches quantities 6 to 10 as 5 plus a quantity, for example 6 = 5 + 1.
2. Avoids counting procedures for finding sums and differences. Teaches five- and ten-based strategies for the facts that are both visual and visualizable.
3. Employs games, not flash cards, for practice.
4. Once quantities 1 to 10 are known, proceeds to 10 as a unit. Temporarily uses the "math way" of naming numbers; for example, "1 ten-1" (or "ten-1") for eleven, "1-ten 2" for twelve, "2-ten" for twenty, and "2-ten 5" for twenty-five.
5. Uses expanded notation (overlapping) place-value cards for recording tens and ones; the ones card is placed on the zero of the tens card. Encourages a child to read numbers starting at the left and not backward by starting at the ones.
6. Proceeds rapidly to hundreds and thousands using manipulatives and place-value cards. Provides opportunities for trading between ones and tens, tens and hundreds, and hundreds and thousands with manipulatives.
7. Teaches mental computation. Investigates informal solutions, often through story problems, before learning procedures.
8. Teaches four-digit addition on the abacus, letting the child discover the paper and pencil algorithm.
9. Introduces fractions with a linear visual model, including all fractions from 1/2 to 1/10. "Pies" are not used initially because they cannot show fractions greater than 1. Later, the tenths will become the basis for decimals.
10. Teaches short division (where only the answer is written down) for single-digit divisors, before long division.

Second Edition

Many changes have occurred since the first RightStart™ lessons were begun in 1994. First, mathematics is used more widely in many fields, for example, architecture, science, technology, and medicine. Today, many careers require math beyond basic arithmetic. Second, research has given us new insights into how children learn mathematics. Third, kindergarten has become much more academic, and fourth, most children are tested to ensure their preparedness for the next step.

This second edition is updated to reflect new research and applications. Topics within each level are always taught with the most appropriate method using the best approach with the child and teacher in mind.

Daily Lessons

Objectives

The objectives outline the purpose and goal of the lesson. Consider the words; "to introduce" is not the same as "to review." When a topic is introduced, it is not expected to be mastered during that lesson. When a topic is reviewed, proficiency should be close.

Materials

The manipulatives needed for the lessons are specially chosen items needed to teach the lessons. Occasionally, common objects, such as scissors, will be needed and will be listed in bold type.

Warm-up

The warm-up provides review, memory work, or an introduction of the day's topics. It can be reduced, modified, or expanded to meet a child's needs.

Activities

Activities are the heart of the lesson. These are the instructions for teaching the lesson. When guided to ask a question, the expected answer from the child is given in square brackets.

Explanations

Special background notes and supporting information for the teacher are provided here.

There are Overview Videos to guide and support you weekly. The provided QR code will direct you to the appropriate video.

Games

Games, not worksheets or flash cards, provide practice. The games, found in the *Math Card Games* book, should be played as many times as necessary until proficiency or memorization takes place. Games are important to learning math, just as books are important to learning reading.

The *Math Card Games* book includes extra games for the child needing more help and more challenging games for the advanced child.

Instructional videos for all the games used in the RightStart™ Mathematics curriculum are available on Vimeo for a small subscription fee.

Instructional game videos

Worksheets

The worksheets are designed to be completed independently in order to demonstrate understanding of the day's lesson. Some lessons, especially in the early levels, have no worksheet.

In conclusion

Each lesson ends with a short summary based on the day's learning.

Timeline

Each RightStart Math level is designed for one school year. This level should be completed in full before beginning the next level.

Joan A. Cotter, Ph.D.
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Lesson 24: Tenths and Hundredths

OBJECTIVES:

1. To review tenths and hundredths
2. To add and subtract tenths and hundredths

MATERIALS:

1. Worksheet 13, Tenths and Hundredths
2. Cotter Abacus
3. *Math Card Games* book, S11
4. Math journal

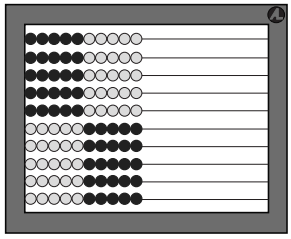
ACTIVITIES FOR TEACHING:

Warm-up. Give the child the worksheet. Tell her to complete just the warm-up problems. Solutions are below.

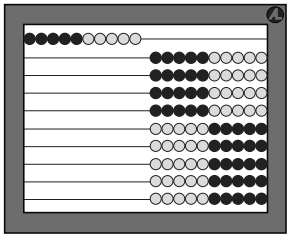
$1^5 + 2^4 + 3^3 = \mathbf{44}$ $\quad 3^3 - 2^4 - 1^5 = \mathbf{10}$

$\frac{3^3}{1^5} - 2^4 = \mathbf{11}$ $\quad \frac{1^5}{3^3} + 2^4 = \mathbf{16\frac{1}{27}}$

Reviewing tenths. Give the child the abacus. Tell her that now all one hundred beads on the abacus will be considered to be one. Tell her to enter one. See the left figure below.



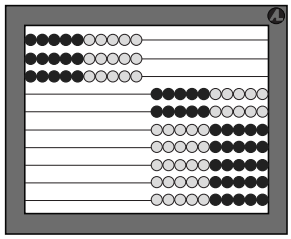
Representing 1.



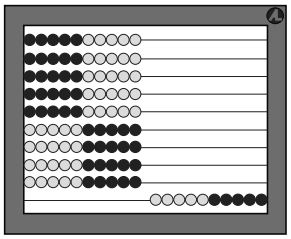
Representing 0.1.

Now tell her to enter one tenth. See the right figure above. Ask: What are two ways to write one tenth? [$\frac{1}{10}$ and 0.1]

Tell her to enter three tenths. See the left figure below. Ask: What are two ways to write it? [$\frac{3}{10}$ and 0.3] Repeat for nine tenths. [$\frac{9}{10}$ and 0.9] See the right figure below.



Representing 0.3.



Representing 0.9.

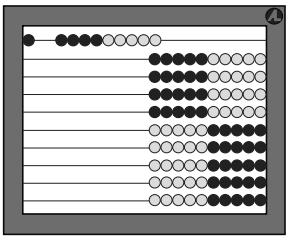
Reviewing Hundredths. Ask: What is one tenth of one tenth? [one hundredth] Tell her to show one hundredth on her abacus. See the left figure on the next page.

EXPLANATIONS:

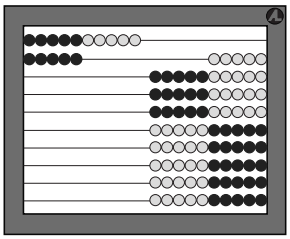
This lesson is similar to Lesson 74 in RightStart™ Mathematics Level E Second Edition, however it provides a foundation for the upcoming lessons.

ACTIVITIES FOR TEACHING CONTINUED:

Tell her to clear the abacus then enter one tenth and five hundredths. See the right figure below. Ask: How many hundredths is this? [15 hundredths]

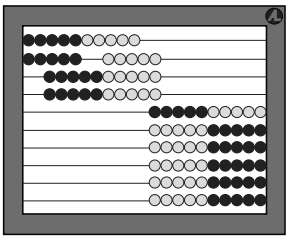


One tenth of one tenth is one hundredth, 0.01.

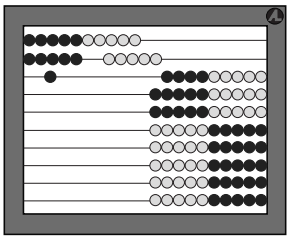


0.1 plus 0.05 = 0.15.

Tell her to add 25 hundredths to the 15 hundredths. See the left figure below. Ask: What are two ways to write the sum using decimals? [0.4 or 0.40]



0.15 + 0.25 = 0.4.



0.15 + 0.06 = 0.21.

Next tell her to clear her abacus then add 15 hundredths and 6 hundredths. [0.21] See the right figure above.

Worksheet 13. Tell the child to complete the worksheet

0.2 + 0.15 = **0.35**	**0.07** + 0.4 = **0.47**	**0.7** + 0.04 = **0.74**
0.56 + 0.04 = **0.6**	**0.38** + 0.15 = **0.53**	**0.82** + 0.18 = **1**
0.79 – 0.06 = **0.73**	**0.44** – 0.2 = **0.24**	**1** – 0.37 = **0.63**

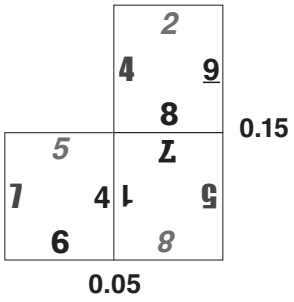
using her abacus. The solutions are below.

Top and Bottom Corners™ with Hundredths game. Play the Top and Bottom Corners™ with Hundredths game, a variation of Top and Bottom Corners™ game, found in *Math Card Games* book, S11. In this game, numbers on the cards are considered to be hundredths. Players take *four* cards to start and take another card after each play.

Record the scores in the math journal. All players start with a score of 5. As usual, players must play to the last card played or to a Corner. They also must play if they can.

In conclusion. Ask: What is the purpose of the decimal point in a number? [It tells where the ones place is.] Which is more, one tenth or ten hundredths? [the same] Which is more, six tenths or sixty hundredths? [the same]

EXPLANATIONS CONTINUED:



Starting with a score of 5 will prevent scores becoming negative.

