RightStart™ Mathematics

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with Kathleen Cotter Lawler
and Maren Stenseth Ehley

LEVEL H LESSONS

Second Edition

Activities for Learning, Inc.

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RIGHTSTART MATHEMATICS OBJECTIVES FOR LEVEL H

Name _____ Year _____

	Trimester 1	Trimester 2	Trimester 3
Numeration			
Understands the difference between rational and irrational numbers*	N/A	N/A	
Understands that subtraction of rational number is the same as adding negative number	N/A	N/A	
Understands and applies absolute value	N/A	N/A	
Understands and applies central numbers, i.e., arithmetic mean, median, and mode	N/A	N/A	
Solving Equations			
Solves problems involving rational numbers using the four operations			
Applies properties of operations, ie., associative, commutative, and distributive, as strategies to solve problems			
Solves problems using positive and negative rational numbers	N/A	N/A	
Converts rational numbers between fractions, decimals, percents, and whole numbers			
Solves multi-step algebraic equations using distributive property including rational number coefficients*	N/A		
Evaluates expressions and writes answers in expanded form or in scientific notation, using positive and negative exponents*	N/A	N/A	
Understands and applies square roots and cube roots to solve problems*			
Problem Solving			
Solves multi-step one-variable equations involving coefficients, exponents, and parentheses			
Persists and finds more than one way to solve problems			
Knows and applies the distance formula to solve real-life problems	N/A		
Solves geometric problems by measuring lengths and computing areas from a scale drawing			
Ratic			
Evaluates proportional relationships to determine equality			
Identifies and analyzes the constant relationship, unit rate, in an equation, table, or graph			
Explores the golden ratio phi, Ø, and sees examples in the real world	N/A		
Applies proportional relationships to real-life situations	N/A		
Computes unit rates associated with ratios	N/A	N/A	
Writes proportional relationship equations	N/A	N/A	
Understands and applies trigonometry ratio calculations, specifically sine, cosine, and tangent	N/A		
Coordinate System			
Understands and plots positive and negative numbers on a line or coordinate plane	N/A		
Creates images and translations on a coordinate plane			
Statistics and Probability			
Understands probability are between 0 and 1, the larger number indicating more likely an event will occur	N/A	N/A	
Understands and applies measures of center and geometric mean to draw informal conclusions	N/A	N/A	
Geometry			
Uses tools to draw geometric shapes with specific conditions given, including side length and angles			
Solves real-world and mathematical problems involving area, surface-area, and volume of shapes and solids			
Understands, finds, and explains planes of symmetry and antiprisms	N/A		
Knows and applies formulas for circumference and area of a circle			
Applies area and circumference of a circle to real-world situations			
Applies Pythagorean theorem to find side length in real-world and mathematical situations*			
Explores and applies Fibonacci sequence and understands how it relates to the Golden Ratio	N/A		
Examines and creates nets and their relationship to solids	N/A		
Study Skills			
Understands and can explain geometric and mathematical terms			
Explores historic and cultural influences in math			
Develops independent learning skills			
Understands the importance of using available resources for independent learning			

IMPORTANT NOTES TO THE STUDENT
Read Before You Start

Welcome to RightStart™ Mathematics Level H Second Edition! This is an exciting time as you will be completing these lessons independently. We have put together some tips to help make this year as smooth as possible. Don't let the length of this section scare you. It's broken down into segments, so read it through and you will be ready to start!

LESSON MANUAL

This is the book you are reading right now. It contains your lessons. Take a moment to flip through the book. Notice the different sections for each lesson. Let's outline them for you.

- **Objectives**: These are the goals of the lesson. Pay close attention to the italicized words; these are terms you need to learn. You will find the explanation of those words in your lesson. Record the definitions in your Math Dictionary, found at the back of the Worksheets. Go find the dictionary now while you are thinking about it. We will talk more about this later.
- **Materials**: This list tells you what you will need for the lesson. Gather the items before you start. A 0.5 mm mechanical pencil is highly recommended. You will find more information on some of the materials used in these lessons as you keep reading.
- **Activities**: This is the lesson.
- **Extras**: This is, well, extras! It helps you with your lessons. It has tips and reminders placed next to the related paragraph in the Activities. Photos are also included showing you how these concepts are applied in real life.

This year, *you* are teaching yourself. You will be reading the lessons and worksheets and learning as you go. Your teacher is a facilitator. They will guide you, help you, and answer questions, but they will not be teaching you. Again, **YOU are teaching yourself**.

MATERIALS

Take a moment to check out the materials you will be using. Get familiar with their names and what they look like so you can quickly grab what you need for the lesson. There will be more detailed instructions in the lessons but for now, here's a list of the materials you will be using.

- **Drawing Board**: This is the surface you will tape your worksheets onto for your work. Using 3M Removable tape is highly recommended.
- **T-square**: The T-square is used to draw horizontal lines. It also provides alignment and support for the triangles so you can draw lines at the proper angles. Some T-squares may have measurements on them, but do not use the T-square as a ruler. It's a T-square!
- **30-60 Triangle and 45 Triangle**: These will be used to help you draw vertical lines and angled lines. The difference between the two triangles is important. The first lesson will identify the differences between the two triangles.

- **4-in-1 ruler**: This ruler has one side with measurements in centimeters and millimeters. The other side has measurements in tenths and sixteenths of an inch. A ruler with inches divided into tenths is unusual and you will find how useful it is!
- **Goniometer**: This tool is used to measure angles. It is best used when lying flat and is easiest to read if you lay it on white paper, such as your worksheet. The goniometer is numbered by tens. The lines between the numbers are by twos. If you are looking for 15°, move the upper arm of the goniometer until the indicator line in the bubble is halfway between the numbers 10 and 20, as well as halfway between the lines representing 4 and 6. Getting the exact measurement takes practice as it can be hard to see some of the numbers. Do your best.
- **mmArc™ Compass**: This tool is used to make circles and arcs. There are instructions in the lessons on how to use this compass.
- **Scientific Calculator**: The Casio Calculator fx-300MS calculator was carefully chosen for you. Lesson 3 will review how to use it.
- ***Math Card Games* book**: This book will tell you how to play the math card games to practice your arithmetic. Almost every lesson will assign a game to play. The games are important because they help you practice your math facts. The games are more fun than review sheets, so remember to play the games assigned.
- **Other materials**: There are other materials that you are most likely familiar with; tangrams, tape, centimeter cubes and Geometry Panels.
- **Items listed in bold**: Every now and then, you will see items listed in bold. These things are not in your materials kit but things you generally have on hand.

ACTIVITIES AND EXTRAS

This section is your lesson and along with the worksheets will provide information for you to explore and learn.

1. **Read the lesson.** Unless it tells you otherwise, read through the lesson. Sometimes it will tell you to complete the worksheet, or part of the worksheet, before reading further. If it doesn't ask you to complete the worksheet before reading further, continue reading the entire lesson. Read it carefully!
2. **Look at the figures and pictures.** These accompanying figures enhance the lesson information. Sometimes a drawing can explain better than words. Pictures are also included and will show how the concepts you are learning are applied.
3. **Read the extras.** Here you will find hints and reminders to help with the lesson. Sometimes they are fun tidbits of information to enhance the lesson.
4. **Summarize.** Can you summarize what you have read? Do you have a good idea of what you will be doing? Great. But before you start on the worksheet you will need to do one more thing.... Don't skip this next step....
5. **Read through the lesson AGAIN!** This time look for terms you need to define. The terms are *italicized words* in the objectives and in the lesson.

It may help to underline or highlight the terms as well as its explanation in the lesson. Then immediately record the definition in your Math Dictionary, found at the back of your worksheets. Writing the definition in your own words helps you understand the terms better. There is also space to draw an example if needed. This may help you remember it better.

It is important to define these terms as you will be referring to these words throughout the lessons. Having the definitions in a central location will make it much easier when you need to check the meaning of something.

6. **Read through the lesson a THIRD time.** Now you are ready to begin the lesson. This will be your third time reading the same information.

   Why are we telling you to read it so many times? No one learns mathematics by reading information only once. Honest. It doesn't matter how smart you are, reading more than once is needed. Often times, you will absorb more information and explanation as you read a second or third time. Sometimes you may skim over an important word the first time or miss a connection, but reading it the second or third time makes it clearer.

Review lessons. There are review lessons sprinkled throughout the year. These are not considered tests, therefore, you may refer to previous lessons if you need to look something up.

Assessments. Assessments are considered tests. You will not be allowed to look at lessons, worksheets, or your dictionary while completing an assessment.

Understanding. Each lesson needs to be understood before going to the next lesson. Don't go on if you don't understand what you are currently doing. The lessons often build on each other. If one lesson is shaky, the next one is going to be more challenging.

Questions. If you are having trouble understanding a lesson, don't just say, "I don't get it." That's not a question! How can anyone understand you and help you? You need to form a question to ask, then ask your facilitator. What exactly do you not understand? Be specific. Learning how to ask questions is an important skill to acquire toward becoming an independent learner.

WORKSHEETS

You have a separate binder that holds the worksheets. You will take the worksheet page out of the book, work on it, then return it to your binder. Each lesson will tell you which worksheet you will need. Some lessons will have one worksheet and other lessons will have more.

There are directions in Lesson 1 on how to tape and align the worksheet to the drawing board. You will be expected to do this with every worksheet, even though it's not written in the lessons.

Use a light hand when drawing on your worksheet. It's easier to erase things if you write lightly. You **will** be erasing things. This is very normal. A lighter line also makes for better accuracy.

Sometimes, when doing the worksheets, you will want to refer back to the lesson. You will be surprised at the hints or clues that make sense when you are actually doing the work.

Keep your completed worksheets in your binder. **Do not throw any worksheets away.** Some lessons will reference a prior worksheet. Sometimes you will want to refer back to them, especially when you get to the assessments.

CONTACT US

If you have read through the lesson at least three times, you have gone to your facilitator for help, and you are still confused, then call or email us directly. The email address, info@RightStartMath.com, and phone number, 888-RS5-MATH (888-775-6284), are on the back of this book. We will help you from 8:30 to 4:30 Central time, Monday through Thursday, and 8:30 to 12:30 on Fridays.

NEED TO KNOW

There is a Need to Know list located at the back of the worksheets, right before the Math Dictionary. These are important things you need to memorize.

The Need To Know list also tells you which lesson has the information. In the lesson, the Need to Know information will have a box around it in the Extras section. You may want to highlight this box with a specific color, different from what you are using for the definitions, as a reminder that this is important and you really NEED TO KNOW it. Plus, it's easier to find when looking for it.

MATH DICTIONARY

The Math Dictionary is located at the back of the worksheets, behind the Need to Know list. The terms are already written in the dictionary for you. It also tells you which lesson or lessons that term was introduced. A few of the terms will be found in more than one lesson because the meaning is expanded. It will be helpful to add new information to your initial definition.

Record the definition for the terms in your own words. Consider drawing a picture to better help you remember the definition.

GRADING

You will be grading your own work based on your persistence, understanding, and results. You are expected to do this after every lesson. In the Worksheet binder, look at the very first pages, titled Level H Worksheet Results. Read the instructions.

Your facilitator has the Solutions manual. They may want to go over your work with you or they may allow you to look at the Solutions manual on your own.

Maybe you weren't totally sure about a concept but then it "clicked" after looking at the solutions. Maybe you want to redo your work. If needed, regrade yourself after looking at the solutions.

SOLUTIONS MANUAL

The Solutions are written to the facilitator. It will show the solutions to the worksheets as well as give additional explanations and insights. Your facilitator will decide how to use the Solutions book with you.

GAMES

Most lessons will have a math card game listed. These games are important because they help you practice your math facts. Make time to play them!

These games don't have to be played during the lesson. You can play the game right away or any time throughout the day. Some students will do their lesson then play the game at a later time. If you don't have someone to play the game with you, modify the game to be a solitaire game. Or you could have your right hand play against your left hand; in other words, play a game against yourself!

The important thing is that you are practicing your math facts. Playing a game for 10 to 15 minutes is the same as doing a worksheet. And it's a whole lot more fun than a drill of math facts!

IT'S TIME TO START.....

Now you're ready to begin. Turn the page and let's go!

Joan A. Cotter, Ph.D.
and Kathleen Cotter Lawler
and Maren Stenseth Ehley

IMPORTANT NOTES TO THE FACILITATOR

Welcome to RightStart™ Mathematics Level H Second Edition! This is an innovative approach for teaching many middle school mathematics topics including fractions, area, volume and surface area, rounding, ratio, pi and phi, tessellations, fractals, basic trigonometry functions, platonic and archimedean solids, pre-algebra with variables and graphing, and more. The student also reviews traditional geometric concepts: angles, rotations, reflections, symmetry, Pythagorean Theorem, and so forth. In this program the student does not write out formal proofs, although they will learn to keep their processes organized and logical.

