Place Value: The Nucleus of Arithmetic

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Abstract

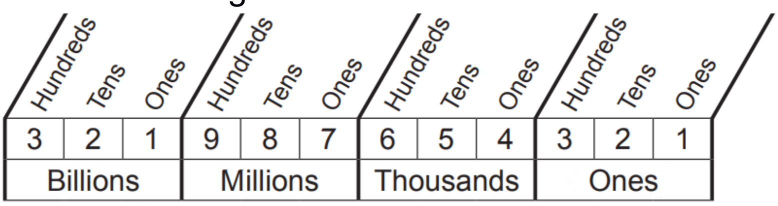
Place value allows numbers to be categorized into tidy components rather than an unending jumble of words. It was considered so important that *Treviso Arithmetic*, written in 1478, referred to place value as one of the basic arithmetic operations. Primary children speaking Eastern Asian languages understand place value years earlier than English-speaking children because the names of the numbers make place value transparent: 26 is two-ten six and 49 is four-ten nine. Thinking of 14 as 14 ones rather than one 10 and four ones interferes with carrying when adding multi-digit numbers. In order to appreciate the pattern that 10 ones equal 10, 10 tens equal 100, 10 hundreds equal 1000, and so forth, students must be allowed to work with numbers into the thousands. Place value is indeed the foundation of arithmetic.

Introduction

Place value organizes numbers into neat packets which lessen the load on working memory. Without place value, we are giving children the numbers as a string of words, each with a name that does not help identify its value. Place value is a necessary ingredient to make sense of any computational algorithms. Without a solid understanding of place value, student math scores drop dramatically around fourth grade and leave students struggling to understand when they get to algebra.

First grade students who have been taught traditionally tend to count on their fingers to answer the question, “What is ten plus three?” Obviously, these children have zero understanding of place value. Those who understand place value, know that 13 is 10 and 3 more, so 10 + 3 is 13. Likewise, they see the relationship between 3 + 4, 30 + 40, 300 + 400 and 3000 + 4000.The value of any digit changes depending upon its position within the number.

Sometimes, we think place value is memorizing and reciting the words: ones, tens, hundreds, thousands, and so forth, as the student sequentially points to each digit in a number starting at the right. It is more than that. Place value must be carefully taught. The value of a digit’s position in a number is not arbitrary, but forms a pattern built around a base. In our usual base-ten system, a ten is 10 ones, a hundred is 10 tens, and a thousand is 10 hundreds. The names of the positions in larger numbers repeat in clusters of three: hundreds, tens, and ones. For example, the number 137,137,137,137 is read as one hundred thirty-seven billion, one hundred thirty-seven million, one hundred thirty-seven thousand, one hundred thirty-seven. Although the cluster concept is not necessary for performing basic arithmetic algorithms, this organization makes large numbers more meaningful.



Place value names in clusters

In general, to recognize and continue a pattern, a minimum of three consecutive elements must be given. Likewise, children need at least three elements to grasp the pattern of place value: 10 ones equal 10; 10 tens equal 100; 10 hundreds equal 1 thousand; and so forth. After students master writing numbers to ten, writing numbers greater than ten is best accomplished with place value understanding. To learn this critical concept, it is imperative that children work with thousands during their first-grade year. Children love large numbers, even 4- and 5-year-olds can work with thousands. Unfortunately, math standards underestimate children's abilities and discourage such work. Children want to understand, and they look for patterns. Without place value, most arithmetic doesn't make sense; it becomes a collection of rules to memorize. Facts and procedures that don't make sense are much harder to learn.

Some mathematics standards for kindergarten require students to work with numbers 11 through 19 to gain a foundation for place value. Working with such a small range of numbers does not provide students with a clue about the real nature of place value. In fact, starting with the teens is problematic because the very number names of teen numbers obscure the tens' structure, so the pattern is further hidden. Good educational practice is to teach the general rule before any exceptions. The teen numbers are exceptions and should be taught **after** the 20s through 90s.