Name: _____

Date: _____

Warm-Up

Evaluate the following expressions.

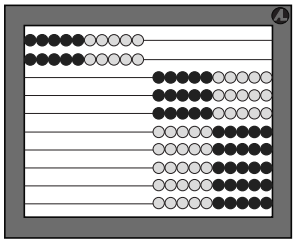
$1^5 + 2^4 + 3^3 =$ _____

$3^3 - 2^4 - 1^5 =$ _____

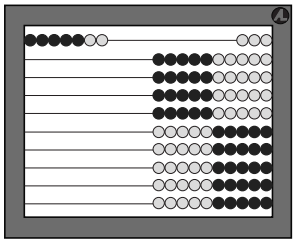
$\frac{3^3}{1^5} - 2^4 =$ _____

$\frac{1^5}{3^3} + 2^4 =$ _____

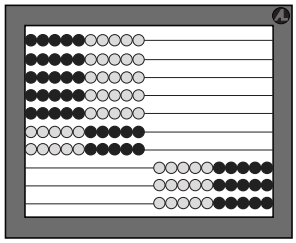
Complete the equations by writing the amount shown on the abacuses in the equation and performing the operations on your abacus. The one hundred beads on the abacus represent 1.



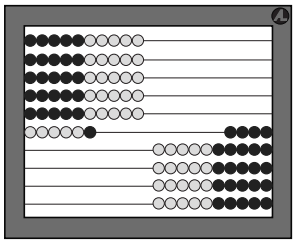
0.2 + 0.15 = _____



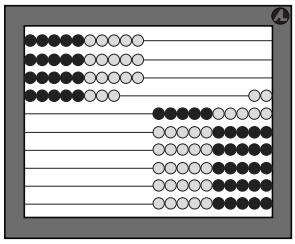
_____ + 0.4 = _____



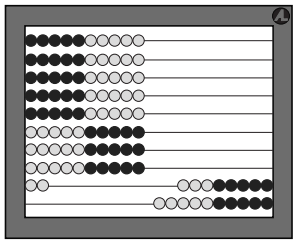
_____ + 0.04 = _____



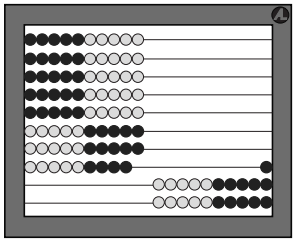
_____ + 0.04 = _____



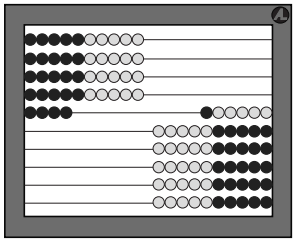
_____ + 0.15 = _____



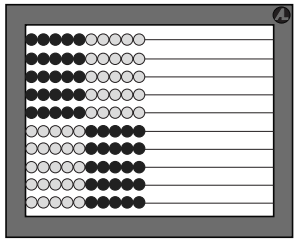
_____ + 0.18 = _____



_____ − 0.06 = _____



_____ − 0.2 = _____



_____ − 0.37 = _____

Lesson 40: Dividing by Tenths

OBJECTIVE:

1. To divide by tenths

MATERIALS:

1. Worksheet 29, Dividing by Tenths
2. Cotter Abacus

ACTIVITIES FOR TEACHING:

Warm-up. Give the child the worksheet. Tell her to complete just the warm-up problems. Solutions are below.

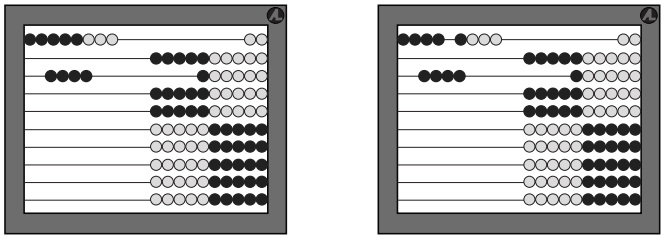
0.85 (4)	38 (2)
× 24 (6)	× 0.49 (4)
340	342
1700	1520
20.40 (6)	18.62 (8)

Dividing whole numbers. Give the child the abacus. Tell her that today she will show division on the abacus in a new way.

Write the expression:

$$\frac{8}{4}$$

and tell her to enter 8 on the top wire. Then tell her to enter 4 on the third wire but to center it under the 8. See the left figure below. Tell her the second wire is similar to the dividing line in the written expression.



With each bead representing 1, the abacuses show eight divided by four, $\frac{8}{4}$.

Ask: How many 4s are in 8? [2] See the second figure above.

Dividing by tenths. Change the expression to:

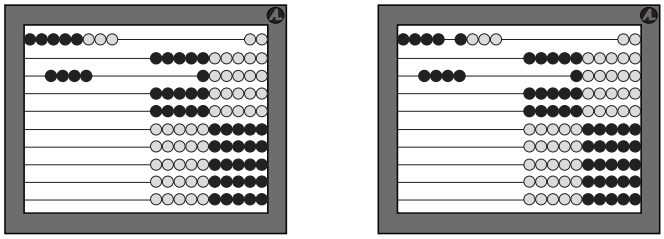
$$\frac{.8}{.4}$$

Tell the child to show this expression on her abacus, with each bead standing for one tenth. See the figures at the top of the next page.

EXPLANATIONS:

Sometimes leading zeros are omitted. Both versions, 0.3 and .3, occur in everyday use.

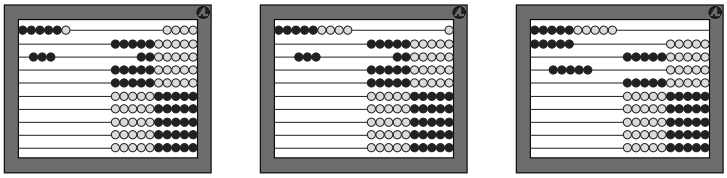
ACTIVITIES FOR TEACHING CONTINUED:



With each bead representing 0.1, the abacuses show eight tenths divided by four tenths, $\frac{.8}{.4}$.

Ask: How many four tenths are in eight tenths? [2] See the right figure above.

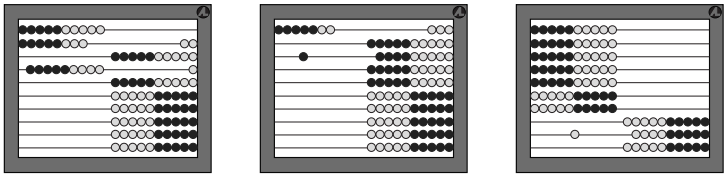
Worksheet 29. Tell the child to complete the first row on the worksheet. The solutions are shown below.



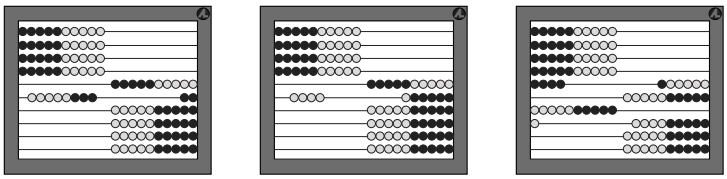
$\frac{.6}{.3} = 2$ $\frac{.9}{.3} = 3$ $\frac{1.5}{0.5} = 3$

Ask her to explain how she found her answer. In the third example, the first row is 1 because ten tenths equal 1.

Then tell her to complete the worksheet. The solutions are below.



$\frac{1.8}{.9} = 2$ $\frac{.7}{.1} = 7$ $\frac{7}{0.1} = 70$



$\frac{4.0}{.8} = 5$ $\frac{4.0}{.4} = 10$ $\frac{4.4}{1.1} = 4$

If each bead in the abacuses above suddenly explodes becoming ten times greater, what happens to your answers?

They stay the same.

In conclusion. Ask: Is 8 tenths divided by 2 tenths the same as 8 divided by 2? [yes] Is 9 tenths divided by 3 tenths the same as 9 divided by 3? [yes] Is 8 hundredths divided by 2 hundredths the same as 8 divided by 2? [yes]

EXPLANATIONS CONTINUED:

If there is additional time following this lesson, play the Subtraction Corners™ with Tenths game, found in *Math Card Games* book, F22.4.

Name: _____

Date: _____

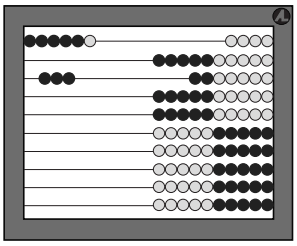
Warm-Up

Multiply the numbers given. Use check numbers to check your work if you like.

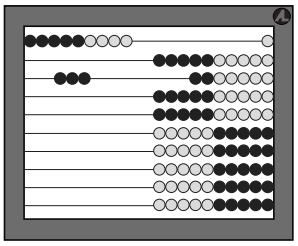
		0	.8	5	( )
		×	2	4	( )
					( )

			3	8	( )
	×	0	.4	9	( )
					( )

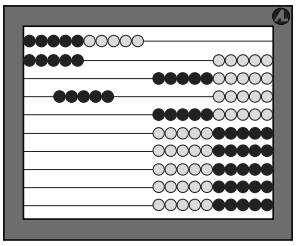
Write the equations shown on the abacuses. Each bead on the abacus represents 0.1.