It is of prime importance to understand mathematics. Approaching mathematics through geometry gives the student an excellent way to understand and remember concepts. The hands-on activities create deeper learning. For example, to find the area of a triangle the student must first construct the altitude, measure it, and then perform the calculation.

The majority of the work is done with a drawing board, T-square, 30-60 triangle, 45 triangle, goniometer (a device for measuring angles), and mmArc™ Compass (a tool for drawing circles and arcs). It is interesting to note that CAD, Computer Aided Design, software is based on these drawing tools.

RightStart™ Mathematics Level H Second Edition also incorporates other branches of mathematics as well as arithmetic and algebra. Some lessons have an art flavor while other lessons have an engineering focus. Even some history of mathematics is woven throughout the lessons.

This text is written with several goals for the student:

a) to use mathematics previously learned,

b) to learn to read math texts,

c) to lay a good foundation for more advanced mathematics,

d) to learn mathematical terms,

e) to discover mathematics everywhere, and

f) to enjoy mathematics.

INDEPENDENT STUDENT LEARNING

These lessons are written for the student to do independently. You are the facilitator, not the teacher. The lessons will teach the student and guide them to understanding. The worksheets are not a review, rather they are the discovery and application for the lesson at hand. When a student has a question, check with the Solutions and see if the notes provide information to guide the student to the solution.

We want our students to become independent learners. You may need to go through a transition time as you both adjust. Each student is different and some may need a little more guidance and assistance at first.

Encourage the students to read the lesson more than once to gain understanding. In the student's notes, they are told to read the lesson at least twice. Learning to read a math textbook is a necessary skill for success in advanced math classes.

One teacher had some students who were delayed readers. She let the students read the lesson alone and do the work. If they did well on their worksheets and could explain what they learned, they moved on. If not, she had them read the lesson and the directions on the worksheets aloud. Mistakes were often made because the students were skimming through the lesson and had missed a vital piece of information. Sometimes they didn't pay attention to the detailed instructions. Because they were delayed readers, the students did not want to read the lesson more than once, so it took encouragement to help them understand it is normal to read the instructions multiple times. Learning to follow directions is a necessary skill for studying and everyday life.

If the student has a serious reading challenge, read the text aloud while they follow along then ask them to explain what they heard. Be sure each concept is understood.

For minor reading challenges, you might model aloud the process of reading a lesson, commenting on the figure, and summarizing the paragraph. Then ask the student to read the lesson aloud on their own. Sometimes the student needs support to overcome frustration, which is inherent in the learning process.

There are terms introduced and defined in the lessons for the student to write in their Math Dictionary. The dictionary is found at the back of the student's worksheets. Encourage the student to use their words and drawings to define the new terms and concepts.

Math should be 95% understood and 5% memorized. Your student will find understanding concepts will help them recall the information. There is a Need to Know page for information or formulas that need to be memorized.

IMPORTANT NOTES TO THE STUDENT

In the student's Lesson book, there are pages right inside the cover to read. Make sure the student reads these before they begin. It would be beneficial for you to read it too. Some students may benefit from reading the pages aloud. This will ensure they have read it, as well as making the expectations clearly understood.

THE SOLUTIONS MANUAL

The solutions manual is written for you, the facilitator. It shows the solutions as well as gives additional insights into the lessons.

Read the Solutions manual for the lesson being worked on **before** the student does their lesson and worksheet. Read any special notes so you are prepared to answer questions the student may ask.

You may want to go over the work with your student or you may hand the manual to them and have them compare their work with the solutions. Some facilitators will let their student go over the solutions themselves then have them explain their work. Being able to point out mistakes is a great step in gaining better understanding and decreases the likelihood of repeating the same error in the future.

Keep in mind that some problems will have more than one solution. We will let you know in the Notes section, although we may not show you all the different solutions.

Also keep in mind that when measuring, your student may not be as accurate as the solutions, therefore, their answers may differ. As long as their results are close to the given answers and the student understood the process, they don't need to redo their work. You may want to double check their arithmetic with a calculator. If there is a significant difference, help them go through the steps to see where they made their mistake. Sometimes it's as simple as measuring in inches rather than centimeters, forgetting part of their formula, or recording the answer incorrectly.

ENRICHMENT LESSONS

There are a few enrichment lessons sprinkled throughout the book. These lessons are optional, meaning it's up to you if you want the student to complete those lessons. The enrichment lessons expand on what the student has previously learned. It's best that they do these lessons, however, depending on the situation, it may not be necessary.

Enrichment lessons may include some new vocabulary. If you choose to not have your student complete these lessons, check if there are new terms introduced.

QUESTIONS, CORRECTIONS, AND SHARING

If you have any questions or concerns please have the **student** email us or call our office during business hours. Our contact information is on the back of this manual as well as the student's Lesson manual. We like feedback from you and the student.

If you or the student finds any errors in the books, please let us know. We do our best to proof this material, but unfortunately there still are errors that get overlooked. If you find a correction that needs to be made, email us at info@RightStartMath.com.

Occasionally, we encourage the student to send us their work so we can display it on our website. The student's parent or guardian must give permission to post the work. Also, indicate how to acknowledge the student: first name only, full name, region, or student's age. Send the work and information to info@RightStartMath.com and we will share their accomplishments.

Thank you for choosing to give your student a RightStart in Math!

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Level H Table of Contents

Level H Table of Contents

Level H Table of Contents

Level H Table of Contents

Level H Table of Contents

Lesson 24: Ratios of Tessellating Triangles

OBJECTIVE:

1. To solve an area and perimeter problem in a tessellation

MATERIALS:

1. Worksheets 24, Ratios of Tessellating Triangles
2. Drawing board, T-square, and 30-60 triangle
3. 4-in-1 ruler
4. Casio Calculator fx-300MS
5. *Math Card Games* book, S18

ACTIVITIES:

Tessellating triangles. As you explored in the last lesson, all triangles tessellate two different ways using rotation and/or reflection. Today you will be creating a triangle tessellation, then exploring the ratios of the area and perimeter of various triangles.

Worksheet 24. You will be tiling a large triangle with 16 smaller similar triangles. If you need a hint for the tessellation, look at your first tessellation on Worksheet 23.

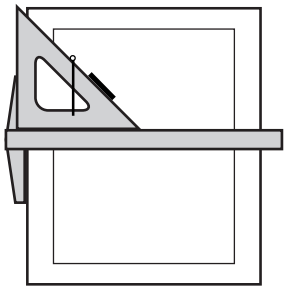
You will need to divide the sides of the large triangle into fourths by dividing them in half twice. If you need a reminder on bisecting lines, keep reading.

Bisecting a horizontal line. Bisecting a line segment is the same thing as finding its midpoint. If you need a review on how to bisect a horizontal line, go back to Lesson 2.

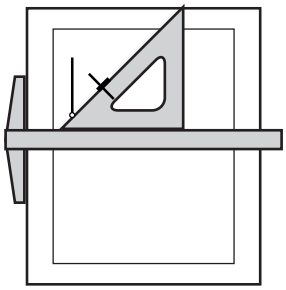
EXTRAS:

Although you could measure a line, divide by two, then measure again and mark the midpoint, using the drawing tools is more accurate and efficient.

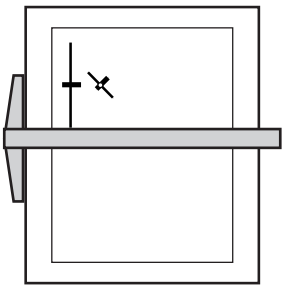
Bisecting a vertical line. To bisect a vertical line, see the figures below.



The first step.



Flip the triangle and repeat.



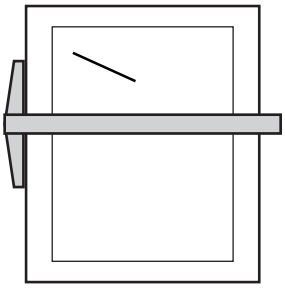
The vertical line is bisected at the tick mark.

When bisecting a line, it does not matter which triangle or angle of the triangle is used as long as the same angle is used for both sides of the line being bisected.

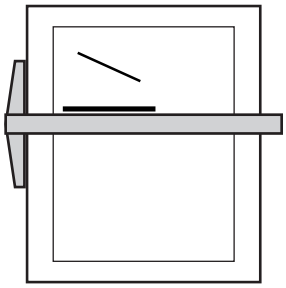
LESSON CONTINUES ON THE NEXT PAGE.

ACTIVITIES:

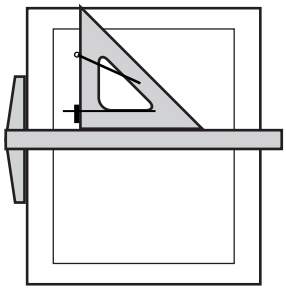
Bisecting a line that is neither horizontal nor vertical. First, you project the line onto the horizontal line, which is shown below. It's similar to finding the line's shadow when the sun is directly overhead. Then you find the midpoint of the "shadow" and project it back to the original line. See the figures below.



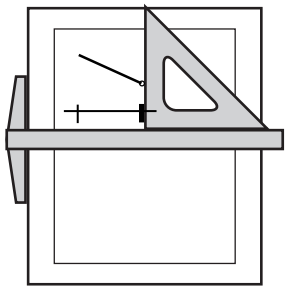
The line segment to be bisected.



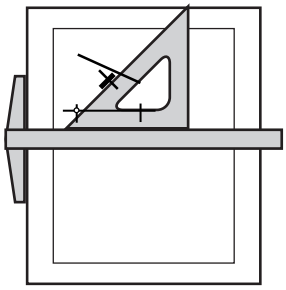
Draw a projection line below the line segment.



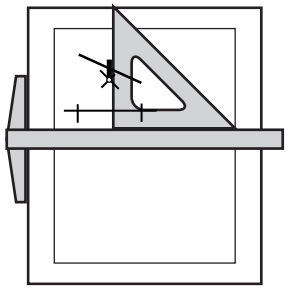
Draw the left tick mark.



Draw the right tick mark.



Bisect the horizontal segment.



Project the midpoint back to the original line.

Today's game. Play the Short Chain Subtraction game, found in the *Math Card Games* book, S18.

EXTRAS:

To project means to make an image. For example, you can project your hand's shadow on the wall to make a rabbit figure.

Using a very sharp pencil as you work will help your accuracy. A 0.5 mm mechanical pencil is best.

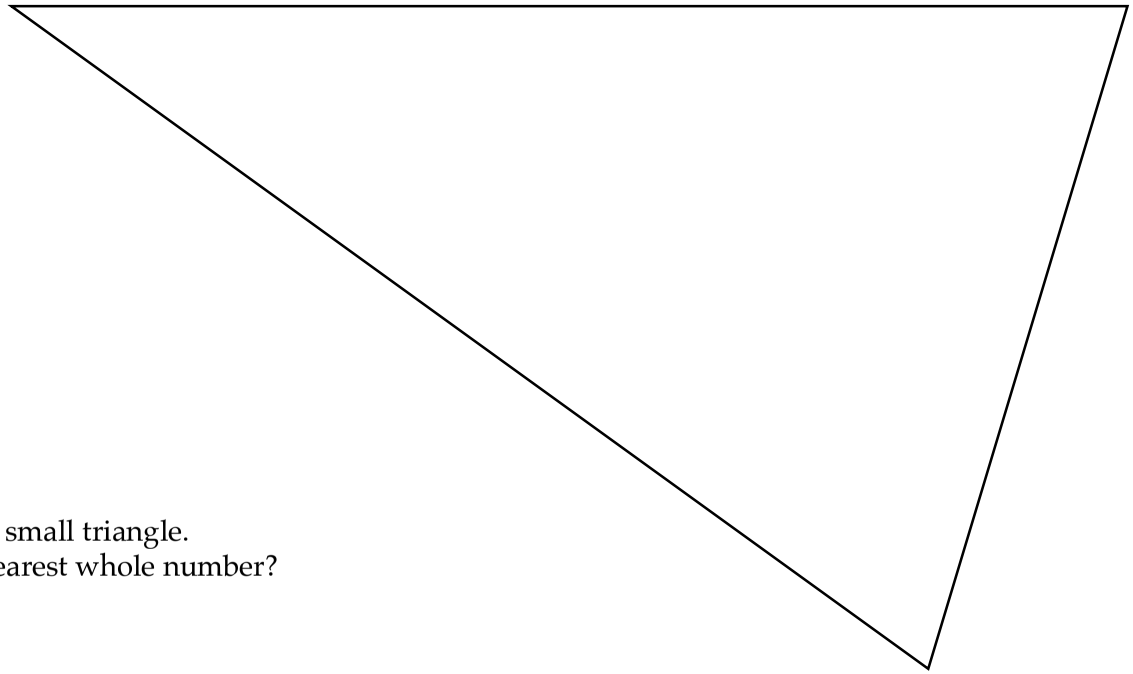
Name: _____

Date: _____

Worksheet 24, Ratio of Tessellating Triangles

1. Tile the large triangle below with 16 smaller similar triangles. See the lesson for a hint. Using your drawing tools, not your ruler.