Solving the Struggle with Place Value

Children often think of 14 as 14 ones, not a ten and 4 ones. The pattern that is needed to make sense of tens and ones is hidden in the names of the numbers. This is true for most western languages. The solution is to temporarily use the *transparent number naming system*.

Transparent number naming

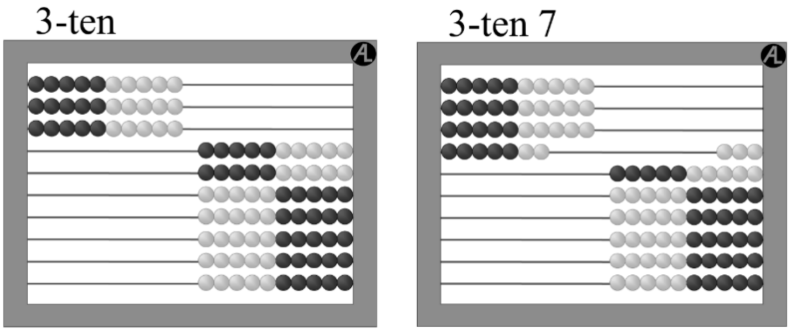
10 = ten	20 = 2-ten
11 = ten 1	21 = 2-ten 1
12 = ten 2	22 = 2-ten 2
13 = ten 3	23 = 2-ten 3
14 = ten 4	. . . .
. . . .	. . . .
19 = ten 9	99 = 9-ten 9

There are at least three benefits with using these transparent number names.

1. They maintain a pattern. We say 3 million, 3 thousand, 3 hundred, and, now we can say, 3 ten.
2. The place value of the number is explicitly stated in the name. How many tens are in sixty-five? It is not clear. How many tens are in 6-ten-five? That is very clear, six!
3. Using transparent number names alleviates the problem of students writing "31" when asked to write 13. Those students heard the "thir" sound in thirteen and thought that the 3 should be written first. When naming 13 as "ten-3" and 31 as "3-ten-1," there is no auditory confusion.

Students in eastern Asia already have this transparent number naming system built into their language and therefore, they understand place value very early. What does that look like? They can add 4-digit numbers with carrying! Even if your students do not speak an eastern Asian language, they can reap the same benefits by using the transparent number names.

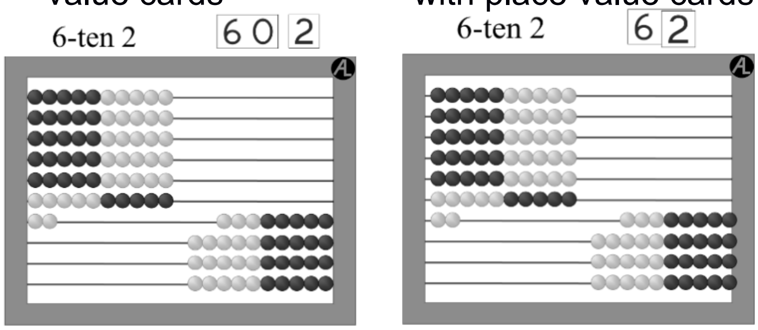
Transparent place value



Place-Value Cards

Combining a visual representation of a quantity, as on the Cotter Abacus, with the transparent number naming system and place-value cards is an excellent three-part approach to teaching number sense.

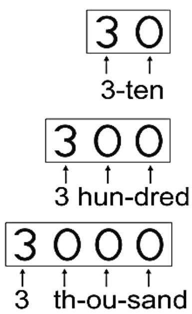
Choosing place value cards — Composing numbers with place value cards



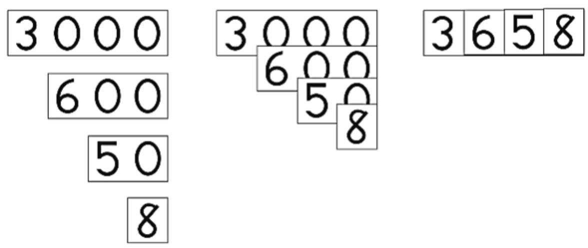
Using place-value cards has several benefits.