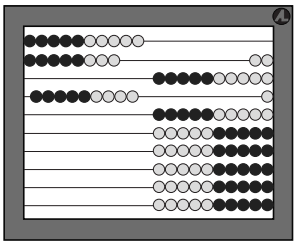
$\frac{0.6}{0.3}$ = _____



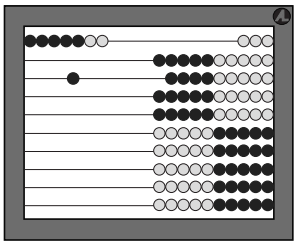
_____ = _____



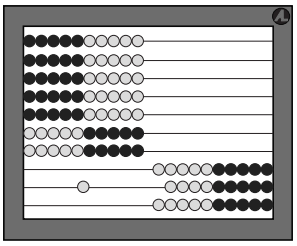
_____ = _____



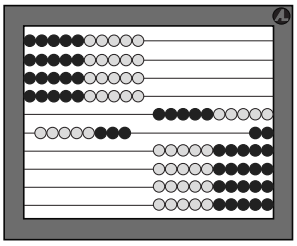
_____ = _____



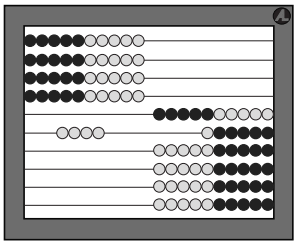
_____ = _____



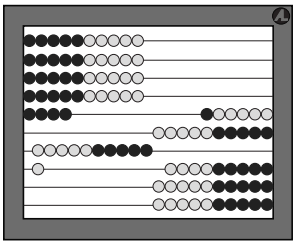
_____ = _____



_____ = _____



_____ = _____



_____ = _____

If each bead in the abacuses above suddenly explodes becoming ten times greater, what happens to your answers? _____

Lesson 51: Remainder Forms after Dividing

OBJECTIVES:

1. To divide by subtracting
2. To write remainders in three forms: whole number, fraction, and decimal

MATERIALS:

1. Worksheet 39, Remainder Forms after Dividing
2. Casio SL-450S calculator

ACTIVITIES FOR TEACHING:

Warm-up. Give the child the worksheet. Tell her to do just the warm-up problems. Solutions are below.

```
     (7) (5)          (4) (2)
      340 r23           85 r29
28 )9543 (3)     73 )6234  (6)
(1)              (1)
```

Dividing by subtracting. Give the child the calculator. Tell her that today she will do some division with a calculator.

Write: $160 \div 32 =$

and tell her she is to use her calculator, but she cannot use the division key. Give her a few minutes to work on the problem independently. Then tell her to discuss the problem.

There are several solutions. One way is to start with 160 and subtract 32s until reaching zero; each subtraction needs to be counted. The constant feature makes it easier. Start by pressing 32, then [−] [−]. Next enter 160 and press [=], which subtracts 32. Continue pressing [=] until the remainder is less than 32, in this case, 0.

Another way is to start with 32 and add 32s until reaching 160, which will be a total of 5 times. Using the constant feature will make this easier, too.

Dividing by a two-digit number using subtraction. Now tell the child to complete the following problem with her calculator, but without using the division key.

$$864 \div 32 =$$

Give her time to solve it before discussing the solution.

While it is possible to subtract 32 twenty-seven times, it is simpler to subtract 320 twice (32 × 10), and then subtract 32 seven times.

Worksheet 39. Tell the child to complete the first problem on the worksheet. Solutions are shown on the next page.

EXPLANATIONS:

Some children may need a hint that they could subtract groups of ten 32s.

ACTIVITIES FOR TEACHING CONTINUED:

414 ÷ 18 = **23**

414 – (18 × 10) – (18 × 10) – 18 – 18– 18

1728 ÷ 54 = **32**

1728 – (54 × 10) – (54 × 10) – (54 × 10) – 54 – 54

Remainders in other forms. Tell her to read and solve Problem 2.

2. In a leap year, 366 days, what is the average number of days in a month? Give the answer in days and a fraction of a day.

Then tell her to discuss the solution.

$$12\overline{)366}\quad 30\ \tfrac{6}{12}$$

Ask: Is there a simpler fraction that is equal $\frac{6}{12}$? [$\frac{1}{2}$] Now tell her to use her calculator to find 366 divided by 12, using the division key. [30.5] Ask: Did you get the same answer? [yes, since $\frac{1}{2}$ is the same as .5]

Tell her to complete Problem 3.

3. In a non-leap year, what is the average number of days in a month? Give the answer in days and a fraction of a day.

$$12\overline{)365}\quad 30\ \tfrac{5}{12}$$

Now tell her to use her calculator to find 365 divided by 12. [30.416666] Ask: Did you get the same answer? [yes, since $\frac{5}{12}$ is the same as .4166666]

Worksheet. Tell the child to complete the worksheet. The solutions are below.

	Remainder as a Whole Number	Remainder as a Fraction	Remainder as a Decimal
7 ÷ 3	2 r1	$2\frac{1}{3}$	2.33
13 ÷ 8	**1 r5**	**$1\frac{5}{8}$**	**1.63**
51 ÷ 7	**7 r2**	**$7\frac{2}{7}$**	**7.29**
1001 ÷ 25	**40 r1**	**$40\frac{1}{25}$**	**40.04**
6983 ÷ 86	**81 r17**	**$81\frac{17}{86}$**	**81.20**
3078 ÷ 12	**256 r6**	**$256\frac{6}{12}$**	**256.50**

In conclusion. Ask: Is it possible to do division without any multiplying? [yes] What operation would you use? [subtraction] Is that the easiest way? [no] What are the three forms for writing remainders? [whole number, fraction, and decimal]

EXPLANATIONS CONTINUED:

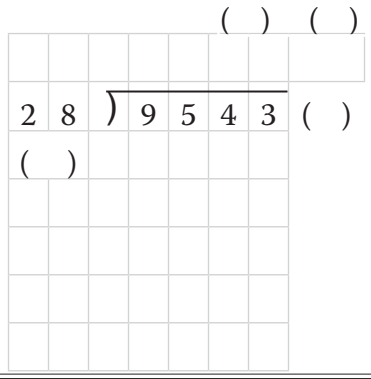
Dividing $\frac{5}{12}$ on a calculator gives 0.4166666. Some children may need to divide 5 by 12 on the calculator to see that $\frac{5}{12}$ is 0.4166666.

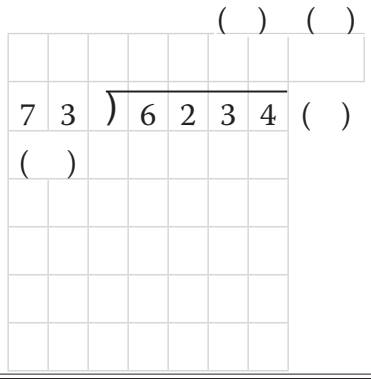
Name: _____

Date: _____

Warm-Up

Divide and check your answers with check numbers.





1. Use your calculator, but not the ÷ key, to find the following. Explain what you did.

414 ÷ 18 _____

_____

1728 ÷ 54 _____

_____

2. In a leap year, 366 days, what is the average number of days in a month? Give the answer in days and a fraction of a day.

3. In a non-leap year, what is the average number of days in a month? Give the answer in days and a fraction of a day.

4. Divide using your calculator as needed. Write the quotients with remainders three different ways. Round the decimals to hundredths.

	Remainder as a Whole Number	Remainder as a Fraction	Remainder as a Decimal
7 ÷ 3	2 r1	$2\frac{1}{3}$	2.33
13 ÷ 8			
51 ÷ 7			
1001 ÷ 25			
6983 ÷ 86			
3078 ÷ 12			

Lesson 62: Area of Parallelograms

OBJECTIVE:

1. To find the area of parallelograms

MATERIALS:

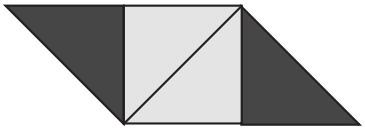
1. Warm-up Practice 4
2. Worksheet 50, Area of Parallelograms
3. One set of tangrams
4. 4-in-1 ruler
5. 45 triangle (or 30-60 triangle), optional

ACTIVITIES FOR TEACHING:

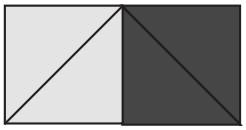
Warm-up. Give the child the warm-up practice sheet. Tell her to complete the second multivide and long division problem. Solutions are on the right.

Area of a parallelogram. Give the child the worksheet and tangrams. Tell her that today's lesson is about finding areas of parallelograms.

Tell her to make a parallelogram with four tangram triangles of the same size, either the large triangles or the small triangles. See the left figure below.



A parallelogram.



Rectangle with same area.

Now tell her to move one piece and to turn it into a rectangle. See the right figure above. Ask: Does the rectangle have the same area as your original parallelogram? [yes] How do you know? [They are the same pieces, just moved.] How can you find the area of the rectangle? [multiply the width times the height] Could we use the width times the height to find the area of the parallelogram? [yes]

Draw a parallelogram as shown below in the left figure.