2. Find the area of the large triangle and the area of a small triangle. Measure to the nearest tenth of a centimeter.
   Record the areas. Find the ratio of the large triangle area to the small triangle area to the nearest tenth of a centimeter in the chart below.



3. Find the perimeter of the large triangle and the small triangle.
   What is the ratio of the two perimeters to the nearest whole number?

	Area	**Perimeter**
Large triangle		
Small triangle		
Ratio of large to small		

1. Tile the large triangle below with 16 smaller similar triangles. See the lesson for a hint. Use your drawing tools, not your ruler.

2. Find the area of the large triangle and the area of a small triangle. Measure to the nearest tenth of a centimeter. Record the areas. Find the ratio of the large triangle area to the small triangle area to the nearest tenth of a centimeter in the chart below.

$A(lg) = \frac{1}{2}wh$

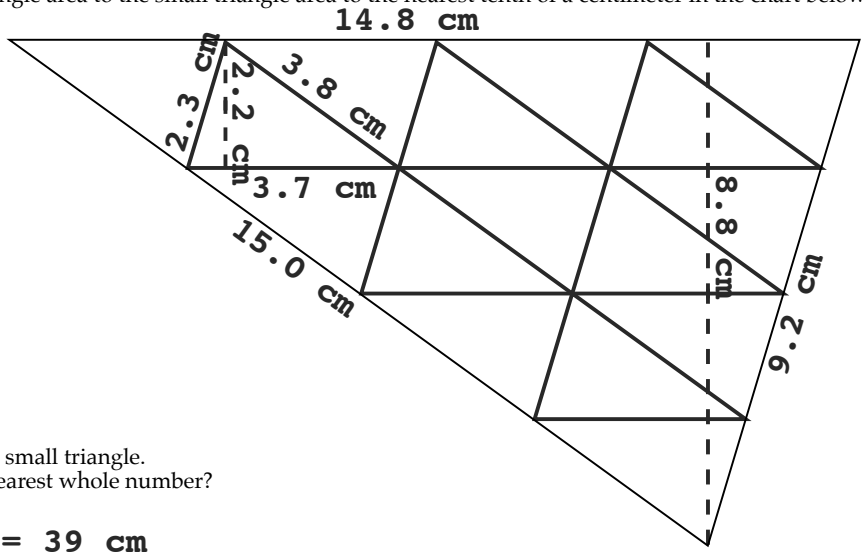
$A(lg) = \frac{1}{2} \times 14.8 \times 8.8$

$A(lg) = 65.1 \text{ cm}^2$

$A(sm) = \frac{1}{2}wh$

$A(sm) = \frac{1}{2} \times 3.7 \times 2.2$

$A(sm) = 4.1 \text{ cm}^2$



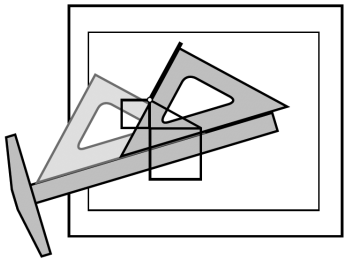
3. Find the perimeter of the large triangle and the small triangle. What is the ratio of the two perimeters to the nearest whole number?

$P(lg) = 14.8 + 9.2 + 15 = 39 \text{ cm}$

$P(sm) = 3.8 + 3.7 + 2.3 = 9.8 \text{ cm}$

	Area	Perimeter
Large triangle	65.1 cm^2	39.0 cm
Small triangle	4.1 cm^2	9.8 cm
Ratio of large to small	16:1	4:1

NOTES: If the student has difficulties creating the smaller triangles, remind them that each line of the triangle will be parallel to one of the sides of the large triangle. All smaller triangles will be congruent.



Some students may use lines parallel to the sides of the larger triangle using the t-square and triangle in an oblique position, as taught in RS2 Level G. See picture on the right. Although the lesson does not mention this method, it is acceptable.

When calculating the area of the triangles, other widths and heights can be used. The calculated area will be approximately the same. Other measurements may vary a bit too. Remind the student to write their formulas.

Many students are surprised to connect the division of the large triangle by 16 pieces and the ratio of the area, 16:1. The ratio of the perimeters of 4:1 is also connected to the division of the triangles because four little triangles fit on each side of the large triangle.

DICTIONARY TERMS: none

Lesson 28: Review and Games 2

OBJECTIVES:

1. To review recent topics
2. To practice skills through playing math card games

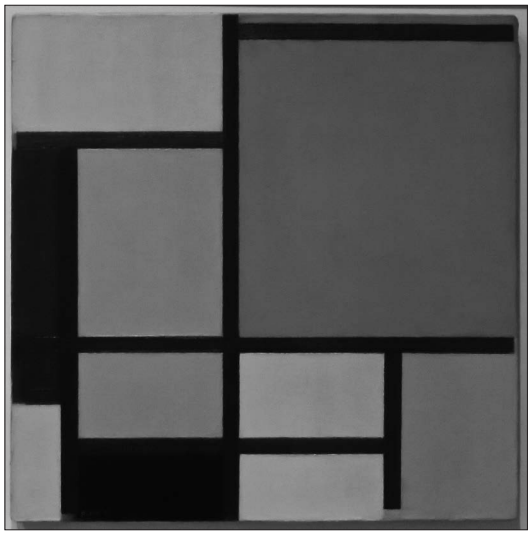
MATERIALS:

1. Worksheets 27-1 and 27-2, Review and Games 2
2. Drawing board, T-square, and triangles
3. 4-in-1 ruler
4. Casio Calculator fx-300MS
5. *Math Card Games* book, P23 and S18

ACTIVITIES:

Review. Today is a review of terms and concepts you have recently learned. Because this is not a test, you may go back to previous lessons if you need to look something up.

Worksheets 27-1 and 27-2. Complete the worksheets.



Oil on canvas, *Composition*, painted in 1921, by Piet Mondrian. On display at the Metropolitan Museum of Art in New York City, NY. To see the painting in color, go to RightStartMath.com/geometry.

Today's game. Play the Crazy Squares game, found in the *Math Card Games* book, P23 and the Short Chain Subtraction game, S18.

EXTRAS:

The games are important because they help you practice your math facts.

Name: _____

Worksheet 28-1,
Review and Games 2

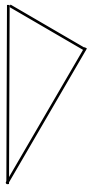
Date: _____

Write the correct term for the following definitions.

1. The name of the artist whose tessellations can be created by starting with a parallelogram and then changing the sides to create a unique repeating pattern. _____

2. The name of the artist who developed art using rectangles of different sizes, then colored them in bright primary colors, black and white. _____

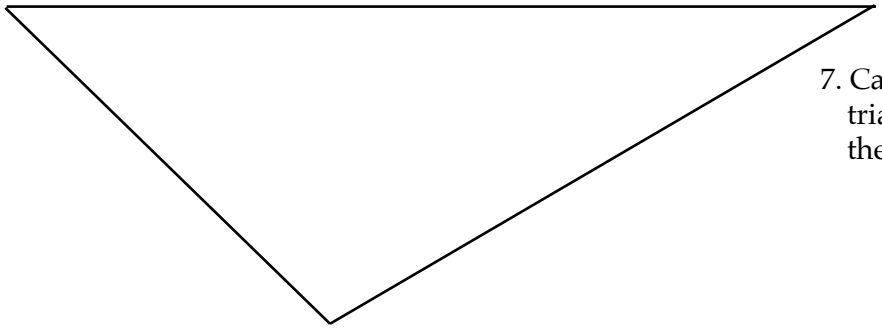
3. Construct a pure tessellation with reflections using your 30-60 triangle, drawing at least 16 triangles. The first triangle is given for you.



4. Sketch the unit.

5. List the angles at the vertexes.

6. Tile the large triangle with four smaller triangles similar to the large triangle. Do not use your ruler.



7. Calculate the perimeters of both the large triangle and a small triangle. Measure to the nearest tenth of a centimeter.

8. Calculate the areas of both the large triangle and a small triangle. Measure to the nearest tenth of a centimeter. Round your answers to the nearest tenth of a centimeter.

9. What is the ratio for the area of the large triangle to a small triangle, rounding to the nearest whole number? _____

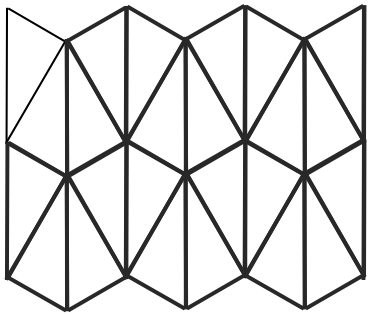
10. What is the ratio for the perimeter of the large triangle to a small triangle to the nearest whole number? _____

Write the correct term for the following definitions.

1. The name of the artist whose tessellations can be created by starting with a parallelogram and then changing the sides to create a unique repeating pattern. **Escher**

2. The name of the artist who developed art using rectangles of different sizes, then colored them in bright primary colors, black and white. **Mondrian**

3. Construct a pure tessellation with reflections using your 30-60 triangle, drawing at least 16 triangles. The first triangle is given for you.

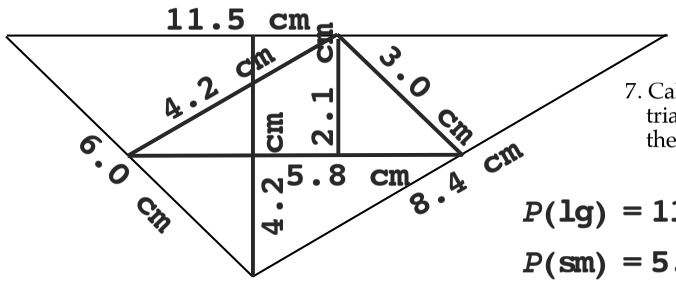


4. Sketch the unit.

5. List the angles at the vertexes.

30, 90, 60, 60, 90, 30
[ORDER MAY VARY.]

6. Tile the large triangle with four smaller triangles similar to the large triangle. Do not use your ruler.



7. Calculate the perimeters of both the large triangle and a small triangle. Measure to the nearest tenth of a centimeter.

$P(\text{lg}) = 11.5 + 8.4 + 6.0 = 25.9 \text{ cm}$

$P(\text{sm}) = 5.8 + 4.2 + 3.0 = 13.0 \text{ cm}$

8. Calculate the areas of both the large triangle and a small triangle. Measure to the nearest tenth of a centimeter. Round your answers to the nearest tenth of a centimeter.

$A(\text{lg}) = \frac{1}{2}wh$

$A(\text{lg}) = \frac{1}{2} \times 11.5 \times 4.2$

$A(\text{lg}) = 24.2 \text{ cm}^2$

$A(\text{sm}) = \frac{1}{2}wh$

$A(\text{sm}) = \frac{1}{2} \times 5.8 \times 2.1$

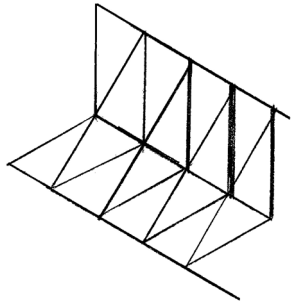
$A(\text{sm}) = 6.1 \text{ cm}^2$

9. What is the ratio for the area of the large triangle to a small triangle, rounding to the nearest whole number? **4:1**

10. What is the ratio for the perimeter of the large triangle to a small triangle to the nearest whole number? **2:1**

NOTES: Some students may struggle with the tessellation in Problem 3. All the lines are 30°, 60°, or vertical. If the student needs assistance, share the solutions on the left which allows them to see the whole image.

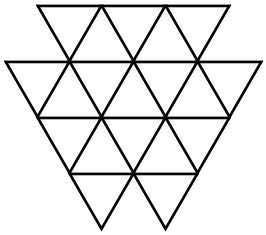
Samuel Uriostegui, RS2 Level H online class student from Illinois, created this pure tessellation.



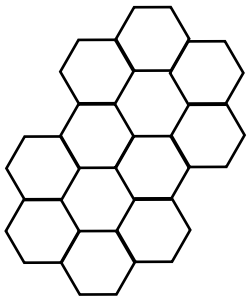
Question 9 the ratio of the area of the large triangle to the small triangle is 24.2:6.1, which is calculated as 3.967:1. Because the instructions ask for the answer rounded to the nearest whole number, the ratio is 4:1.

