Mastering Multiplication and Division Facts:
New Ways of Learning the Facts
with Card Games and the Short Multiplication Table

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Abstract

Most students get overwhelmed with math worksheets. Students who do not understanding a concept will not benefit from additional worksheets. Flashcards merely reinforce what a student does not know and can become another source of frustration, creating feelings of failure. Rather than worksheets or flashcards, games are a successful method to learn, to apply, and to master the facts.

When a student learns to read, they can apply the new skills by reading for pleasure. In the same way, math card games combine practice with pleasure. Although learning math requires hard work, it can be enjoyable.

The Short Multiplication table is a derivative of the complete multiplication table. It utilizes the commutative property to remove duplicate facts; also, it is shaded into groups of fives to assist finding products and quotients.

Introduction

We know more now than we did 50 years ago, even 20 years ago. We know more about how children learn and how to teach more effectively. Yet, we have failed to modify our methods. We need to change the way we teach.

Multiplication and division facts are obstacles that some students never overcome. We need a way to help students make sense of multiplication and division, then apply it to higher level mathematics.

Worksheets are not the answer. If it was the answer, then we could give the students more worksheets, and the facts would be memorized. We have tried this in various forms; with paper, computers, and apps. It is not working.

Multiplication and division are hard to learn when rote memorization is the only avenue for learning. The 100 facts to memorize are often presented as random unrelated numbers with little or no understanding attached. It is an exercise in frustration for all involved: students, teacher, and parents. This approach is killing our children's mathematical futures.

Emotions and Learning

Along with the information recorded in our memories are the feelings we experienced when we learned it. These feelings are often recalled along with the fact or experience. For this reason, information stored with negative feelings tends to be forgotten. Children who associate math with feelings of failure, inadequacy, or anxiety will find learning difficult, and worse yet, they

will ignore applications to daily life. This is limiting the students. This limits their future careers, which limits the future of our world!

Yet, if a student has an enjoyable time learning, then positive emotions will replace past negative emotions. Positive emotions have been connected to the acquisition of skills that foster academic success, increase student engagement, improve trust and social engagement, and improve well-being (Hart, 2021).

When children recall feelings of discovery and success, they will want to continue learning and will apply that knowledge to other areas. Therefore, it is important that learning be a pleasant experience.

There is another reason to make mathematics enjoyable for children. When a child learns to read, they can practice those skills by reading for pleasure. In the same way, these card games combine practice with pleasure.

Although learning math requires hard work, it can be enjoyable. When a person is interested in and loves their work, they can more easily tackle the challenging segments found in any activity.

Math Card Games

Games are generally recognized as a fun activity. Most people have fond memories of playing games with family and friends. What if we incorporate math card games with learning and practicing? This will bring the positive emotions into the learning arena. Then, when we also provide a platform to learn while playing the games, we have a environment ripe for success.

Games will hone skills and help the student become more confident and fluid in their thinking. The more games are played, the more students learn. If a concept is not solid, play the game again and again. Also, playing previously-played games will allow the student to see their growth and master their facts.

These games allow children of various ages and abilities to play together. It does away with anxiety-producing flash cards, which cast the parent or teacher in the role of judge. It does away with isolating and overwhelming worksheets. Games create a stress-free atmosphere that allows them to learn at their own pace. Then the parent or teacher becomes a partner in the learning process.

What makes a good game? A good math game includes a method to find the facts while playing, rather than relying on prior knowledge to play. It is more than a reward; rather, it is essential for practice time so that mastery of the facts occurs.

Along with the facts that they will learn, they will also learn social skills: taking turns, winning and losing graciously, and helping others. Game play is an environment where children will be equals without concerns for equity, gender, race, or disability.

Games, when done right, are more than just review. A good game will teach during play. Games provide interesting repetition needed for automatic

responses in a social setting. More importantly, games provide an application for the new information.