Say: Let's find the area of the parallelogram. First turn it into a rectangle having the same area. Draw a line from the top left corner to the base as shown in the second figure above. Ask: To where do we need to move the triangle? [to the right side] See the third figure. Shade in the rectangle as shown in the fourth figure. Ask: Does this rectangle have the same area as the parallelogram? [yes]

EXPLANATIONS:

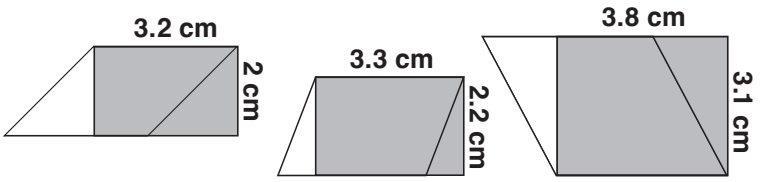
A parallelogram is a quadrilateral with opposite sides parallel.

```
      468  (0)
  × 0.24   (6)
     1872
     9360
   112.32  (0)
  × 0.72   (0)
    22464
   786240
0.6)80.8704  (0)
0.4)134.784  (0)
0.8)336.96   (0)
0.9)421.2    (0)
      468


      (3)  (8)
       30  r8
82)2468   (2)
(1) 246
      08
       0
       8
```

ACTIVITIES FOR TEACHING CONTINUED:

Worksheet 50, Problem 1. Tell the child to solve the first row on the worksheet. The solutions are below.



$A = 3.2 \times 2$
$A = 6.4 \text{ cm}^2$

$A = 3.3 \times 2.2$
$A = 7.3 \text{ cm}^2$

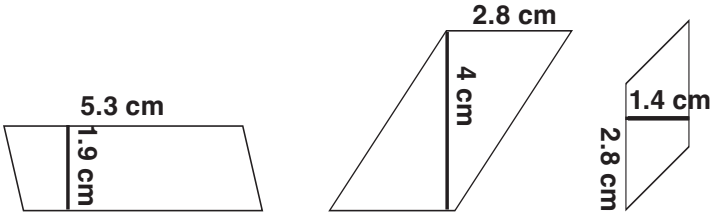
$A = 3.8 \times 3.1$
$A = 11.8 \text{ cm}^2$

Tell her to explain why the area of a parallelogram can be found by multiplying the width by the height. [The area of the rectangle has the same area as the parallelogram.]

Finding the heights in a parallelogram. Draw a parallelogram as shown in the left figure and ask: How can you find the height of this parallelogram without drawing a rectangle? [Draw a line perpendicular to the width.] Draw several heights as shown in the right figure and ask: Which height should we use? [any of them]

A parallelogram A parallelogram with heights.

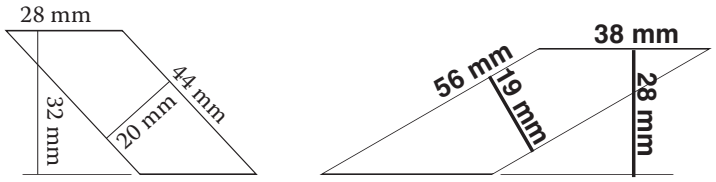
Problems 2 and 3. Tell the child to complete the worksheet. The solutions are below.



$A = 5.3 \times 1.9$
$A = 10.1 \text{ cm}^2$

$A = 2.8 \times 4$
$A = 11.2 \text{ cm}^2$

$A = 1.4 \times 2.8$
$A = 3.9 \text{ cm}^2$



$A = 28 \times 32 = 896 \text{ mm}^2$
$A = 44 \times 20 = 880 \text{ mm}^2$

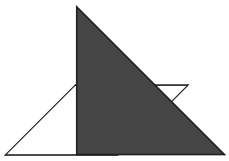
$A = 38 \times 28 = 1064 \text{ mm}^2$
$A = 56 \times 19 = 1064 \text{ mm}^2$

Ask: Why do you think the two areas for the first figure in Problem 3 are not identical? [Measurements are not exact.]

In conclusion. Ask: How do you find the area of a parallelogram? [multiply width times height] What is the height? [line that is perpendicular to the width]

EXPLANATIONS CONTINUED:

To be more accurate in drawing the perpendicular lines, the child could use either a tangram triangle or a triangle from the drawing set. See below.



Measurements given here are accurate, but worksheet measurements may vary and will affect the final answers.

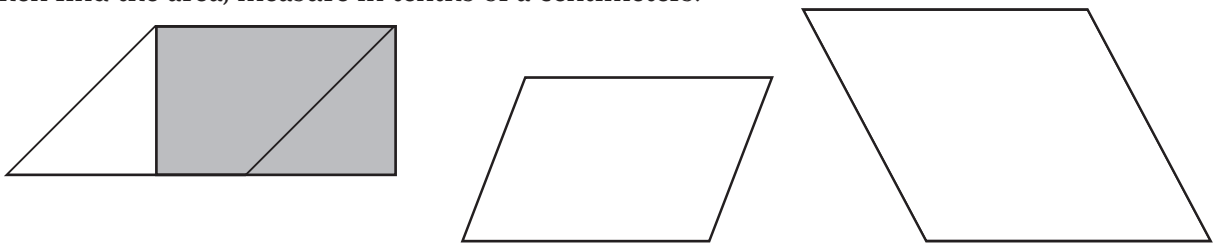
The fact that the area calculations for the same figure do not always give the identical answer is often surprising and sometimes upsetting to some people.

If there is additional time following this lesson, play the Old Main Squares game, found in *Math Card Games* book, P22.

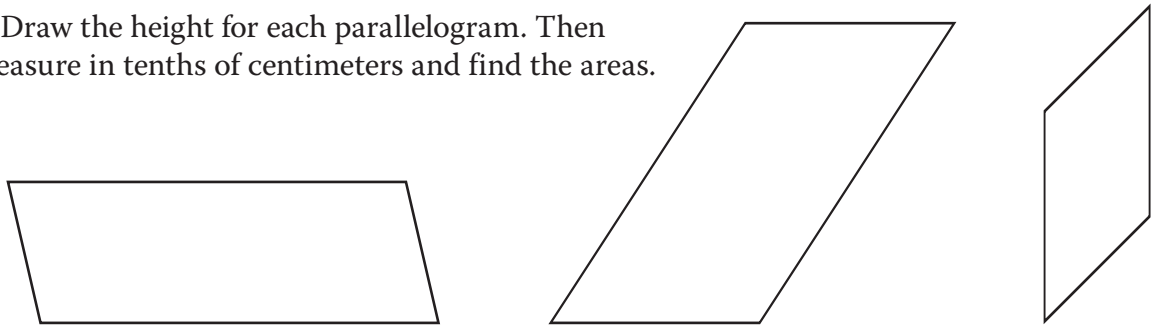
Name: _____

Date: _____

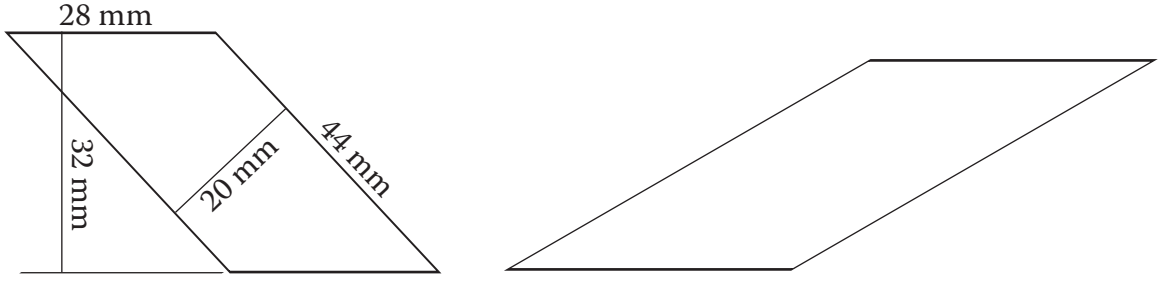
1. Turn the following parallelograms into rectangles. Hatch or shade the rectangle. Then find the area; measure in tenths of a centimeters.



2. Draw the height for each parallelogram. Then measure in tenths of centimeters and find the areas.



3. Find the area of the parallelograms in two different ways. Use millimeters.



Lesson 74: Volume of Prisms

OBJECTIVES:

1. To learn the term *net*
2. To use nets to make and visualize prisms
3. To find the volumes of prisms

MATERIALS:

1. Warm-up Practice 5
2. Worksheet 62, Volume of Prisms
3. Geometry panels*

ACTIVITIES FOR TEACHING:

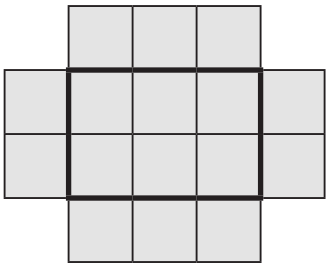
Warm-up. Give the child the warm-up practice sheet. Tell him to complete the second multivide and long division problem. Solutions are on the right.

Review that a prism is a 3D solid that has two congruent parallel polygons connected with faces that are parallelograms. Ask: Is a cube a prism? [yes]

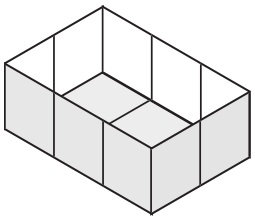
Worksheet 62, Problem 1. Give the child the worksheet and geometric panels.