Name: _____

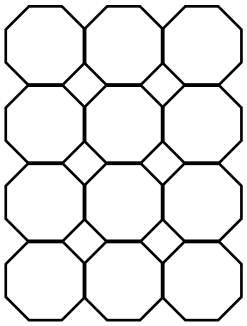
Date: _____



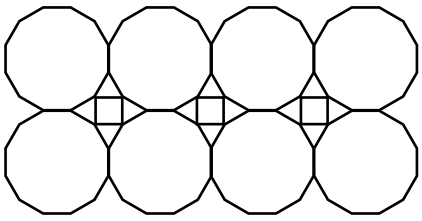
(a)



(b)



(c)



(d)

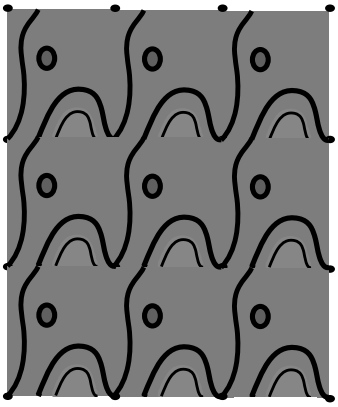
11. Which figure(s) above are demiregular tessellations? _____

12. Which figure(s) above are regular tessellations? _____

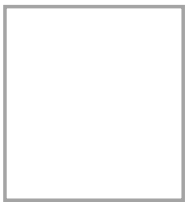
13. Which figure(s) above are pure tessellations? _____

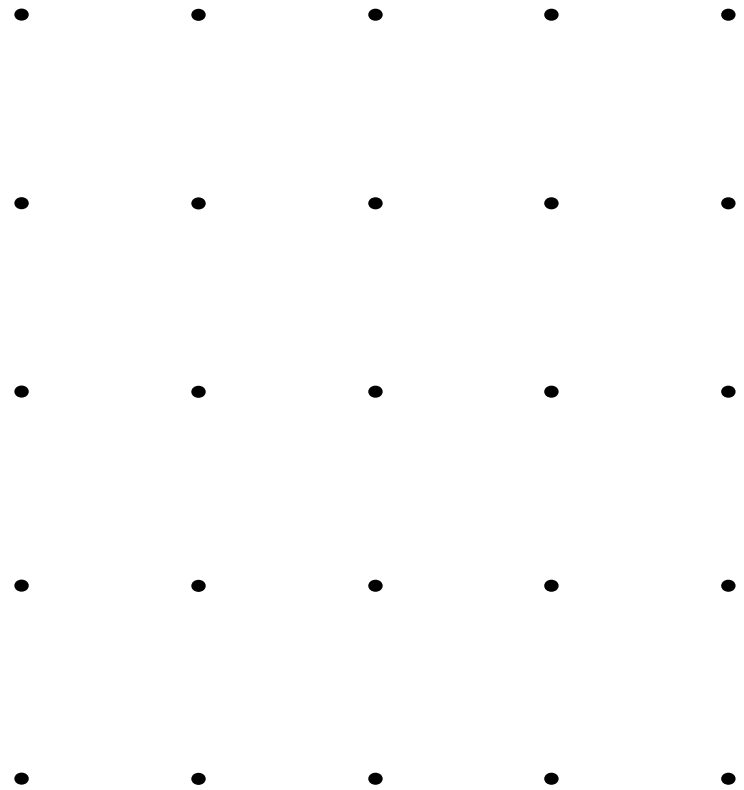
14. Which figure(s) above are semiregular tessellations? _____

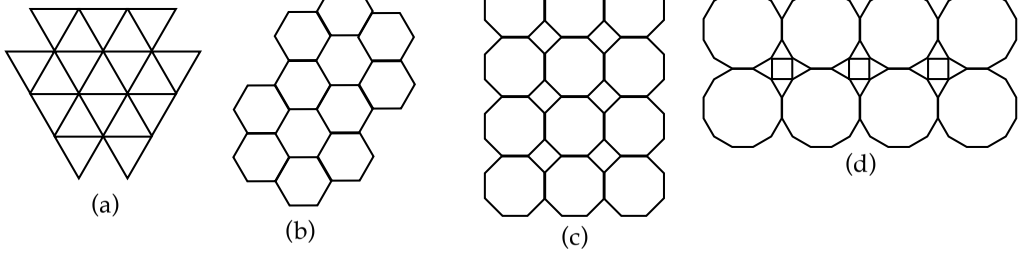
15. Bonus: Create an Escher-type tessellation, starting with a rectangular tessellation. The points for the vertices are given below. Color and add details to make it creative. An example is shown here.



Use this quadrilateral as your template.







11. Which figure(s) above are demiregular tessellations? **d**

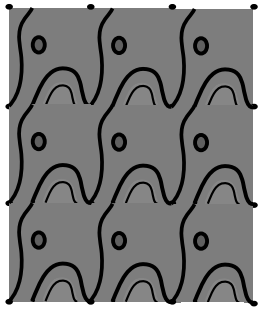
12. Which figure(s) above are regular tessellations? **a, b**

13. Which figure(s) above are pure tessellations? **a, b**

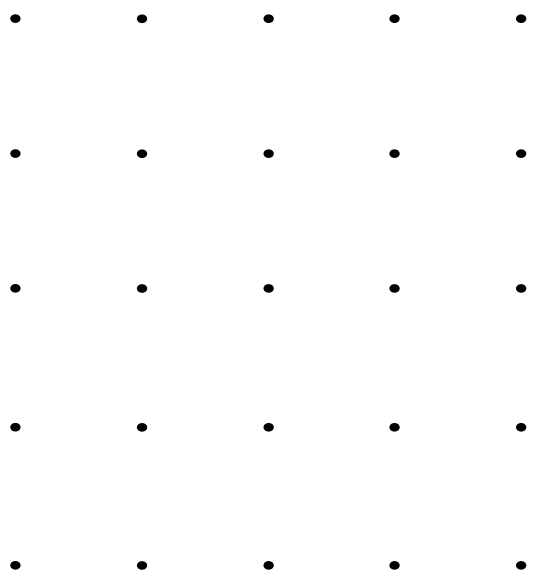
14. Which figure(s) above are semiregular tessellations? **c**

15. Bonus: Create an Escher-type tessellation, starting with a rectangular tessellation. The points for the vertices are given below. Color and add details to make it creative. An example is shown here.

[DESIGNS WILL VARY.]



Use this quadrilateral as your template.



NOTES: The bonus question at the end is an extra challenge for the student. You may decide how to use it.

DICTIONARY TERMS: none

Lesson 46: Fibonacci Numbers and Phi

OBJECTIVES:

1. To discover the relationship between Fibonacci numbers and ϕ
2. To discover relationships within the Fibonacci numbers

MATERIALS:

1. Worksheet 46, Fibonacci Numbers and Phi
2. Casio Calculator fx-300MS
3. **Basic calculator, optional**
4. *Math Card Games* book, F22.1

ACTIVITIES:

Fibonacci numbers and phi. In the last lesson, you compared the Fibonacci spiral and the golden spiral. You noticed that the greatest differences occurred at the smaller squares. The larger squares were similar to each other.

In 1753, the Scottish mathematician, Robert Simson, discovered a relationship between consecutive Fibonacci numbers and the golden ratio. Today's worksheet will explore that relationship.

Worksheet 46. The first problem has you write the first twelve Fibonacci numbers. Then you will compare the ratio of consecutive Fibonacci numbers using the format of the golden ratio, the ratio of the longer to the shorter, or in this situation, the larger number to the smaller number. To get the ratios in decimal form, divide and round to five places. The first few are done for you.

Next, compare how much the ratios differ from ϕ, the golden ratio. A quick way to find these differences is to store 1.61803 in memory on your calculator to use it repeatedly. Summarize your findings in Question 3. How do these numbers compare to your spiral drawings? Do these problems now, then continue reading below.

Problem 4. In Problem 4, n refers to the **position** in the Fibonacci sequence and f_n read as "f sub n" refers to the value, or actual number, of the nth term in the Fibonacci sequence. For example, f_2 refers to second Fibonacci number, which is 1.

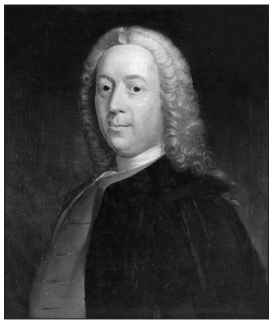
n	f_n
1	1
2	1
3	2
4	3
5	5
6	8
7	13

Look at the chart on the right. When n is 3, the third position in the sequence, f_n refers to the third Fibonacci number, which is 2. When n is 4, the fourth position in the sequence, f_n refers to the fourth Fibonacci number, which is 3.

It is helpful to think about it like this:

If $n = 2$, what is f_n? That is, what is f_2? $f_2 = 1$
If $n = 4$, what is f_n? That is, what is f_4? $f_4 = 3$

EXTRAS:



Robert Simson, professor of mathematics at the University of Glasgow, Scotland.

Lesson 41 discussed the meaning of phi, ϕ. Review now if needed.

For the third column of Problem 2, you will need to use a **basic calculator** (like the Casio 450) or do the subtraction by hand. If you use your scientific calculator, the answers will be in scientific notation.

Lesson 3 reviews using memory with a calculator.

In f_n, the f stands for Fibonacci.

LESSON CONTINUES ON THE NEXT PAGE.

ACTIVITIES:

In the 1800s, Edouard Lucas, a French mathematician, made an interesting observation regarding adding consecutive Fibonacci numbers.

The third column of the chart for Problem 4 asks for the sum of the first n terms. When $n = 2$, the first two terms, f_1 and f_2, are 1 and 1. The sum of these terms is 2. When $n = 3$, the sum of the first three terms, $f_1 + f_2 + f_3$, is 1 + 1 + 2, which is 4.

n	f_n	Sum of the first n terms
1	1	1
2	1	2
3	2	

When $n = 4$, what do you think the sum of the first 4 terms is? You will answer this on the worksheet.

You will also work with f_{n+1}. When $n = 3$, $f_3 = 2$. So, what is f_{n+1}? Substituting 3 for n, $f_{3+1} = f_4$ and $f_4 = 3$. This gives the value of the fourth term in the Fibonacci sequence!

Let's say n is 4. What is f_{n+1}? It is f_{4+1}, or f_5, the fifth term in the Fibonacci sequence, which is 5.

Let's say n is 5. What is f_{n+1}? The answer is at the bottom of the page.

Notice that f_{n+1} is not the same as $f_n + 1$. The first expression, f_{n+1}, advances the n position one place to the next term in the Fibonacci sequence. The second expression, $f_n + 1$, **adds 1** to the **actual** Fibonacci number. Compare f_{n+1} and $f_n + 1$ when n is 4.

Complete Problem 4 now. Then continue reading.

Problem 5. When finding the correct equation for Problem 5, use the chart you made in Problem 4. Pick a number for n and try it in the three equations to see which one is correct. It may help to write down your calculations. When you think you have a correct answer, try another n to check your work.

Complete the worksheet.

Today's game. Play the Corners™ with Eighths game, found in the *Math Card Games* book, F22.1.

EXTRAS:

Edouard Lucas. He was a professor of mathematics at the Lycée Saint Louis and the Lycée Charlemagne in Paris. He was also interested in recreational mathematics and invented puzzles and games.

Answer: 8

Name: _____

Date: _____

1. Write the Fibonacci sequence.

1 1 ____ ____ ____ ____ ____ ____ ____ ____ ____ ____

2. Using the format of the golden ratio, find the ratios of consecutive Fibonacci numbers to 5 decimal places. Then find how much they differ from phi using a **basic calculator** or by hand.

Ratio of Consecutive Fibonacci Numbers		
Fraction form	**In decimal form (5 places)**	**Difference from ϕ, 1.61803**
$\frac{1}{1}$	1.00000	0.61803
$\frac{2}{1}$	2.00000	
$\frac{3}{2}$	1.50000	
$\frac{5}{3}$		

3. In the chart you just completed, explain what the third column shows.

_____

_____

_____

CONTINUE READING THE LESSON.

4. Fill in the chart below.

n	f_n	**Sum of the first n terms**	**Sum of the first n terms + 1**
1	1	1	2
2	1	2	3
3	2	4	
4	3		
5	5		
6	8		
7	13		

CONTINUE READING THE LESSON.

5. Circle the correct equation.

Sum of the first n terms + 1 = f_n

Sum of the first n terms + 1 = f_{n+1}

Sum of the first n terms + 1 = f_{n+2}

1. Write the Fibonacci sequence.

1 1 2 3 5 8 13 21 34 55 89 144

2. Using the format of the golden ratio, find the ratios of consecutive Fibonacci numbers to 5 decimal places. Then find how much they differ from phi using a **basic calculator** or by hand.

Ratio of Consecutive Fibonacci Numbers		
Fraction form	In decimal form (5 places)	Difference from ϕ, 1.61803
$\frac{1}{1}$	1.00000	0.61803
$\frac{2}{1}$	2.00000	–0.38197
$\frac{3}{2}$	1.50000	0.11803
$\frac{5}{3}$	1.66667	–0.04864
$\frac{8}{5}$	1.60000	0.01803
$\frac{13}{8}$	1.62500	–0.00697
$\frac{21}{13}$	1.61538	0.00265
$\frac{34}{21}$	1.61905	–0.00102
$\frac{55}{34}$	1.61765	0.00038
$\frac{89}{55}$	1.61818	–0.00015
$\frac{144}{89}$	1.61798	0.00005

3. In the chart you just completed, explain what the third column shows.

The higher the Fibonacci numbers the closer the ratios are to ϕ.