Short Multiplication Table

The Short Multiplication table is a modified multiplication table. The duplicate facts from the full multiplication table are removed. The Short Multiplication table was found in a 12th-century manuscript from Toledo, Spain. It is also shown in the first arithmetic book, Treviso Arithmetic, printed in 1478. See Figure 1.

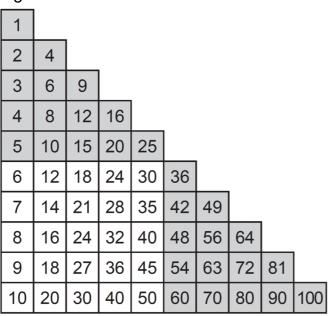| 1									
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8	16	24	32	40	48	56	64		
9	18	27	36	45	54	63	72	81	
10	20	30	40	50	60	70	80	90	100

Figure 1. Short Multiplication table

To find the multiples of 6, see the first six multiples in the 6-row. The rest of the 6s continue down in the 36-column to 60. See Figure 2.

Look for 6 × 8. See Figure 3. Next, find 8 × 6. Notice that product is in the same cell as 6 × 8. This occurs because of the commutative property.
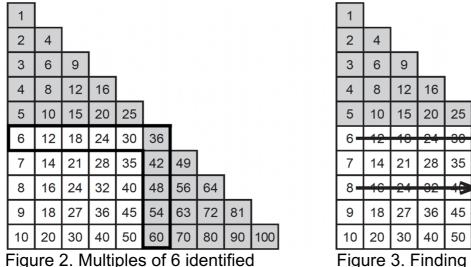
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9	18	27	36	45	54	63	72	81	
10	20	30	40	50	60	70	80	90	100

Figure 2. Multiples of 6 identified

1									
2	4								
3	6	9							
4	8	12	16						
5	10	15	20	25					
6	12	18	24	30	36				
7	14	21	28	35	42	49			
8	16	24	32	40	48	56	64		
9	18	27	36	45	54	63	72	81	
10	20	30	40	50	60	70	80	90	100

Figure 3. Finding 6 × 8 and 8 × 6

Notice that no counting is necessary on the Short Multiplication table since the cells are grouped by fives with dark and light cells. With 8 × 6, the first five cells are white, and the sixth cell is shaded. Recognition of numbers is quick because of the shading in groups of fives.

The Short Multiplication table will help the students support their learning and provide a platform for practicing their multiplication facts during game play. It give the students a reference to rely on and will be set aside when it is no longer needed.

Ring Around the Products Game

This multiplication game provides players with an opportunity to learn and practice the facts. Two to four players may play. Encourage the students to use the Short Multiplication table as needed.

Use the 100 multiplication cards from 1 × 1 to 10 × 10. Also use basic number cards: 10 of each number from 1 through 10 for a total of 100 cards. Form two separate stocks. In the center of the playing area, place six multiplication cards face up, in two rows of three. Ring these cards with 14 basic number cards face up.

Figure 4.

The object of the game is to collect the most multiplication cards. Two basic number cards are multiplied together to find the product on the multiplication card.
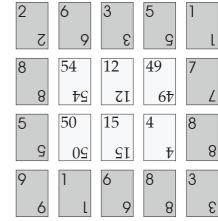
The first player checks the outside cards for pairs whose product equals an inside card. The player removes the corresponding cards and lays each card face up in front of them in three separate piles. This provides visual confirmation of the facts as well as checks the player's work. Collect as many facts as possible during each turn. In Figure 4, the player can collect three facts: 54 = 6 × 9, 12 = 2 × 6, and 15 = 3 × 5.

Once the player's turn is over, fill in the missing cards from the respective card stocks, and the next player takes their turn. Play continues until the multiplication card stock is exhausted. When the basic number card stock is exhausted, reuse the basic number cards from the players' piles.

If a player cannot find any facts, they skip their turn and replace the basic number cards in the four corners with cards from the stock. The winner is the player with the most multiplication cards.