Tell him that in the previous lesson he found some volumes made with centimeter cubes. In this lesson he will find the volumes of some prisms made with the geometry panels.

Tell him to think what a net of a solid is. A *net* of a solid is the connected faces laid flat. Then tell him to read the first problem on the worksheet and make the net. See the net below.



Tell him to fold it up and find its volume. See the solution below.



$V = Bh$
$V = 3 \times 2 \times 1$
$V = 6 \text{ dm}^3$

EXPLANATIONS:

* If the panels have not been used yet, the edges need to be creased. Bend the edges along the perforated lines toward the colored side. Place the panel on a hard surface and bend gently. Bending two panels at a time works well.

3456 **(0)**
× 2.8 **(1)**
27648
69120
9676.8 (0)
× 4.8 **(3)**
774144
3870720
4)46448.64 (0)
0.7)11612.16 (0)
6)16588.8 (0)
0.8)2764.8 (0)
3456

(6) (3)
123 r12
28)3456 **(0)**
(1) 28
65
56
96
84
12

ACTIVITIES FOR TEACHING CONTINUED:

Problem 2. Tell the child to complete the second problem. The solution is shown below. Tell him he can make the net first with the geometry panels.

$V = Bh$
$V = 4 \times 3 \times 2$
$V = 24 \text{ dm}^3$

Ask: How many times greater is the second prism compared to the first prism? [24 ÷ 6 = 4] How could you imagine the four smaller prisms fitting in the larger prism? [two turned sideways on the bottom row and two more on the upper row]

Problem 3. Tell the child to complete the third problem. The solution is below.

$V = Bh$
$B = A(\text{tri}) = \frac{1}{2}wh = \frac{1}{2} \times 1 \times 0.866 = 0.433 \text{ dm}^2$
$V = 0.433 \times 4 = 1.732$
$V = 1.7 \text{ dm}^3$

Problems 4 and 5. Ask: How many equilateral triangles are in a hexagon? [6] Tell him to complete the worksheet. The solutions are below.

$V = Bh$
$B = 4A(\text{tri}) = 4 \times \frac{1}{2}wh = 4 \times \frac{1}{2} \times 1 \times 0.866 = 1.732 \text{ dm}^2$
$V = 1.732 \times 3 = 5.196$
$V = 5.2 \text{ dm}^3$

$V = Bh$
$B = A(\text{tri}) = 6 \times \frac{1}{2}wh = 6 \times \frac{1}{2} \times 1 \times 0.866 = 2.598 \text{ dm}^2$
$V = 2.598 \times 1 = 2.598$
$V = 2.6 \text{ dm}^3$

In conclusion. Ask: How do you find the volume of a box? [Multiply the area of the base by the height.] Does it matter which part of the box is the base? [no]

EXPLANATIONS CONTINUED:

Since there are only 30 yellow squares in a set of geometry panels, two squares will be missing to make the prism for Problem 2.

Some children may need to physically fold the panels rather than doing it mentally.

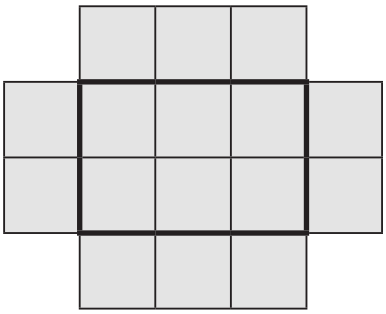
If there is additional time following this lesson, play Distribution Corners™ game, found in *Math Card Games* book, P37.

Name: _____

Date: _____

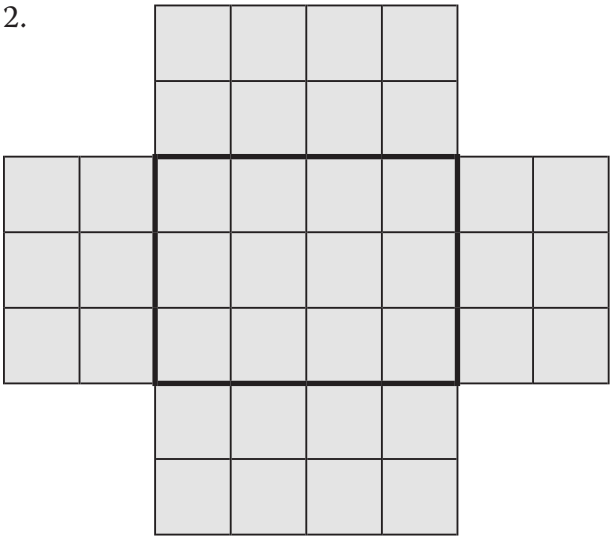
1. Make the net shown with geometry panels.

Then turn the net over and fold it up to make a rectangular prism. Find the volume in cubic decimeters. Each little square represents a square decimeter.



2-5. The side of each small figure is 1 dm. Make each net and mentally fold it on the bold lines to make a prism. Then find the volume to a tenth of a cubic decimeter. Height of an equilaterial triangle is 0.866 × the width.

2.



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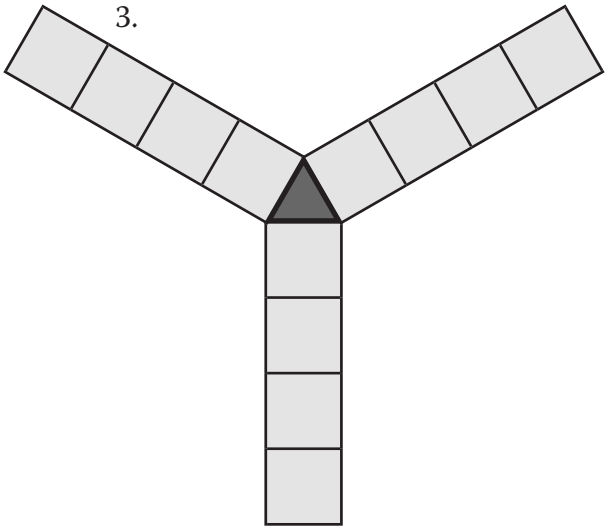

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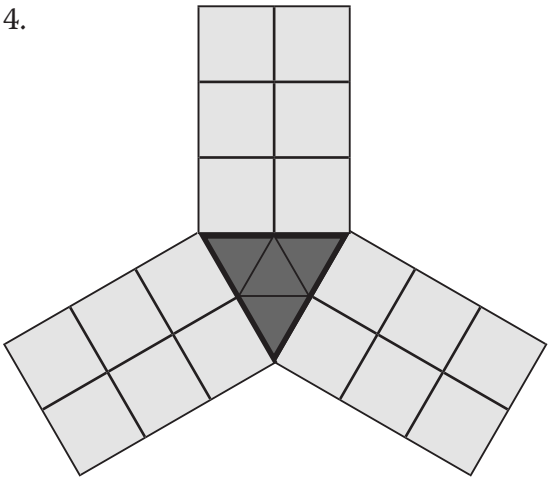

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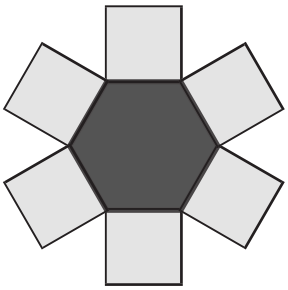


Lesson 85: Equivalent Fractions on Multiplication Table

OBJECTIVES:

1. To use the multiplication table to simplify fractions
2. To practice simplifying fractions

MATERIALS:

1. Fraction chart
2. Worksheet 68, Multiplication Table
3. *Math Card Games* book, F23.1

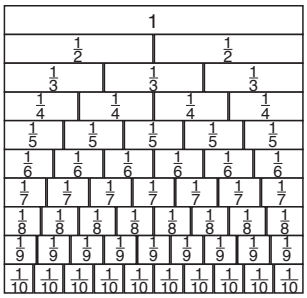
ACTIVITIES FOR TEACHING:

Warm-up. Ask: Two thirds plus what equals one? [one third] Two thirds plus what equals two? [four thirds] Nine eighths minus what equals one? [one eighth]

Fractions on the multiplication table. Give the child the fraction chart. Have him refer to Worksheet 68, Multiplication Table from the previous lesson. Tell him that the multiplication table can be used for simplifying fractions.

Tell him to look at his fraction chart and name the fractions that are equal to one half. [$\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$]

Now tell him to look on his multiplication table and find a 1 and a 2 in the same column. This represents $\frac{1}{2}$. See the right figure below.



The fraction chart.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Showing one half on the multiplication table.

Ask: Can you find two fourths? Touch the 2 and 4 cells with your index finger and thumb. See left figure below.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20

Showing two fourths.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20

Showing three sixths.

Continue with three sixths. See the right figure above. Tell him to keep going to the tenths. See figures below.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20

Showing four eighths.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20

Showing five tenths.

EXPLANATIONS:

Overview Video QR code for Lessons 85 through 88.

A Multiplication Table can also be found in Appendix p. 3.

ACTIVITIES FOR TEACHING CONTINUED:

Tell him to name and touch more fractions in the top two rows that are equivalent to one half. See below.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20

More fractions equivalent to one half.