1. They are stacked and read from left to right.
2. They can be easily separated if the student forgets the value of a number.
3. They make reversing numbers almost impossible and provide a reliable model to copy when writing the number.
4. The syllables in the transparent number name, in combination with the place-value cards, emphasizes how many digits follow the initial number.

Syllables help identify place value.



Building numbers with place value cards.



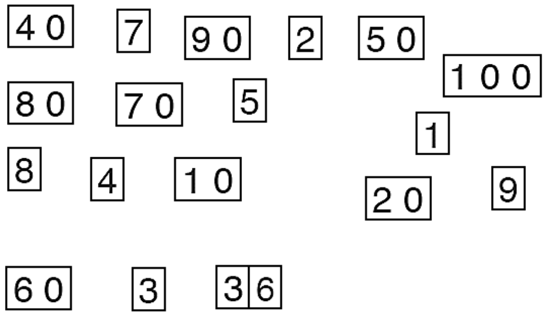
Activities to Support a Strong Understanding of Place Value

Can You Find Game — Level One

This game provides practice in naming tens and ones using the transparent number names. The object of the game is to quickly find the place-value card(s) for the stated number. Lay out the place-value cards face up in random order. Explain to the child that you are going to say a number for which they are to find the card as quickly as possible.

Start by asking for one-card numbers: Can you find 6-ten? Can you find 3? Can you find 2 ones? Can you find 10-ten?

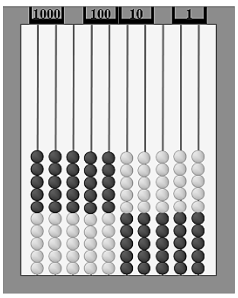
When the child is comfortable with the first set of numbers ask: Can you find 3-ten 6? Can you find 8-ten 3? Can you find 5-ten 4? Continue until all the cards are picked up.



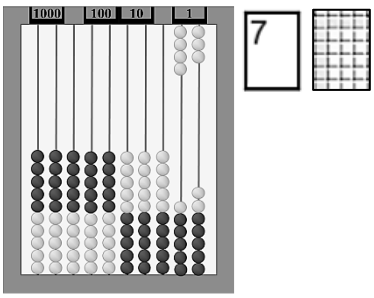
Play the game again, but this time enter the quantities on the abacus, have the child read the quantity, and then find the corresponding cards.

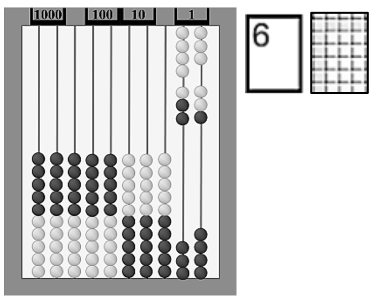
Bead Trading Game

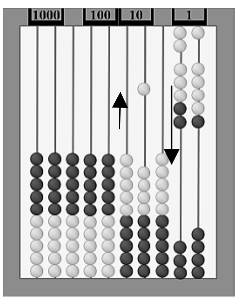
Another well-loved game emphasizing place value is the Bead Trading Game. You will need the Cotter Abacus and a stack of basic number cards from 1 through 9.

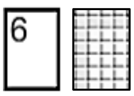


Object: Reach 1000 by adding the numbers on the cards.

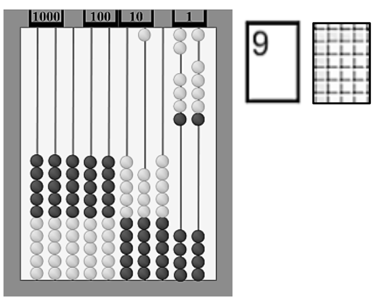








Trade 10-ones for 1-ten



- In the bead trading activity, trading 10 ones for 1 ten occurs frequently; 10 tens for 1 hundred, less often; and 10 hundreds for 1 thousand, rarely.
- Bead trading helps the child experience the greater value of each column from left to right.
- To detect a pattern, there must be at least three examples in the sequence. To experience place value as a pattern, the thousands are needed.

