Fractions:
Seeing is Believing

based on the work of Dr. Joan A. Cotter

Fractions

- Fractions generally have a bad reputation.

PEANUTS CHARLES SCHULZ

- Often viewed as incomprehensible and unpredictable.
- Sometimes perceived as scary!

Fractions

- This is a huge misunderstanding.
- Fractions are necessary and useful.
- Fractions are amazing!

Fraction History

- Latin "frangere" meaning "to break."
- Considered only as part of a whole.
- Could never be equal or greater than 1.

Fraction History

- In the 1600s, the concept of fractions expanded.
- Now included a division perspective.
- Fractions could be equal to or more than 1.

$$\frac{1}{3}$$

Fraction History

- In the 1600s, the concept of fractions expanded.
- Now included a division perspective.
- Fractions could be equal to or more than 1.

$$\frac{2}{2}$$

Fraction History

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$$\frac{2}{3}$$

1

Fraction History

- In the 1600s, the concept of fractions expanded.
- Now included a division perspective.
- Fractions could be equal to or more than 1.

$$\frac{2}{3}$$

Fraction History

Old Fractions	New Fractions
Always less than one	Can be equal or greater
$\frac{1}{3}$	$\frac{1}{3}$ $\frac{3}{3}$ $\frac{4}{3}$
Whole is fractured	Viewed as division
Familiar = "proper"	Unfamiliar = "improper"
Limited view	Supports understanding

Fraction Model: Circles
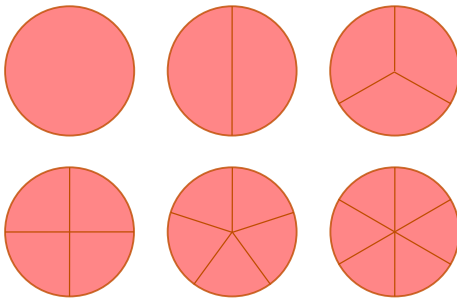
Are we comparing angles, arcs, or area?

Fraction Model: Circles
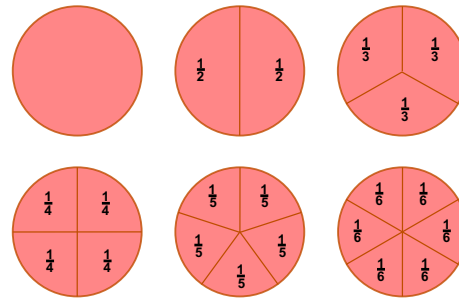
Try to compare 4/5 and 5/6 with this model.
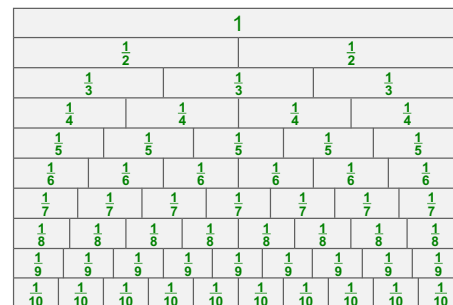

Fraction Model: Circles

- Experts in visual literacy say that comparing quantities in pie charts is difficult because most people think linearly. It is easier to compare along a straight line than compare pie slices.   askoxford.com
- Specialists also suggest refraining from using more than one pie chart for comparison.   statcan.ca
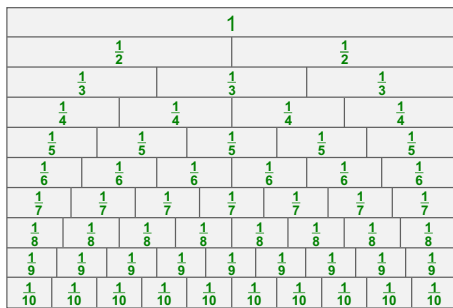