Next tell him to use the multiplication table to find three fifths and some equivalent fractions. See below.

3	6	9	12
4	8	12	16
5	10	15	20

3	6	9	12
4	8	12	16
5	10	15	20

3	6	9	12
4	8	12	16
5	10	15	20

3	6	9	12
4	8	12	16
5	10	15	20

Showing equivalent fractions for three fifths.

Simplifying fractions. Tell the child that he can also use the multiplication table to simplify fractions. It is just the opposite. Say: To simplify $\frac{3}{9}$, first find a column with both 3 and 9. [3s column] Then slide all the way to the left. Ask: What does $\frac{3}{9}$ simplify to? [$\frac{1}{3}$] See below.

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20

Simplifying three ninths to one third.

Repeat for $\frac{12}{16}$. Ask: What column has both 12 and 16? [4s column] Then slide all the way to the left. Ask: What does $\frac{12}{16}$ simplify to? [$\frac{3}{4}$] See the two left figures below.

1	2	3	4
2	4	6	8
3	6	9	12
4	8	12	16

1	2	3	4
2	4	6	8
3	6	9	12
4	8	12	16

Simplifying twelve sixteenths to three fourths.

1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16

1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16

Ask: Supposing you had used the 2s column for the 12 and 16, what would it simplify to? [$\frac{6}{8}$] Say: Since $\frac{6}{8}$ is not simplified, put it into the 2s column again, to be simplified to $\frac{3}{4}$. See the two right figures above.

Simplifying with the Multiplication Table game. Play the Simplifying with the Multiplication Table game, found in *Math Card Games* book, F23.1.

In conclusion. Ask: If two fractions are equivalent, what do we call the fraction with the lower numbers? [simplified] What does 10 twentieths simplify to? [one half] What does 20 fortieths simplify to? [one half]

EXPLANATIONS CONTINUED:

Both the numerator and denominator of the fraction must be in the same column, but they need not be adjacent.

Some children may benefit from seeing these fractions on the fraction chart.

Name: _____

Date: _____

Fill in the multiplication table as instructed in the lesson.

Multiplication Table

1	2	3	4	5	6	7	8	9	10
2									
3									
4									
5									
6									
7									
8									
9									
10									

Lesson 122: Analyzing Patterns

OBJECTIVES:

1. To generate data from a mathematical relationship
2. To graph the patterns
3. To analyze the patterns

MATERIALS:

1. Warm Up Practice 14
2. Worksheet 103, Analyzing Patterns
3. 4-in-1 ruler or other straightedge

ACTIVITIES FOR TEACHING:

Warm-up. Give the child the warm-up practice sheet. Tell him to complete the second section. Solutions are on the right.

Worksheet 103. Give the child the worksheet and straightedges. Tell him that today's lesson is about plotting relationships on a graph.

Problems 1 and 2. Tell the child to complete the first two problems on the worksheet.

1. Ari plays three math card games every week. Jordan plays six math games every week and Cy plays two. Fill in the table to represent the number of games that they played.

The completed table is shown below.

	Total Number of Math Games Played		
Weeks	Ari	Jordan	Cy
0	0	**0**	**0**
1	3	**6**	**2**
2	6	**12**	**4**
3	**9**	**18**	**6**
4	**12**	**24**	**8**
5	**15**	**30**	**10**
6	**18**	**36**	**12**

2. How much did you add to each week's sum for:
Ari **3** Jordan **6**
Cy **2**

Problem 3. Tell the child to complete the third problem, plotting the points and connecting them. The completed graphs are shown on the next page.

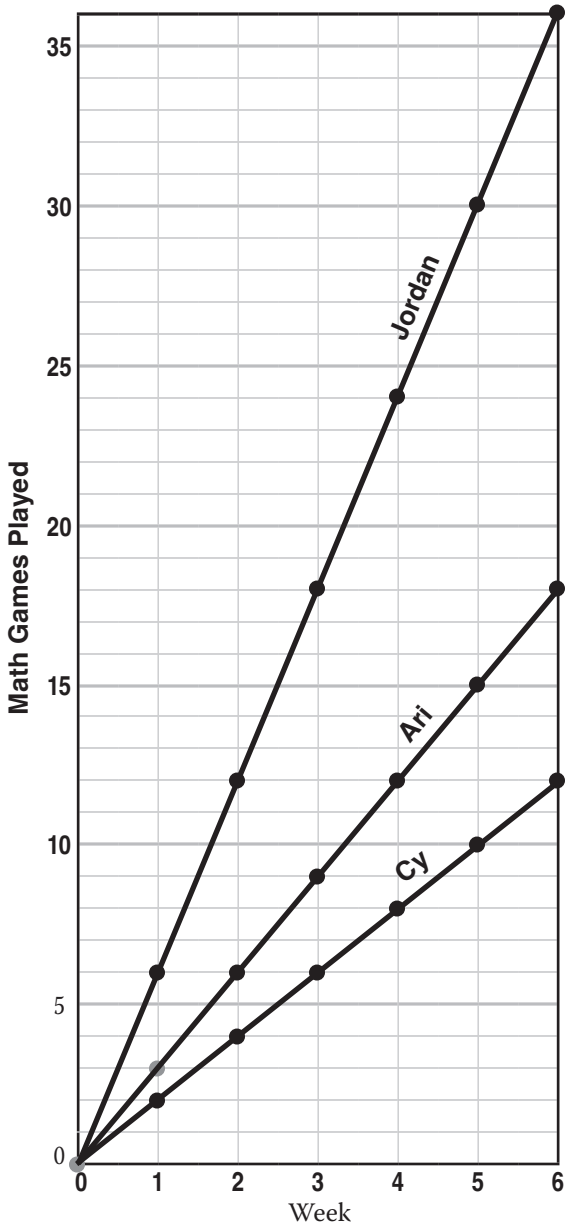
Problem 4. Tell him to answer the questions. The solutions are shown on the next page.

EXPLANATIONS:

4.68 **(0)**
× 42 **(6)**
936
18720
196.56 (0)
× 27 **(0)**
137592
393120
42: 7)**5307.12** **(0)**
6)**758.16** **(0)**
27: 9)126.36 **(0)**
3)14.04 **(0)**
4.68

(8)
1.07
23)24.61 **(4)**
(5) 23
161
161

ACTIVITIES FOR TEACHING CONTINUED:



What is the shape of the three graphs? **straight lines**

At Week 2, Jordan has played how many times more games than Ari? **two times more**

At each week, Ari has played what fraction of the number of games that Jordan has played? $\frac{1}{2}$

At each week, Cy has played what fraction of the number of games that Jordan has played? $\frac{1}{3}$

At what week has Jordan played 12 games? **2**

At what week has Ari played 12 games? **4**

At what week has Cy played 12 games? **6**

In conclusion. Ask: Which makes it easier to see data, tables or graphs? [Answers may vary.]

EXPLANATIONS CONTINUED:

If there is additional time following this lesson, play the One Hundred Percent game, found in *Math Card Games* book, F50.

Name: _____

Date: _____

1. Ari plays three math card games every week. Jordan plays six math games every week and Cy plays two. Fill in the table to represent the number of games that they played.

2. How much did you add to each week's sum for:

Ari _____ Jordan _____

Cy _____

	Total Number of Math Games Played		
Weeks	Ari	Jordan	Cy
0	0		
1	3		
2	6		
3			
4			
5			
6			

3. Label the graph with numbers and titles. Along the bottom, write the number of weeks from 0 to 6. Along the left side, write the number of games played.

Then plot the total number of games each person played from the table above. Connect the points for each player and label with the player's name.

4. What is the shape of the three graphs?

_____

At Week 2, Jordan has played how many times more games than Ari?

_____

At each week, Ari has played what fraction of the number of games that Jordan has played? ____

At each week, Cy has played what fraction of the number of games that Jordan has played? ____

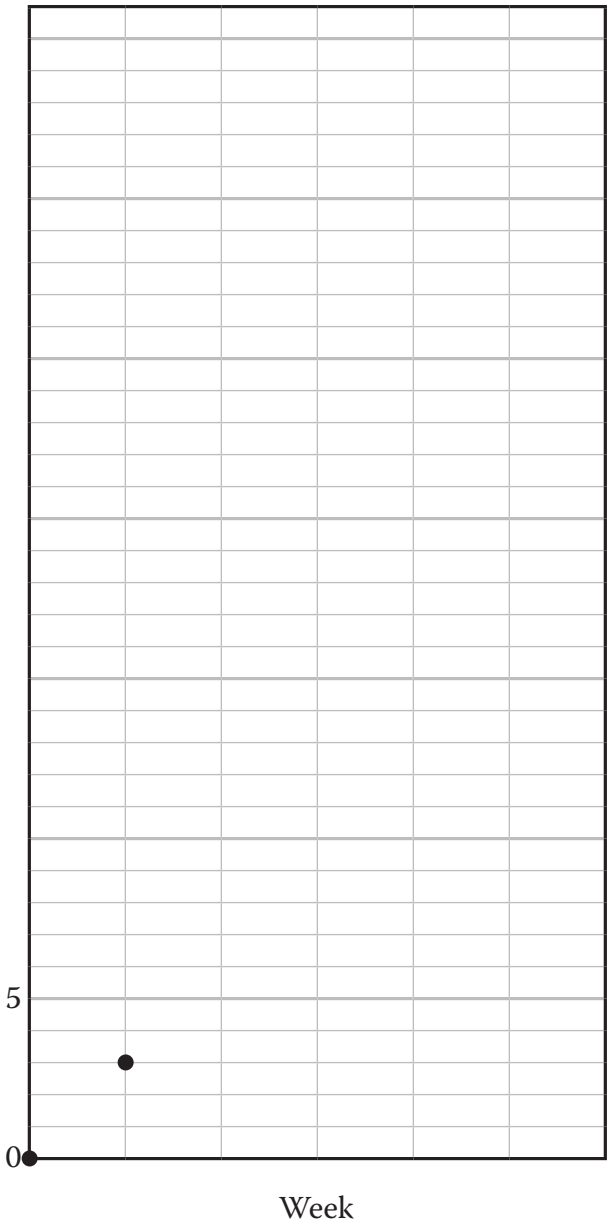
At what week has Jordan played 12 games? ____

At what week has Ari played 12 games?