Reliance on the Short Multiplication Table

Think of the Short Multiplication table compared to a recipe. The first few times the food is made, the recipe is carefully referred to. Then, over time, when the chef becomes more comfortable, they will reference the recipe less often, only checking when they are unsure. Finally, after repeated successes, the recipe card is unneeded.

This analogy can be applied to the Short Multiplication table. Initially, the Short Multiplication table will be used consistently. Then, over time, it will be only used to verify information. Finally, it will be set aside and no longer needed.

Division on the Short Multiplication Table

The Short Multiplication table can also be used for division. To find 18 ÷ 3, start at 3, move to the end of the row, turn the corner, and proceed until 18 is reached. This is the 6th cell; 18 ÷ 3 = 6. See Figure 5 on the next page.

Another method to find 18 ÷ 3 is to see that 18 is in the 6-row and is 3 cells in. See the second arrow starting at 18 and moving three cells to the left.

To find 30 ÷ 4, start at row 4, move to the end of the row, and then down to the largest number that is not more than 30, which is 28. Find the difference between 28 and 30 to calculate the remainder: 30 ÷ 4 = 7 with a remainder of 2. See Figure 6.
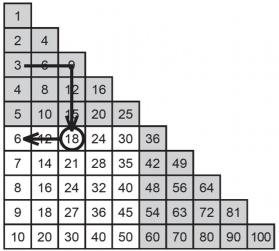
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9	18	27	36	45	54	63	72	81	
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Figure 5. 18 ÷ 3 = 6

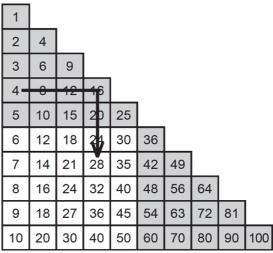| 1									
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Figure 6. 30 ÷ 4 = 7 r2

Division War Game

Two players can play this game. If more players are available, teams can formed. Use the multiplication cards greater than 9 and the basic number cards without the 1s. Encourage the players to use the short multiplication table as needed.

Divide the multiplication cards equally between the two players or two teams. Also, divide the basic number cards equally. Players lay their two stacks face down in front of them.

Each player turns over a basic number card and a multiplication card. They divide the multiplication card by the basic number card. The player with the greater quotient is the winner of that round.

Some students may need to have the multiplication card be no more than ten times the basic number card. For example, if the basic number card is 3, the multiplication card must be 30 or less.

The first player has 16 ÷ 3 which is 5 with a remainder of 1. The second player has 81 ÷ 10 which is 8 with a remainder of 1. The second player has the greater quotient, and therefore takes all four cards.

A war occurs when the quotients, ignoring the remainders, are equal. When this happens, each player places a card from each stack face down, then

turns over two more cards and finds the quotient as before. The player with the greater quotient takes all twelve cards.

Continue playing with each player turning over a card from each of their decks. The object of the game is to capture all the cards. For a shorter game, the winner is the player who has the most cards when the initial stacks are exhausted.

Figure 7. 16 ÷ 3 = 5 r1 and 81 ÷ 10 = 8 r1;
second player with the higher quotient, 8, wins this round

Division Remainder War Game
This game is a variation of the Division War game. Rather than the player with the greater quotient being the winner of a round, it is now the player with the greater remainder.

Figure 8. 25 ÷ 7 = 3 r4 and 49 ÷ 8 = 6 r1;
first player with the higher remainder of 4 wins this round

Conclusion
The Short Multiplication table provides a resource in which students can rely upon until they internalize the information and no longer need to reference it. Practice is needed for the facts to become automatic. Ten to 15 minutes of a card game are equivalent to a worksheet. Rather than flash cards or traditional worksheets, provide games for the child to learn, apply what they have learned, and explore new applications for their knowledge.

Maya Angelou, author and poet, says "Do the best you can until you know better. Then when you know better, do better."

We know better. Math card games provide interesting repetition needed for automatic responses in a social setting. More importantly, games provide an application for the new information.

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