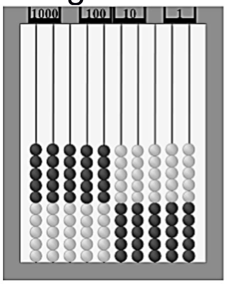
4-Digit Addition on the Cotter Abacus

On the second side of the Cotter Abacus, each bead is worth the value of the number above the column. Beads on the two wires under 1000 are each worth one thousand. Beads on the two wires under 100 are each worth one hundred, and so on.

When entering the beads, keep the beads as even as possible so that it is obvious when you have entered ten, all of one color. This makes it easier to see when you need to trade.

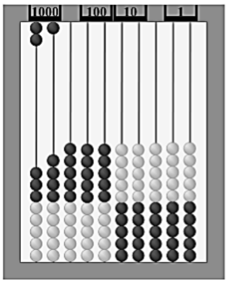
Follow the examples below:

4-Digit Addition



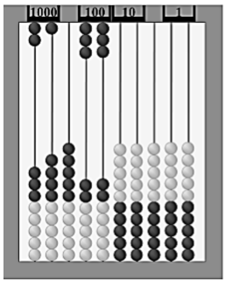
3658
+ 2738

Enter 3-thousand



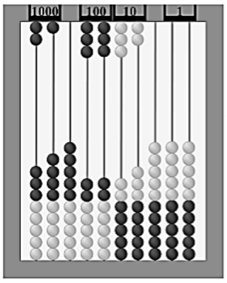
3658
+ 2738

Enter 6-hundred



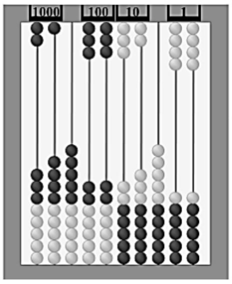
3**6**58
+ 2738

Enter 5-ten



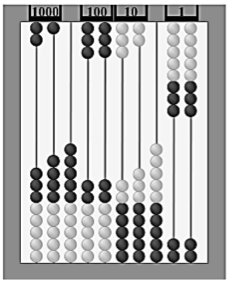
36**5**8
+ 2738

Enter 8-ones



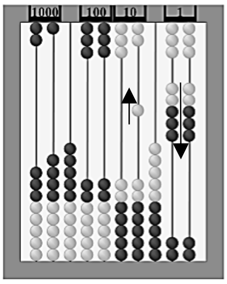
365**8**
+ 2738

Add 8



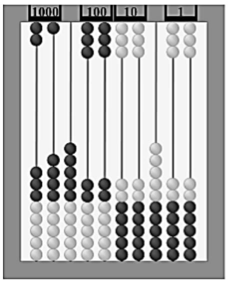
3658
+ 273**8**

Trade 10 ones for 1-ten



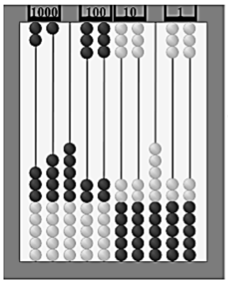
3658
+ 273**8**

Write the ones



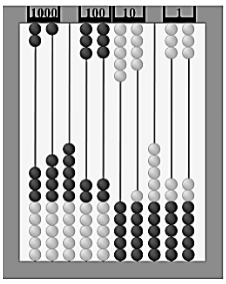
3658
+ 2738
6

Write the trade



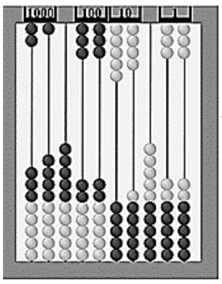
1
3658
+ 2738
6

Add 3-ten



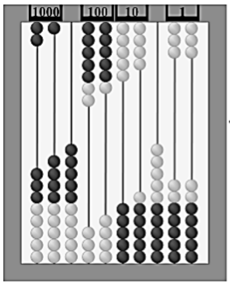
1
3658
+ 27**3**8
6

Write the tens



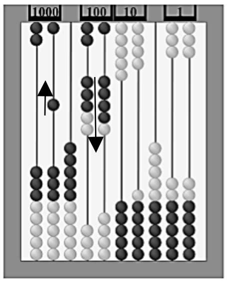
1
3658
+ 2738
96

Add 7-hundred



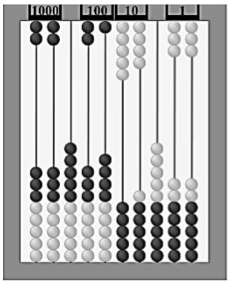
1
3658
+ 2**7**38
96

Trade 10-hundreds



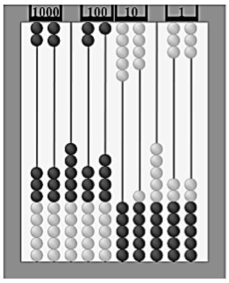
$$\begin{array}{rcccc} & & & 1 & \\ & 3 & 6 & 5 & 8 \\ + & 2 & \mathbf{7} & 3 & 8 \\ \hline & & & 9 & 6 \end{array}$$