____

At what week has Cy played 12 games?

____



Lesson 129: Solving for Unknowns on the Math Balance

OBJECTIVES:

1. To understand that the two sides of an equation are equal
2. To physically solve for an unknown in an equation using the math balance

MATERIALS:

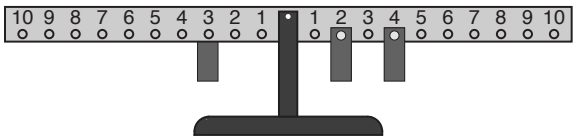
1. Math Balance
2. Worksheet 110, Solving for Unknowns on the Math Balance

ACTIVITIES FOR TEACHING:

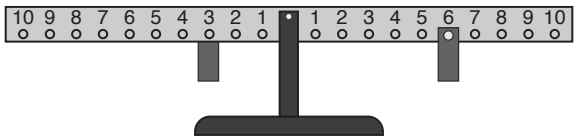
Warm-up. Ask: What is the most important property of an equation? [The two sides are equal.] What does the word equation means? [equal] If you add a weight on the 4-peg, what must be done to the other side to make it balance? [add a weight on the 4-peg] What is a second way you could do this? [add two weights to the 2-peg]

Mystery 1. Tell the child that today's lesson is about finding a mystery number on the math balance.

Set a math balance where he can see only the front. Place two weights on the right side, one at 2 and one at 4. Also place two weights on the back side at the left. Do not tell him how many weights are on the 3. See the left figure below.



$3x = 2 + 4$



$3x = 6$

Say: We want to find out how many weights are on the 3. Ask: If you replace the weights at 2 and 4 with one weight, where would you put it? [at the 6] See the right figure above.

Ask: Now can you tell the number of weights at 3? [6 ÷ 3 = 2] Show him the two weights from the back of the left 3-peg.

Say: Let's write the equations from each step. We will use an x for the mystery number. If necessary, repeat the math balance activity while writing the equations.

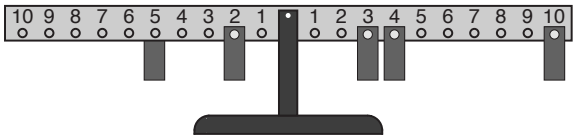
$$3x = 2 + 4$$
$$3x = 6$$
$$x = 2$$

Mystery 2. Put weights at the 3, 4, and 10 on the right side of the math balance. On the left side put a weight at the 2-peg and three weights behind 5-peg. Again, do not let the child see the number of weights on the 5-peg. See the figure on the next page.

EXPLANATIONS:

Overview Video QR code for Lessons 129 through 132.

ACTIVITIES FOR TEACHING CONTINUED:

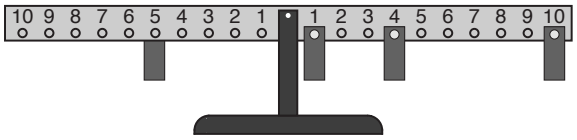


$5x + 2 = 3 + 4 + 10$

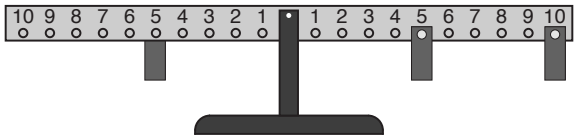
Ask: What is the equation? Write it together:

$$5x + 2 = 3 + 4 + 10$$

Ask: How can we remove the weight at 2 and still stay in balance? [Remove the 2 from the left side. Remove 2 on the right side by subtracting 2 from 3, resulting in moving the weight on the 3-peg to the 1-peg.] See the left figure below.



$5x = 1 + 4 + 10$



$5x = 15$

Ask: What is the next equation? Write it together:

$$5x = 1 + 4 + 10$$

Ask: What is the next step? [Combine the weights on the right side.] See the figure above on the right. The equations are:

$$5x = 15$$
$$x = 3$$

Worksheet 110, Problem 1. Give the child the worksheet and math balance. Tell him to read the instructions on the worksheet. Ask: How can you use your math balance to check your answers? [Put weights on according to the figure and the extra weights on the back side according to your answer.] Tell him to complete the worksheet. Solutions are below.

1. **$9 + 2x = 5 \times 3$**
   $2x = 6$
   $x = 3$

2. **$6 \times 4 + 2x = 6 + 10 \times 2$**
   $24 + 2x = 26$
   $2x = 2$
   $x = 1$

3. **$3 \times (6 + 4) = 5x + 10$**
   $30 = 5x + 10$
   $20 = 5x$
   $x = 4$

4. **$9 \times 2 + 6x = 2 + 8 \times 4 + 10 \times 2$**
   $18 + 6x = 54$
   $6x = 36$
   $x = 6$

5. **$10 \times 2 + 5 + 2x = 10 \times 3$**
   $25 + 2x = 30$
   $2x = 5$
   $x = \frac{5}{2}$

6. **$4 \times 4 + x(2 + 1) = 1 + 3 \times (3 + 5)$**
   $16 + 3x = 25$
   $3x = 9$
   $x = 3$

In conclusion. Ask: What do you call two expressions that are equal? [equation] What does it mean when checking an equation if the two sides are not equal? [A mistake was made.]

EXPLANATIONS CONTINUED:

The child's equations may vary slightly.

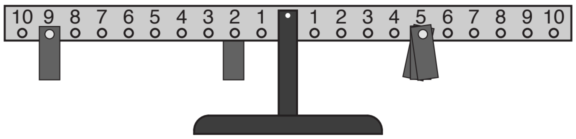
If time remains, play either the Negative Corners™ game or Top and Bottom Corners™ game found in *Math Card Games* book, S10 or S11.

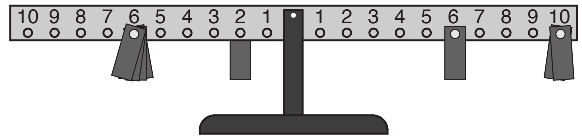
Name: ______________________________

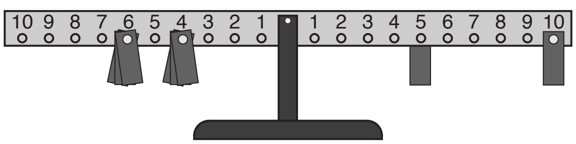
Date: ______________________________

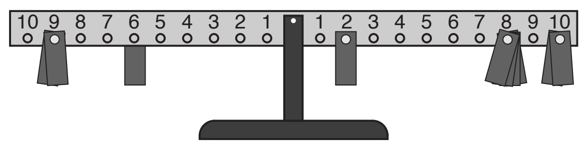
Find the mystery number, the number of weights on the back side of the math balance, in each figure. Write out the equations as you solve them.

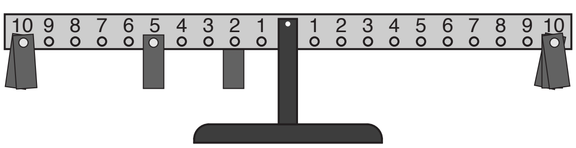
Use your math balance to check your work. If x is greater than 5, lay the extra weights across the 5 weights as shown on the right.



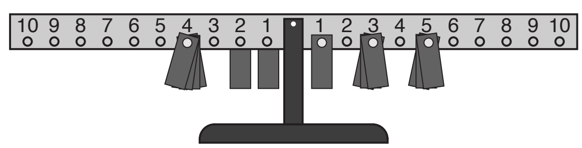








The answer will be a fraction. It cannot be verified with the math balance.



The same number of weights are on both the left 2-peg and the left 1-peg.

Lesson 144: Drawing a Regular Polygon in a Circle

OBJECTIVES:

1. To review angles
2. To measure angles with the goniometer
3. To divide a circle into equal areas
4. To construct regular polygons in a circle

MATERIALS:

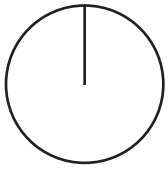
1. Worksheet 125, Drawing a Regular Polygon in a Circle
2. Drawing board, T-square, 45 triangle and 30-60 triangle
3. Goniometer

ACTIVITIES FOR TEACHING:

Warm-up. Ask: What is a regular polygon? [all of the sides are equal and all angles are equal] If you are facing the front of the room and turn 180°, where will you be facing? [back] If you are facing the front of the room and turn 360°, where will you be facing? [front]

Angles in a circle. Tell the child that today's lesson is about drawing regular polygons inside circles.