CONTINUE READING THE LESSON.

4. Fill in the chart below.

n	f_n	Sum of the first n terms	Sum of the first n terms + 1
1	1	1	2
2	1	2	3
3	2	4	5
4	3	7	8
5	5	12	13
6	8	20	21
7	13	33	34

CONTINUE READING THE LESSON.

5. Circle the correct equation.

Sum of the first n terms + 1 = f_n

Sum of the first n terms + 1 = f_{n+1}

(Sum of the first n terms + 1 = f_{n+2})

NOTES: The ratios of consecutive Fibonacci numbers in Problem 2 use the format of the golden ratio, that is the longer number to the shorter number. This is explained in the lesson; guide the student to the first paragraph under the Worksheet 46 heading if needed. If the student is unsure what consecutive means, define it as the next number in the sequence.

Finding the fraction form of the ratio in the table follows the pattern using the consecutive Fibonacci numbers. The decimal form is found by dividing the two numbers.

The chart in Problem 4 uses n and the f_n. This is explained in the lesson with a variety of scenarios.

DICTIONARY TERMS: none

Lesson 60: Basic Trigonometry

OBJECTIVES:

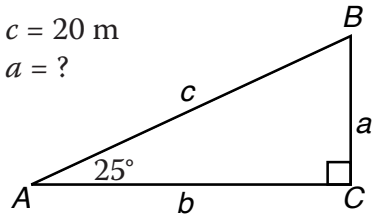
1. To learn the meaning of *trigonometry*
2. To learn the terms *opposite, adjacent, sine* (*sin*), *cosine* (*cos*), and *tangent* (*tan*)
3. To calculate sin, cos and tan of angles

MATERIALS:

1. Math Dictionary
2. Worksheets 60-1 and 60-2, Basic Trigonometry
3. Casio Calculator fx-300MS
4. Goniometer
5. *Math Card Games* book, P37

ACTIVITIES:

A problem. In the problem below, you are given c and $\angle A$ and asked to find a. If you knew the ratio, $\frac{a}{c}$, you could solve the problem as you did in the previous lesson.

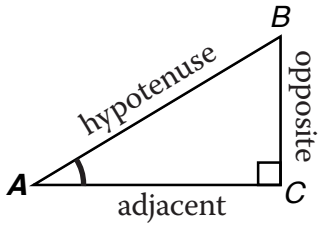


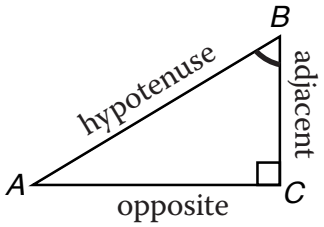
Trigonometry. Solving problems using right triangle **ratios** involves a branch of mathematics called *trigonometry* (TRIG-ah-NOM-ah-tree). The name is often shortened to trig. The derivation of the word is quite simple. *Tri* means three, *gon* means angle, and *metry* means measure.

The Babylonians, Greeks, Egyptians, Indians, Chinese, and Arabs all contributed to the field of trigonometry. Surveyors and astronomers today use it extensively for finding distances.

Trig terms. Basic trig uses six terms; one you know, two are familiar, and three will be new. Look at the first triangle below. The sides are labeled based on their **relationship to $\angle A$**. The leg *opposite* $\angle A$ is $\overline{BC}$. The leg *adjacent to*, or next to, $\angle A$ is $\overline{AC}$. The hypotenuse is opposite the right angle.

When recording these definitions, a diagram may be more helpful.





Look at the second triangle above. The sides are now labeled based on their **relationship to $\angle B$**. When looking at $\angle B$, $\overline{BC}$ is adjacent to $\angle B$, and $\overline{AC}$ is opposite $\angle B$. The hypotenuse is always across from the right angle!

EXTRAS:

LESSON CONTINUES ON THE NEXT PAGE.

ACTIVITIES:

These next three terms are new. The trig names for the most common ratios are given in the first rectangle below. They are usually abbreviated as shown in the second rectangle. However, the abbreviations are pronounced like the original words, *sin* (SIGN), *cos* (KOH-sign), and *tan* (TAN-jent).

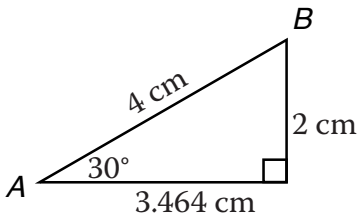
$$\text{sine } (A) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{cosine } (A) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \text{tangent } (A) = \frac{\text{opposite}}{\text{adjacent}}$$

or

$$\sin (A) = \frac{\text{opp}}{\text{hyp}} \qquad \cos (A) = \frac{\text{adj}}{\text{hyp}} \qquad \tan (A) = \frac{\text{opp}}{\text{adj}}$$

The important thing to remember is that the trig functions, sin (*A*), cos (*A*), and tan (*A*), are ratios based on the angle being considered, in this case, $\angle A$.

Let's say the measurement of $\angle A$ is 30°. See the triangle below. The ratios are shown on the right.

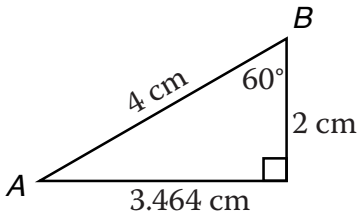


$$\sin (30) = \frac{\text{opp}}{\text{hyp}} = \frac{2}{4} = 0.500$$

$$\cos (30) = \frac{\text{adj}}{\text{hyp}} = \frac{3.464}{4} = 0.866$$

$$\tan (30) = \frac{\text{opp}}{\text{adj}} = \frac{2}{3.464} = 0.577$$

If your reference angle is $\angle B$, then the ratios will change as shown.



$$\sin (60) = \frac{\text{opp}}{\text{hyp}} = \frac{3.464}{4} = 0.866$$

$$\cos (60) = \frac{\text{adj}}{\text{hyp}} = \frac{2}{4} = 0.500$$

$$\tan (60) = \frac{\text{opp}}{\text{adj}} = \frac{3.464}{2} = 1.732$$

Worksheets 60-1 and 60-2. Today, you will be making a trig table for angles from 5° to 85°. Use the Pythagorean theorem and calculate the ratios to 3 decimal places. On Worksheet 60-2, measure the angles with a goniometer. Calculate the ratios and enter them in the table on Worksheet 60-1.

Today's game. Play the Distribution Corners™ game, found in the *Math Card Games* book, P37.

EXTRAS:

Trigonometry and its ratios only work with **right** triangles.

You might like this for remembering the trig ratios: soh-cah-toa.

SOH stands for sin, opp, hyp,
CAH stands for cos, adj, hyp,
TOA stands for tan, opp, adj.

Although we show the sin (*A*), cos (*A*), and tan (*A*) in regards to $\angle A$, it could be with $\angle A$ or $\angle B$ or whatever angle you are looking at. For example, sin (*B*). The (*A*) is a variable and can be replaced with any angle being considered.

One student, Connie, remembers the order of the opposite, adjacent, and hypotenuse in these trig ratios like this:

opp is always on top,
hyp is always on the bottom,
and *adj* jumps around.

Remember what you know about equilateral triangles to find the length of a hypotenuse.

You will use Worksheet 60-1 in the next few lessons.

Name: _____

Worksheet 60-1,
Basic Trigonometry

Date: _____

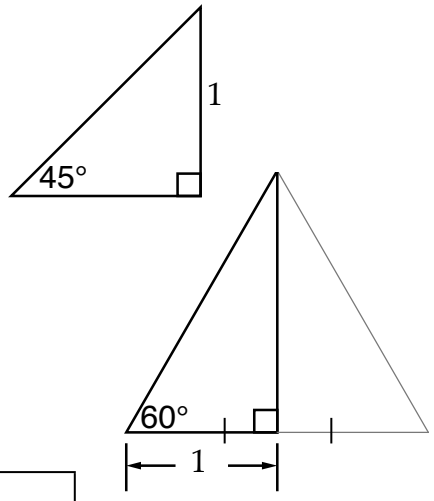
1. Calculate and label the sides of the two triangles on the right. Use the Pythagorean theorem. Keep your answers in square root form.

2. Use the 45 triangle on the right to find sin (45), cos (45), and tan (45). Record your answers to 3 decimal places in the chart.

3. Use the 30-60 triangle to find sin (60), cos (60), tan (60). Also find sin (30), cos (30), and tan (30). Record your answers in the chart.

4. Complete Worksheet 60-2.

5. Calculate and record the remaining values in the chart to 3 decimal places.



Angle	**sin = $\frac{\text{opp}}{\text{hyp}}$**	**cos = $\frac{\text{adj}}{\text{hyp}}$**	**tan = $\frac{\text{opp}}{\text{adj}}$**
5			
10			
15			
20			
25			
30			
35			
40			
45			
50			
55			
60			
65			
70			
75			
80			
85			

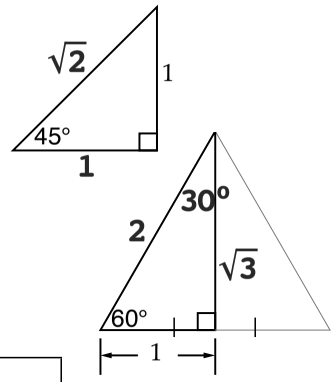
1. Calculate and label the sides of the two triangles on the right. Use the Pythagorean theorem. Keep your answers in square root form.

2. Use the 45 triangle on the right to find sin (45), cos (45), and tan (45). Record your answers to 3 decimal places in the chart.

3. Use the 30-60 triangle to find sin (60), cos (60), tan (60). Also find sin (30), cos (30), and tan (30). Record your answers in the chart.

4. Complete Worksheet 60-2.

5. Calculate and record the remaining values in the chart to 3 decimal places.



Angle	$\sin = \frac{opp}{hyp}$	$\cos = \frac{adj}{hyp}$	$\tan = \frac{opp}{adj}$
5	$\frac{0.87}{10.00} = 0.087$	$\frac{9.96}{10.00} = 0.996$	$\frac{0.87}{9.96} = 0.087$
10	$\frac{1.74}{10.00} = 0.174$	$\frac{9.85}{10.00} = 0.985$	$\frac{1.74}{9.85} = 0.177$
15	$\frac{2.59}{10.00} = 0.259$	$\frac{9.66}{10.00} = 0.966$	$\frac{2.59}{9.66} = 0.268$
20	$\frac{3.42}{10.00} = 0.342$	$\frac{9.40}{10.00} = 0.940$	$\frac{3.42}{9.40} = 0.364$
25	$\frac{4.23}{10.00} = 0.423$	$\frac{9.06}{10.00} = 0.906$	$\frac{4.23}{9.06} = 0.467$
30	$\frac{1.00}{2.00} = 0.500$	$\frac{1.732}{2.00} = 0.866$	$\frac{1.00}{1.732} = 0.577$
35	$\frac{5.74}{10.00} = 0.574$	$\frac{8.19}{10.00} = 0.819$	$\frac{5.74}{8.19} = 0.701$
40	$\frac{6.43}{10.00} = 0.643$	$\frac{7.66}{10.00} = 0.766$	$\frac{6.43}{7.66} = 0.839$
45	$\frac{1.00}{1.414} = 0.707$	$\frac{1.00}{1.414} = 0.707$	$\frac{1.00}{1.00} = 1.000$
50	$\frac{7.66}{10.00} = 0.766$	$\frac{6.43}{10.00} = 0.643$	$\frac{7.66}{6.43} = 1.191$
55	$\frac{8.19}{10.00} = 0.819$	$\frac{5.74}{10.00} = 0.574$	$\frac{8.19}{5.74} = 1.427$
60	$\frac{1.732}{2.00} = 0.866$	$\frac{1.00}{2.00} = 0.500$	$\frac{1.732}{1.00} = 1.732$
65	$\frac{9.06}{10.00} = 0.906$	$\frac{4.23}{10.00} = 0.423$	$\frac{9.06}{4.23} = 2.142$
70	$\frac{9.40}{10.00} = 0.940$	$\frac{3.42}{10.00} = 0.342$	$\frac{9.40}{3.42} = 2.749$
75	$\frac{9.66}{10.00} = 0.966$	$\frac{2.59}{10.00} = 0.259$	$\frac{9.66}{2.59} = 3.730$
80	$\frac{9.85}{10.00} = 0.985$	$\frac{1.74}{10.00} = 0.174$	$\frac{9.85}{1.74} = 5.661$
85	$\frac{9.96}{10.00} = 0.996$	$\frac{0.87}{10.00} = 0.087$	$\frac{9.96}{0.87} = 11.448$

$$c^2 = a^2 + b^2$$
$$c^2 = 1^2 + 1^2 = 2$$
$$c = \sqrt{2} \approx 1.414$$

$$c^2 = a^2 + b^2$$
$$2^2 = 1^2 + b^2$$
$$b^2 = 4 - 1 = 3$$
$$b = \sqrt{3} \approx 1.732$$

NOTES: When finding the measurement of the height of the equilateral triangle, half the base is 1, therefore the full base is 2. Since it is an equilateral triangle, all the other sides are also 2.