Fraction Chart

Fraction Chart

1									
$\frac{1}{2}$					$\frac{1}{2}$				
$\frac{1}{3}$			$\frac{1}{3}$			$\frac{1}{3}$			
$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$			$\frac{1}{4}$		
$\frac{1}{5}$	$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$			$\frac{1}{5}$	
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$	
$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$		$\frac{1}{7}$		$\frac{1}{7}$	$\frac{1}{7}$	
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$		$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$		$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Ask the children to put it together like a puzzle.
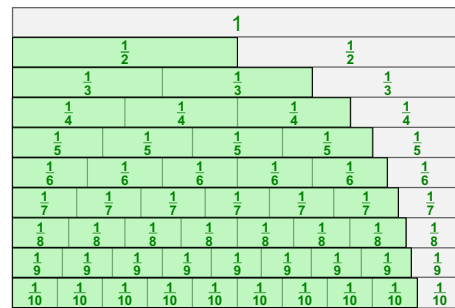
Fraction Chart

1									
$\frac{1}{2}$					$\frac{1}{2}$				
$\frac{1}{3}$			$\frac{1}{3}$			$\frac{1}{3}$			
$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$			$\frac{1}{4}$		
$\frac{1}{5}$	$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$			$\frac{1}{5}$	
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$	
$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$		$\frac{1}{7}$		$\frac{1}{7}$	$\frac{1}{7}$	
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$		$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$		$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

An interesting pattern.

Fraction Stairs

$\frac{1}{10}$
$\frac{1}{9}$
$\frac{1}{8}$
$\frac{1}{7}$
$\frac{1}{6}$
$\frac{1}{5}$
$\frac{1}{4}$
$\frac{1}{3}$
$\frac{1}{2}$
1
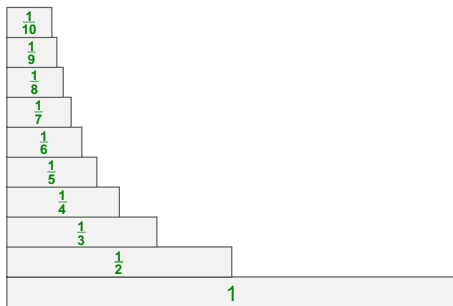
Games

$$\frac{\text{Games}}{\text{Math}} = \frac{\text{Books}}{\text{Reading}}$$

Games provide instant feedback.

Games provide interesting repetition needed for automatic responses in a social setting.

More importantly, games provide an application for the new information!
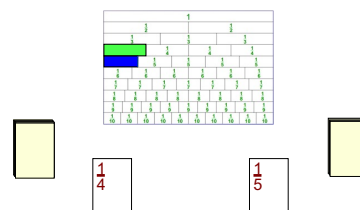

Unit Fraction War

Purpose: Practice for naming and comparing unit fractions.
To help the children realize a unit fraction decreases as the denominator increases.

Goal: To collect all, or most, of the cards by comparing unit fractions.
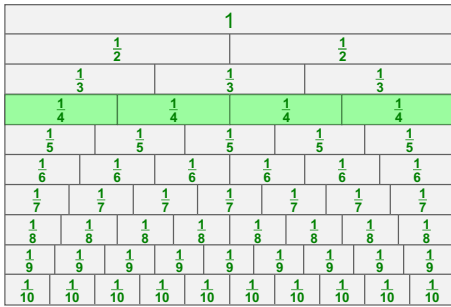

Unit Fraction War

$\frac{1}{4}$ $\frac{1}{5}$

Fraction Chart

1			
1/2		1/2	

1/3 1/3 1/3

1/4 1/4 1/4 1/4

1/5 1/5 1/5 1/5 1/5

1/6 1/6 1/6 1/6 1/6 1/6

1/7 1/7 1/7 1/7 1/7 1/7 1/7

1/8 1/8 1/8 1/8 1/8 1/8 1/8 1/8

1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9

1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10

How many fourths in a whole?

Fraction Chart

1			
1/2		1/2	

1/3 1/3 1/3

1/4 1/4 1/4 1/4

1/5 1/5 1/5 1/5 1/5

1/6 1/6 1/6 1/6 1/6 1/6

1/7 1/7 1/7 1/7 1/7 1/7 1/7

1/8 1/8 1/8 1/8 1/8 1/8 1/8 1/8

1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9

1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10
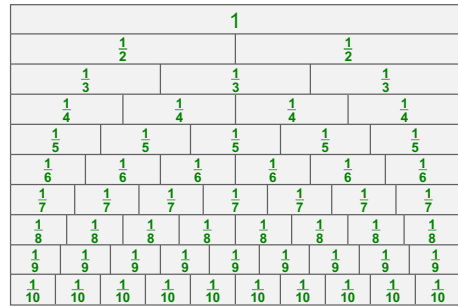
How many fourths in a whole? How many fifths? Eighths?