Sketch a circle with a vertical radius as shown below on the left. Ask: If this were a clock and the radius was the minute hand, how many degrees would the hand move through until it was at the beginning of the next hour? [360°]

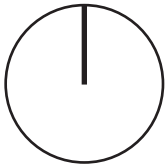


Circle with one radius.

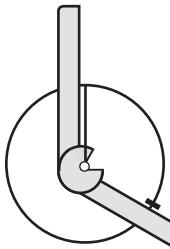
Circle divided into thirds.

Draw the second figure shown above on the right. Ask: How could you find the angle between these lines, which are evenly spaced? [360 ÷ 3 = 120°]

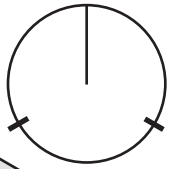
Worksheet 125, Problem 1. Give the child the worksheet, drawing tools, and goniometer. Tell him to tape the worksheet to his drawing board. First draw the vertical radius with his T-square and triangle as shown in the first figure below. To draw another radius, set the goniometer at 120° and align it to the first radius and draw a tick mark, a short line, on the edge of the circle. See the second figure. In the same way, draw the other tick mark. Then draw the radii. See the third figure.



First radius.



Drawing a tick mark.



Drawing all tick marks.

Drawing all radii.

EXPLANATIONS:

The word *radius* comes from the same word as ray, like the rays of the sun. The plural of radius is *radii* or *radiuses*.

If the two parts of the goniometer come apart, they can be snapped back together. Align the part with the bump on top of the other part and press down.

These radii could also be drawn with the 30-60 triangle.

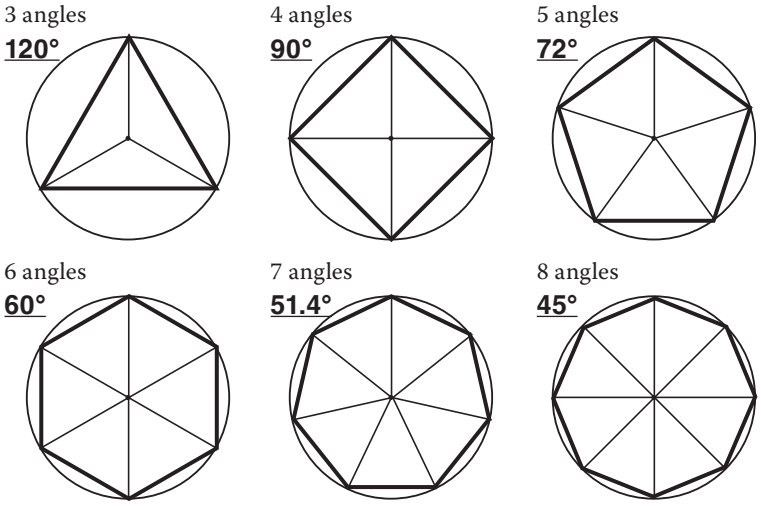
ACTIVITIES FOR TEACHING CONTINUED:

Lastly have him connect the ends of adjacent radii. See the figure on the right. Ask: What figure do the connection lines make? [equilateral triangle] Ask: Is it a regular polygon? [yes]

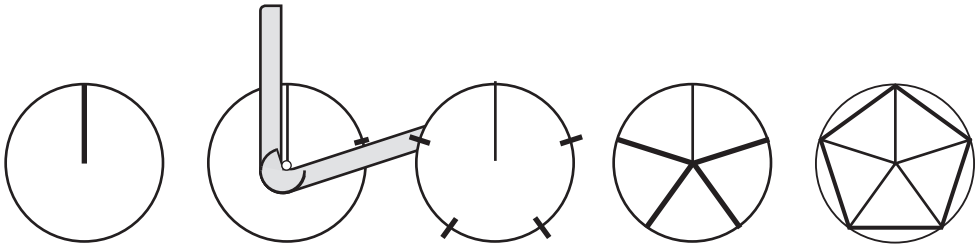
Equilateral triangle.

Problem 2. Drawing the radii for four equal angles is almost a trivial problem. Let the child do it his own way. The connected radii form a square.

Problems 3-6. Tell the child to complete the worksheet. Solutions are below.



For a child needing more help for Problem 3, ask: How can you find the angle between the radii? [360 ÷ 5 = 72°] Set the goniometer to 72° and draw the tick marks. See the second and third figures below. Draw the radii as shown below in the fourth figure.



First radius. **Drawing a tick mark.** **Drawing all tick marks.** **Drawing the radii.** **The pentagon.**

Connect the ends of adjacent vertices as shown above in the last figure.

In conclusion. Ask: If you draw 10 equally spaced radii in a circle, what is the angle between the radii? [36°, 360 ÷ 10] If you draw nine equally spaced radii in a circle, what is the angle between the radii? [40°, 360 ÷ 9]

EXPLANATIONS CONTINUED:

If the child is having difficulities seeing the equilateral triangle, draw the lines in a different colored pencil.

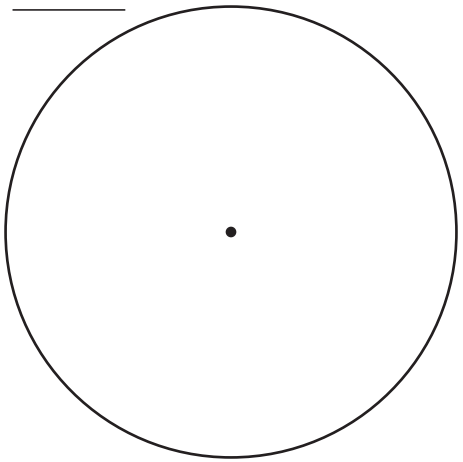
It is good practice to check if the angle between the last tick mark and the vertical radius is correct. If an adjustment is necessary, tick marks are easier to change than radii.

Name: _____ Date: _____

Draw regular polygons by first drawing radii that have the same angle between each radius.
Then connect the ends of the radii. Write the angle between the radii on the line near each circle.

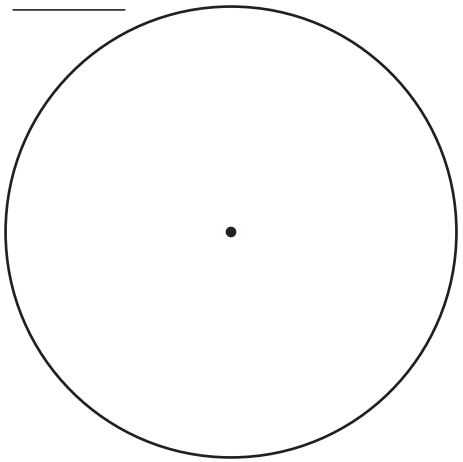
3 angles

_____



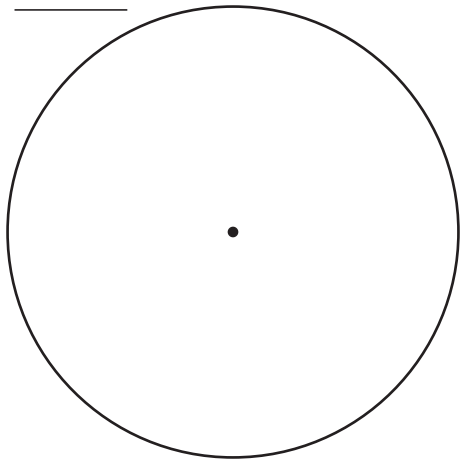
4 angles

_____



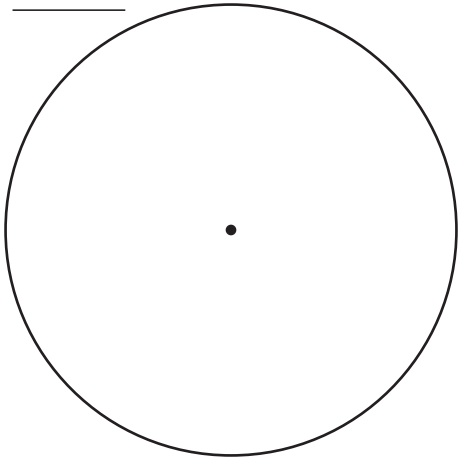
5 angles

_____



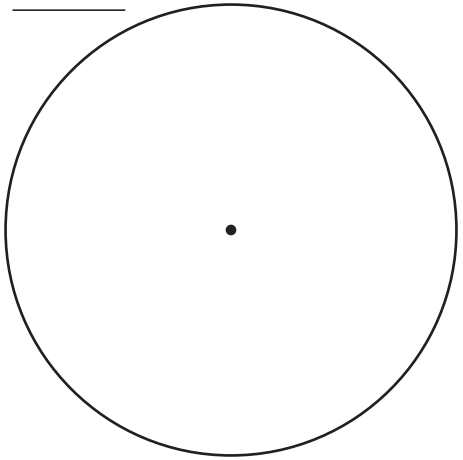
6 angles

_____



7 angles

_____



8 angles

_____