Write the hundreds



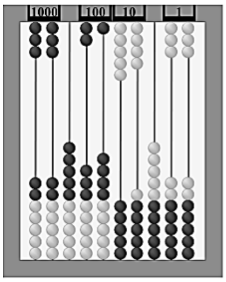
$$\begin{array}{rcccc} & & & 1 & \\ & 3 & 6 & 5 & 8 \\ + & 2 & 7 & 3 & 8 \\ \hline & & \mathbf{3} & 9 & 6 \end{array}$$

Write the trade



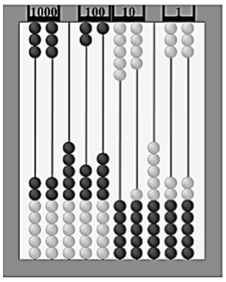
$$\begin{array}{rcccc} & \mathbf{1} & & 1 & \\ & 3 & 6 & 5 & 8 \\ + & 2 & 7 & 3 & 8 \\ \hline & & \mathbf{3} & 9 & 6 \end{array}$$

Add 2-thousand



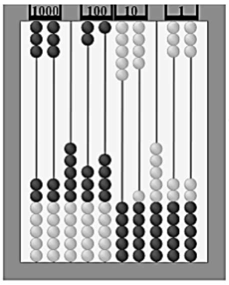
$$\begin{array}{rcccc} & 1 & & 1 & \\ & 3 & 6 & 5 & 8 \\ + & \mathbf{2} & 7 & 3 & 8 \\ \hline & & 3 & 9 & 6 \end{array}$$

Write the thousands



$$\begin{array}{rcccc} & 1 & & 1 & \\ & 3 & 6 & 5 & 8 \\ + & 2 & 7 & 3 & 8 \\ \hline & \mathbf{6} & 3 & 9 & 6 \end{array}$$

Final answer



$$\begin{array}{rcccc} & 1 & & 1 & \\ & 3 & 6 & 5 & 8 \\ + & 2 & 7 & 3 & 8 \\ \hline & 6 & 3 & 9 & 6 \end{array}$$

Summary

Mastering place value, the nucleus of arithmetic, ensures a solid foundation. Follow these steps:

1. Introduce quantities and operations in a visualizable manner using the Cotter Abacus.
2. Use the transparent number naming system temporarily to make place value an obvious part of the number name.
3. Introduce the written numerals using place value cards.
4. Use math games and activities to reinforce the concepts and keep mathematics an enjoyable experience.

Reference

Clayton, K. C., Cotter, J. A. (2022). *RightStart™ Tutoring: Number Sense.* Activities for Learning, Inc.

Cotter, J. A. *Skip the Counting and Other Innovative Practices for Teaching Elementary Mathematics*. In press.