If the student uses $\sqrt{2}$ and $\sqrt{3}$ in their calculations, rather than 1.414 and 1.732, it is more accurate, although the resulting answers will not vary.

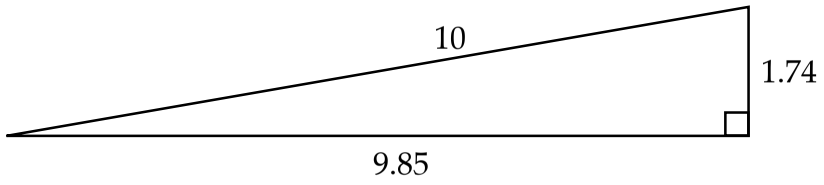
Make sure the student watches for the opposite and the adjacent sides, given the angle being considered.

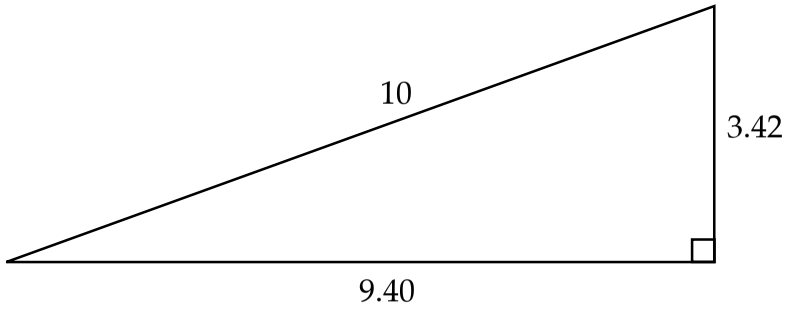
Colin Egger, an 11 year old student, cleverly approached finding tangents like this:

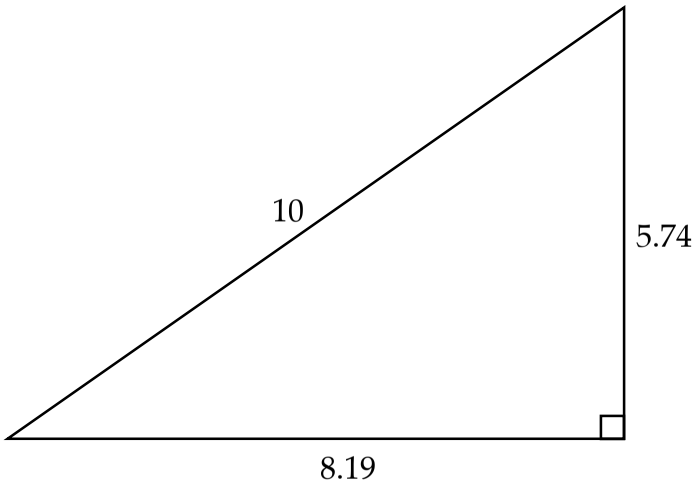
$$\tan = \frac{\sin}{\cos} = \frac{opp \div hyp}{adj \div hyp} = \frac{opp}{adj}$$

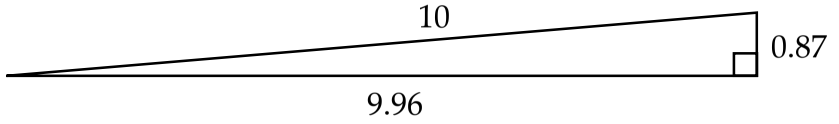
Interesting thought process, Colin!

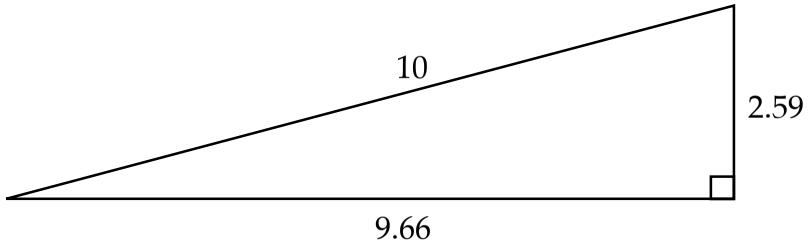
Use these triangles to find the sin, cos, and tan of the angles in the table on Worksheet 60-1. Measure the angles with the goniometer and label.

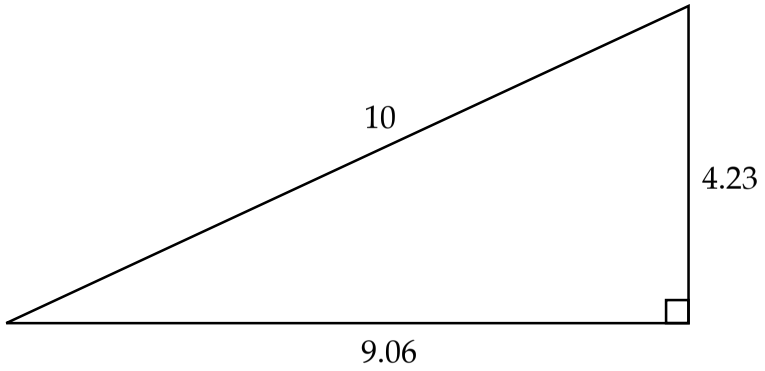


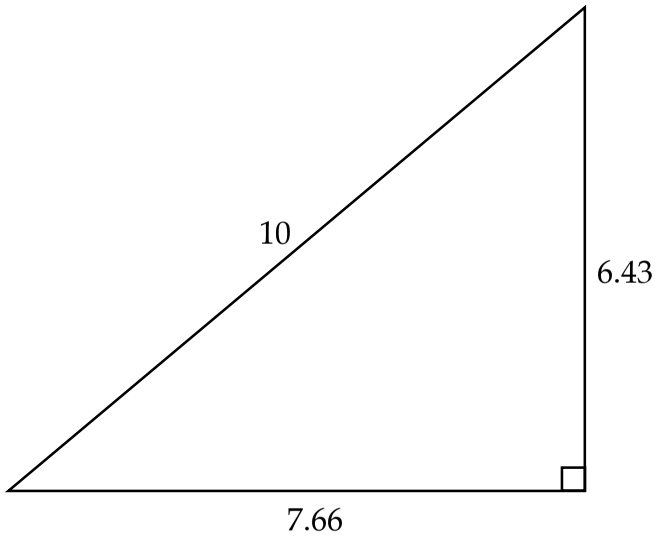










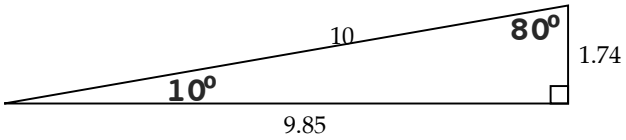


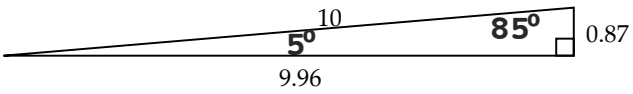
Name: _____

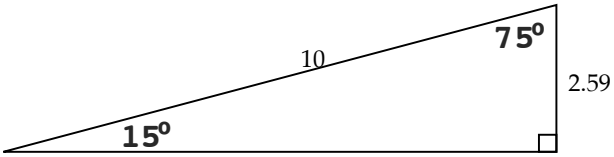
Date: _____

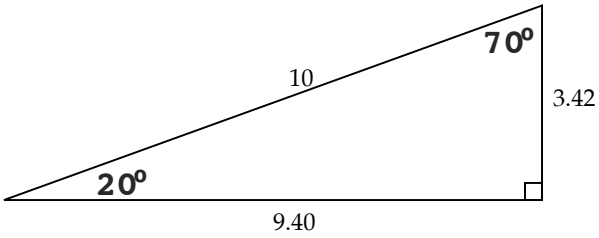
Worksheet 60-2, Basic Trigonometry

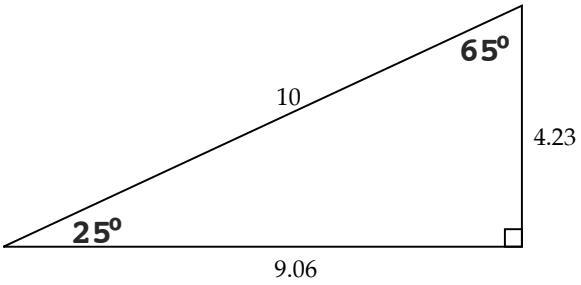
Use these triangles to find the sin, cos, and tan of the angles in the table on Worksheet 60-1. Measure the angles with the goniometer and label.

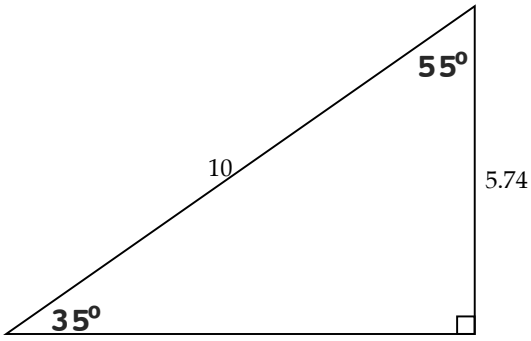


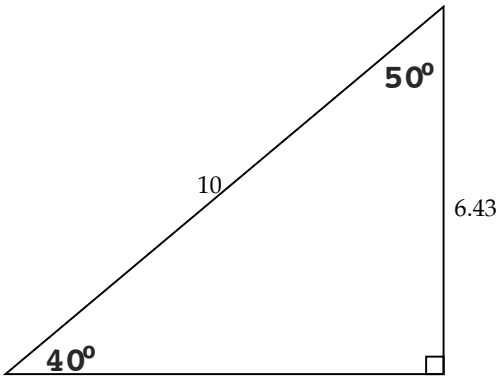












NOTES: Some students may measure all the angles, whereas others will quickly see that the two angles will total 90°.

DICTIONARY TERMS: trigonometry, opposite, adjacent, sine (sin), cosine (cos), tangent (tan)

Lesson 73: Volume of Prisms

OBJECTIVES:

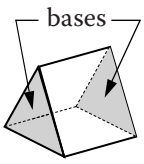
1. To learn the term *prism* and the names of several prisms: *rectangular prism*, *right prism*, and *oblique prism*
2. To calculate volumes and surface areas of prisms

MATERIALS:

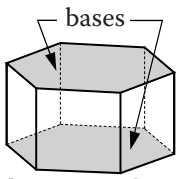
1. Math Dictionary
2. Worksheet 73, Volume of Prisms
3. Geometry panels
4. Casio Calculator fx-300MS
5. *Math Card Games* book, S24

ACTIVITIES:

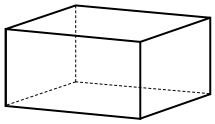
Prisms. A *prism* is a polyhedron with two congruent parallel polygons called bases. The connecting faces are always parallelograms. The shape of the base determines the name of the prism. See the figures below with the names of several prisms.



triangular prism



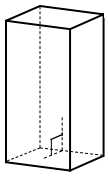
hexagonal prism



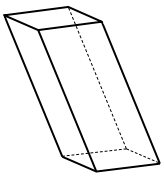
rectangular prism

Notice that the boxes from the previous lesson will now be called by their geometrical name, *rectangular prisms*.

If the bases of a prism are at right angles to the other faces, the polyhedron is a *right prism*. Otherwise, it is an *oblique prism*. See the prisms on the right.



right prism



oblique prism

Is a cube a prism? Is a cube a right prism? Is a cube a rectangular prism? The answers are at the bottom of the page.

Worksheet 73, problem 1. After you construct the four prisms shown on the worksheet, fill in the table. Since you know the volume of a geometry panel cube, the volumes of the first two prisms should be easy. Compare prism R with prisms P and Q to make an estimate. Likewise, compare prism S to prism R. Most of the calculations for this problem you can do mentally.

Problem 2. This is the important result of this lesson. If you have a mental image of a prism and understand what volume is, the general volume formula will make sense to you.

Problems 3 and 4. These problems will apply what you have learned.

Today's game. Play the Subtraction Bingo game, found in the *Math Card Games* book, S24.

EXTRAS:

Remember to refer to your Math Dictionary for terms you are unsure of their meanings.

For the rectangular prism, there are three sets of rectangles that can be the bases.

When you complete your worksheet, remember to grade your work on Persistence, Understanding, and Results.

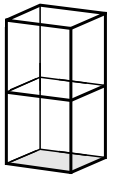
Answers: yes, yes, yes

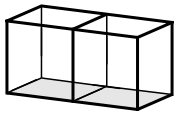
Name: _____

Date: _____

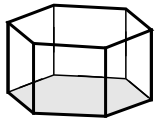
Worksheet 73, Volume of Prisms

1. Construct the prisms shown with the geometry panels. Then fill in the table for each prism. Use centimeters. Show your work for finding the base areas for R and S below on the right.

P. 

Q. 

R. 

S. 

Prism Name	Estimated Volume	Area of Base, *B*	Height, *H*	Volume, *V*	Surface Area, *S*
P.					
Q.					
R.					
S.					

2. What is the formula for the volume, *V*,

   of a prism using *B* and *H*? _____

3. If prisms Q and S were models of buildings,

   which prism has more floor space? _____

   Which prism has more roof space? _____

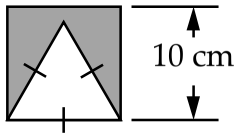
   Which needs more paint for the outside walls? _____

   What is the angle of a corner wall in prism Q? _____

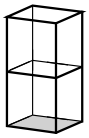
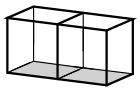
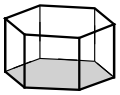
   What is the angle of a corner wall in prism S? _____

4. What is the area of the shaded section

   of the square? _____



1. Construct the prisms shown with the geometry panels. Then fill in the table for each prism. Use centimeters. Show your work for finding the base areas for R and S below on the right.

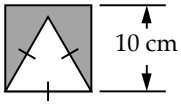
P. 
Q. 
R. 
S. 

Prism Name	Estimated Volume	Area of Base, *B*	Height, *H*	Volume, *V*	Surface Area, *S*
P. rectangular		10 × 10 = 100 cm²	20 cm	2000 cm³	1000 cm²
Q. rectangular		20 × 10 = 200 cm²	10 cm	2000 cm³	1000 cm²
R. triangular		43.3 cm²	10 cm	433 cm³	386.6 cm²
S. hexagonal		259.8 cm²	10 cm	2598 cm³	1119.6 cm²

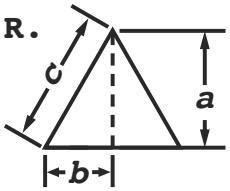
2. What is the formula for the volume, *V*, of a prism using *B* and *H*? ___*V = BH*___

3. If prisms Q and S were models of buildings,
which prism has more floor space? __S__
Which prism has more roof space? __S__
Which needs more paint for the outside walls? __same__
What is the angle of a corner wall in prism Q? __90°__
What is the angle of a corner wall in prism S? __120°__

4. What is the area of the shaded section of the square? __56.7 cm²__



100 − 43.3 = 56.7 cm²

R. 

$a^2 + b^2 = c^2$
$a^2 = c^2 - b^2$
$a^2 = 10^2 - 5^2$
$a^2 = 100 - 25 = 75$
$a = \sqrt{75}$
$a \approx 8.66$ cm

$B\triangle = \frac{wh}{2} = \frac{10 \times 8.66}{2}$
$B\triangle = 43.3$ cm²

S. $B\hexagon = 6 \times B\triangle$
$B\hexagon = 259.8$ cm²

NOTES: The first prism, P, can also be identified as a square prism.

Area of the bases for prisms P and Q is width × height. The triangular prism base calculations, on the bottom right of the solutions above, 43.3 cm², needs the height of the triangle, 8.66 cm. The area of the hexagonal prism base is six times the area of the triangular base and is also on the bottom right of the solutions above.

Using trig with the equilateral triangle is another approach to finding the height of the triangle. Calculations will be as follows: $\sin(60) = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{10}$, so opposite, or height, is 8.66 cm.

The volume, *V*, is found by multiplying the third column, Area of Base, *B*, by the fourth column, Height, *H*.

The surface area, *S*, calculations, where *w* is width, *h* is height and *d* is depth, are as follows:
P. rectangular prism: $S = 2wh + 2dh + 2wd = 2 \times 10 \times 10 + 4 \times 10 \times 20 = 200 + 800 = 1000$ cm²
Q. rectangular prism: $S = 2wh + 2dh + 2wd = 4 \times 20 \times 10 + 2 \times 10 \times 10 = 800 + 200 = 1000$ cm²
R. triangular prism: S = 2 triangles + 3 sides $= 2 \times \frac{1}{2}wh + 3 \times wh = 2 \times 43.3 + 3 \times 10 \times 10 = 86.6 + 300 = 386.6$ cm²
S. hexagonal prism: S = 2 hexagons + 6 sides $= 2 \times 6 \times \frac{1}{2}wh + 6 \times wh$
$= 2 \times 6 \times 43.3 + 6 \times 10 \times 10 = 519.6 + 600 = 1119.6$ cm²

Question 3 asks about the amount of paint for the walls. Both Q and S use six 10 × 10 panels.

DICTIONARY TERMS: prism, rectangular prism, right prism, oblique prism

Lesson 107: Negative Numbers

OBJECTIVES:

1. To review adding and subtracting negative numbers
2. To practice adding negative numbers

MATERIALS:

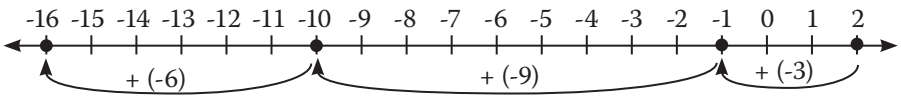
1. Worksheet 107, Negative Numbers
2. *Math Card Games* book, S10

ACTIVITIES:

Negative numbers. In the last lesson, you learned about natural numbers, 1, 2, 3, ..., whole numbers, 0, 1, 2, 3, ..., and about integers. Integers include whole numbers as well as their negative numbers. Let's have a quick review of negative numbers.

Adding and subtracting negative numbers. It is winter in a cold place and the temperature outside is 2°. If it gets 3° colder, what would the temperature be? -1°. It gets colder and the temperature goes down 9 more degrees. Now what is the temperature? -10°. It gets even colder! It drops another 6 degrees; now what is the temperature? -16°.

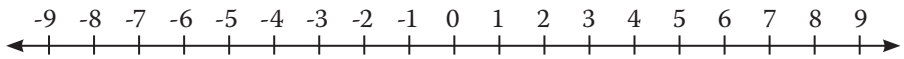
It may help you to find the answers using a number line.



Now, consider the above temperatures using equations:

2 + (-3) = -1	is the same as	2 – 3 = -1
-1 + (-9) = -10	is the same as	-1 – 9 = -10
-10 + (-6) = -16	is the same as	-10 – 6 = -16

Think of it as walking along the number line. Addition and subtraction will tell you which way to face, left or right. The sign of the number, positive or negative, will tell you if your steps will be forward or backward, regardless of which way you are facing.



So look at 0 + 8. Starting at zero, face right (because you are adding) and move forward 8 steps (because it's a positive 8). You have added 8 to 0, landing you on 8. Use your finger to make these moves on the number line above.

Now consider 8 – 5. Turn around and facing the left (because you are subtracting), and walk 5 steps forward (because it's a positive 5).

Next consider 8 + (-5). Starting back on 8, face right (because you are adding) but walk backward 5 steps (because the sign is negative). You're back on 3 again.

EXTRAS:

The word "degree" is indicated frequently with the ° symbol and has multiple meanings. It can refer to angle measurements as well as temperature measurements. It can also mean levels, like college degrees, or the intensity of a burn.

While the term *negative 6* is commonly used in mathematics classes, *minus 6* is often used in discussing the weather.

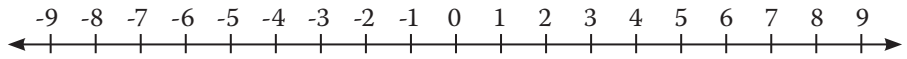
The parentheses are not necessary around negative numbers, however they make it easier to read.

Rather than memorizing the rules of adding and subtracting, think about what is logical. Look at the answer. Ask yourself if it makes sense.

LESSON CONTINUES ON THE NEXT PAGE.

ACTIVITIES:

Let's do another example. Consider 0 – 6. Start at zero, face left (because you are subtracting) and move forward 6 steps (because it's a positive 6). You have subtracted 6 from 0, landing you on -6.



What about -6 – (-2)? Start at -6, face left (because you are subtracting) and walk backward 2 steps (because the sign is negative). Did you land on -4? Is this the same as -6 + 2? It is!

Try this one on your own: -3 – (-4). Did you face left and walk backward? Or did you face right and walk forward, turning the expression into addition, -3 + 4 = 1? Which is easier for you?

Summary. Notice how 7 – 5 = 2 is the same as 7 + (-5) = 2. Do you see how subtraction is really the same as adding the opposite of the number? Opposites are what you can add together to get zero. The opposite of 3 is -3, the opposite of -4 is 4, and the opposite of x is $-x$.

So what happens when we subtract a negative number? It is the same as adding the opposite of the negative number. Therefore,

$$3 - (-4) = 7 \quad \text{is the same as} \quad 3 + 4 = 7$$

Worksheet 107. On this worksheet you will practice adding and subtracting negative numbers while plotting a dot-to-dot drawing on a graph.

Remember the first number of an ordered pair refers to the location along the x-axis; the second number refers to the location along the y-axis.

Today's game. Play the Negative Corners game, found in the *Math Card Games* book, S10.

EXTRAS:

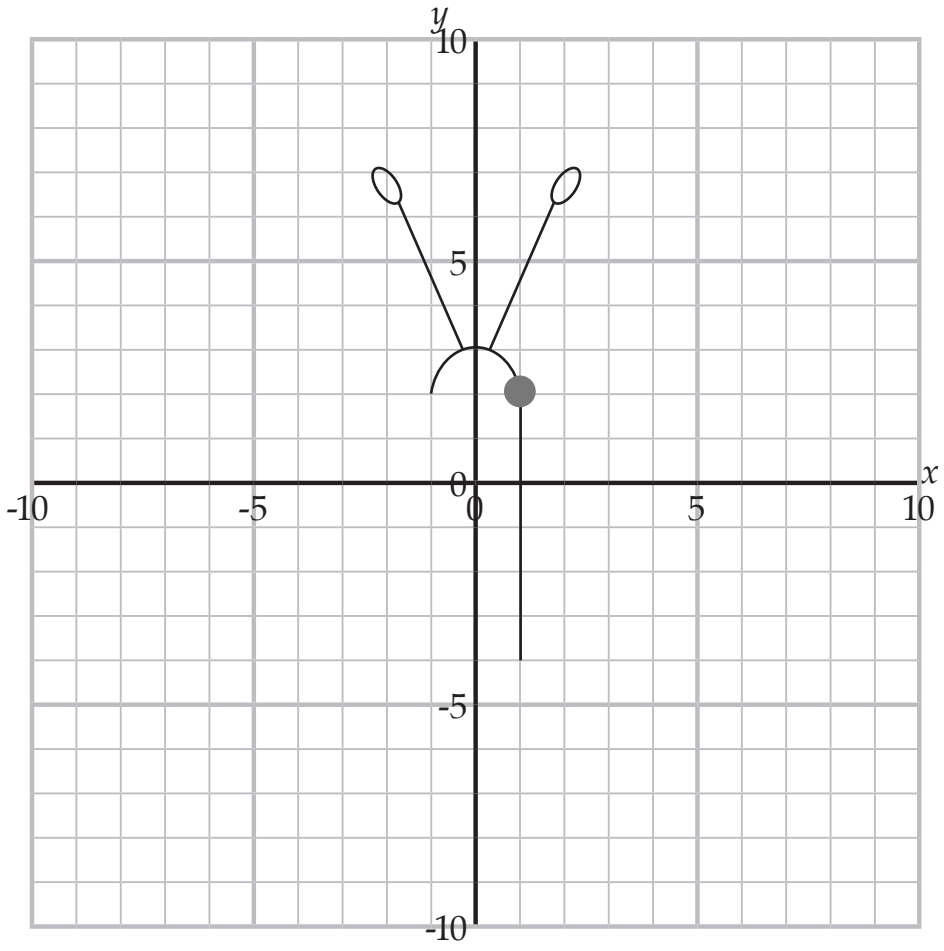
Lesson 18 reviews coordinate systems, absolute coordinate positions, and translation values. Go back to refresh if needed.

Name: _____

Date: _____

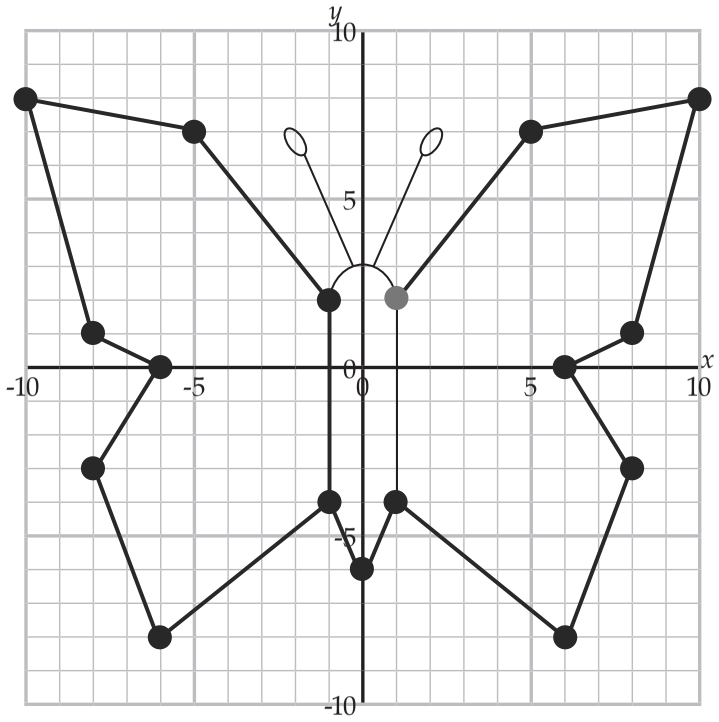
Calculate each equation, create an ordered pair, then plot on the graph below. The first one is done for you. Connect the dots as you go in order to discover the image.

	***x* value**	***y* value**	**Ordered Pair**		***x* value**	***y* value**	**Ordered Pair**
1	4 + (-3)	4 + (-2)	**(1, 2)**	10	8 – 9	72 – 76	
2	8 – 3	-2 + 9		11	19 + (-25)	-4 – 4	
3	14 + (-4)	17 – 9		12	1 – 9	-17 + 14	
4	-4 + 12	-6 – (-7)		13	-21 + 15	19 + (-19)	
5	-3 – (-9)	-11 + 11		14	-2 + (-6)	-1 – (-2)	
6	-26 + 34	17 – 20		15	12 – 22	1 – (-7)	
7	-10 – (-16)	-20 – (-12)		16	-19 + 14	20 + (-13)	
8	-13 + 14	-9 – (-5)		17	-11 + 10	-64 + 66	
9	5 + (-5)	-16 + 10		18	85 + (-86)	-17 – (-13)	



Calculate each equation, create an ordered pair, then plot on the graph below. The first one is done for you. Connect the dots as you go in order to discover the image.

	x value	*y* value	Ordered Pair		*x* value	*y* value	Ordered Pair
1	4 + (-3)	4 + (-2)	(1, 2)	10	8 – 9	72 – 76	**(–1, –4)**
2	8 – 3	-2 + 9	**(5, 7)**	11	19 + (-25)	-4 – 4	**(–6, –8)**
3	14 + (-4)	17 – 9	**(10, 8)**	12	1 – 9	-17 + 14	**(–8, –3)**
4	-4 + 12	-6 – (-7)	**(8, 1)**	13	-21 + 15	19 + (-19)	**(–6, 0)**
5	-3 – (-9)	-11 + 11	**(6, 0)**	14	-2 + (-6)	-1 – (-2)	**(–8, 1)**
6	-26 + 34	17 – 20	**(8, –3)**	15	12 – 22	1 – (-7)	**(–10, 8)**
7	-10 – (-16)	-20 – (-12)	**(6, –8)**	16	-19 + 14	20 + (-13)	**(–5, 7)**
8	-13 + 14	-9 – (-5)	**(1, –4)**	17	-11 + 10	-64 + 66	**(–1, 2)**
9	5 + (-5)	-16 + 10	**(0, –6)**	18	85 + (-86)	-17 – (-13)	**(–1, –4)**



NOTES: Rather than memorizing the rules of adding and subtracting, remind the student to think about what is logical. Ask them if their answer makes sense.

If needed, remind the student to look back to Lesson 18 to review coordinate systems, absolute coordinate positions, and translation values.

DICTIONARY TERMS: none

Lesson 118: Tables and Graphing

OBJECTIVES:

1. To graph a series of inequalities
2. To read information on the graph

MATERIALS:

1. Straightedge
2. Casio Calculator fx-300MS
3. Worksheet 118, Tables and Graphing

ACTIVITIES:

Water rates. Water systems are needed for many essentials such as drinking, cooking, bathing, cleaning, fire fighting, gardens, lawns, and manufacturing. Cities have many different ways of charging residents for the cost of supplying the water. Some cities charge a monthly rate while others charge a higher rate if you use more.

Units of water. On Worksheet 113, did you wonder why a unit of water is 748 gallons? The reason is that a unit of water is 100 cubic feet, abbreviated ccf. To help you visualize that amount of water, let's imagine a column of water. Look at a corner in a room that is 8 ft high. Now mentally construct a rectangular prism that extends from floor to ceiling with a square base that is a meter, or about three feet, on a side. That column of water is 1 unit, or 748 gallons.

Worksheet 118. This worksheet asks you to graph the equations you wrote on Worksheet 113. First, you will need to fill in the tables. Let's take a look at the first table where:

$$c = 1.42u$$

For the values of u, calculate the corresponding values of c and write them in the table.

For the second table, the equation is:

$$c = 5.68 + 2.87(u - 4)$$

To complete the table, substitute the values, 4, 6, and 8, for u. Complete the other tables in the same way.

Graphing the values. To graph the results, first find the coordinate for the u value. Then go up to where you think the c value is and plot a point. This requires approximating. When you have the points plotted for each table, draw a line connecting the points for that table. Notice the higher the rate, the steeper the line.

Today's game. Play Corny Corners, a variation of the Corners™ game, A9 in the *Math Card Games* book. The instructions for this game are in a previous lesson.

EXTRAS:

There are also other charges on a water bill, such as maintenance costs and sewer.

Sometimes hcf is used for hundred cubic feet instead of ccf.

CCF stands for "centum cubic feet," which means 100 cubic feet or 748 gallons of water.

Name: _____

Date: _____

1. Fill in the tables below to find the cost, c, of the monthly water usage. Round to the nearest penny. Then graph the values.

Number of units, u	**Price per unit of water**
0–4	$1.42
5–8	$2.87
9–18	$5.29
19–24	$6.68
≥ 25	$7.63

For $c \le 4$:
$c = 1.42u$

u	c
0	0
2	2.84
4	

For $4 \le c \le 8$:
$c = 5.68 + 2.87(u - 4)$

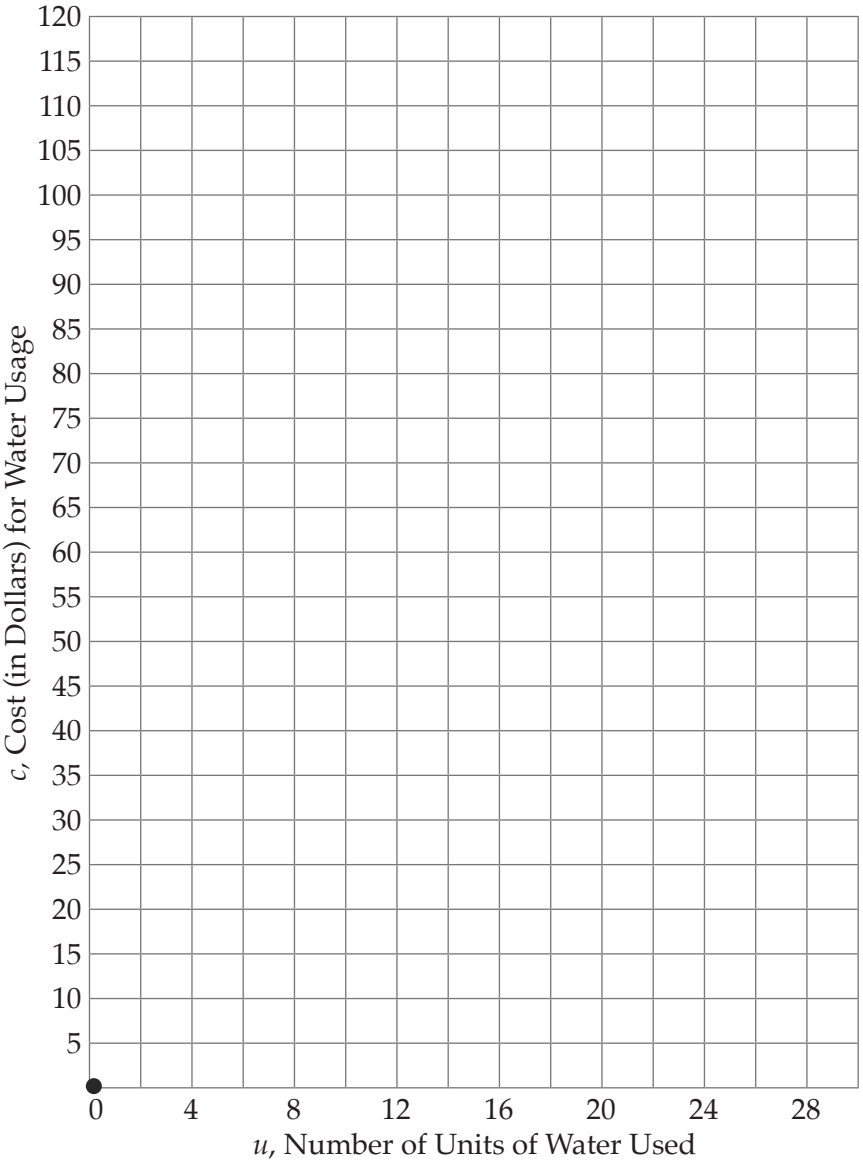
u	c
4	
6	
8	

For $8 \le c \le 18$:
$c = 17.16 + 5.29(u - 8)$

u	c
8	
12	
16	
18	

For $18 \le c \le 24$:
$c = 70.06 + 6.68(u - 18)$

u	c
18	
20	
24	



Use your graph for the following question.

2. The average family of four uses 12,000 gallons of water a month. What would they pay per month for their water? Per year?

1. Fill in the tables below to find the cost, c, of the monthly water usage. Round to the nearest penny. Then graph the values.

For $c \le 4$:
$c = 1.42u$

u	c
0	0
2	**2.84**
4	**5.68**

For $4 \le c \le 8$:
$c = 5.68 + 2.87(u - 4)$

u	c
4	**5.68**
6	**11.42**
8	**17.16**

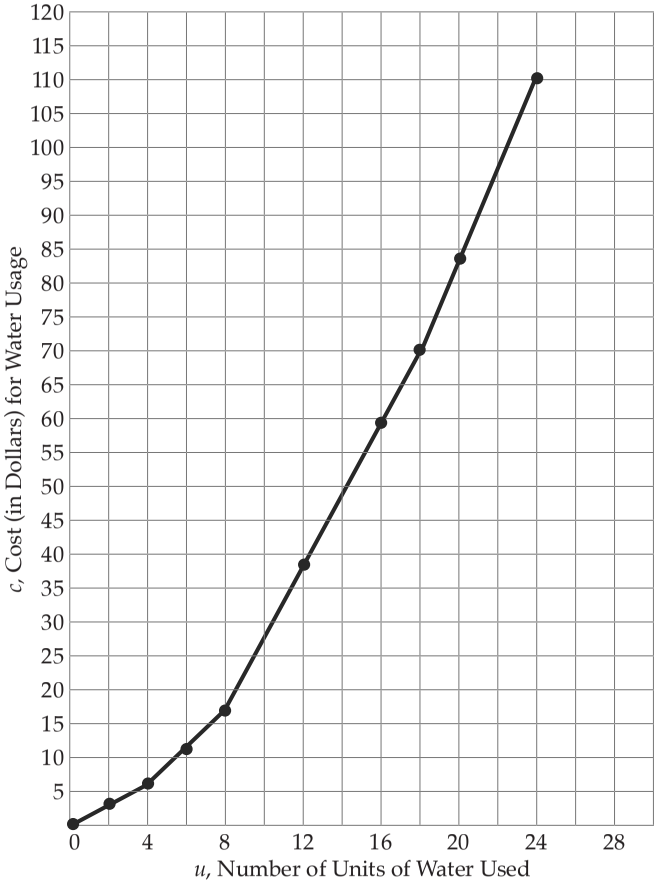
For $8 \le c \le 18$:
$c = 17.16 + 5.29(u - 8)$

u	c
8	**17.16**
12	**38.32**
16	**59.48**
18	**70.06**

For $18 \le c \le 24$:
$c = 70.06 + 6.68(u - 18)$

u	c
18	**70.06**
20	**83.42**
24	**110.14**

Number of units, u	Price per unit of water
0–4	$1.42
5–8	$2.87
9–18	$5.29
19–24	$6.68
≥ 25	$7.63



Use your graph for the following question.

2. The average family of four uses 12,000 gallons of water a month. What would they pay per month for their water? Per year?

12,000 gallons = 16.04 units. They would pay about $60 a month. $720 a year.

NOTES: Question 2 has the student uses the graph to answer the question. It might be worth a discussion with the student as to why the question is asking to use the graph. Answers may be that it is faster, that the formula is unavailable or cumbersome, or perhaps because an exact amount is not necessary.

If the student uses the formula, the monthly cost is $59.48 and annual cost is $713.76.

DICTIONARY TERMS: none