Concentrating on One

Purpose: To help the children realize that 5 fifths, 8 eighths, and so forth, make a whole.

Goal: To find the pairs that make a whole.
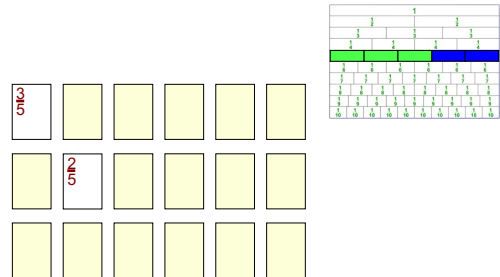
Concentrating on One

3/5

2/5

Fraction Chart

1			
1/2		1/2	

1/3 1/3 1/3

1/4 1/4 1/4 1/4

1/5 1/5 1/5 1/5 1/5

1/6 1/6 1/6 1/6 1/6 1/6

1/7 1/7 1/7 1/7 1/7 1/7 1/7

1/8 1/8 1/8 1/8 1/8 1/8 1/8 1/8

1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9

1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10

Which is more, 3/4 or 4/5?

Fraction Chart

1			
1/2		1/2	

1/3 1/3 1/3

1/4 1/4 1/4 1/4

1/5 1/5 1/5 1/5 1/5

1/6 1/6 1/6 1/6 1/6 1/6

1/7 1/7 1/7 1/7 1/7 1/7 1/7

1/8 1/8 1/8 1/8 1/8 1/8 1/8 1/8

1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9

1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10
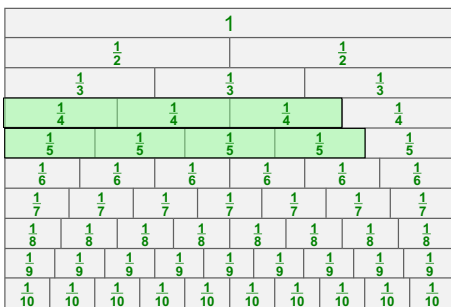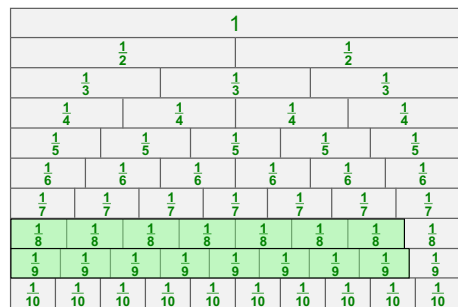
Which is more, 7/8 or 8/9?
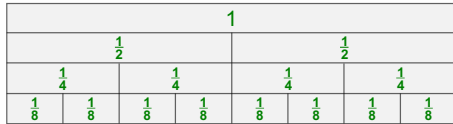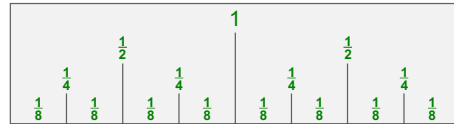
Partial Chart

Partial Chart

Partial Chart
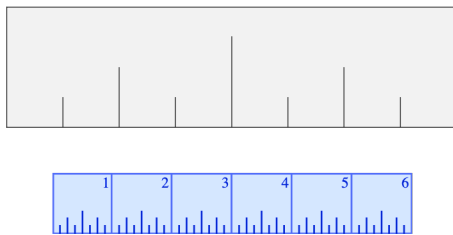
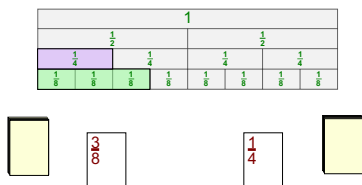## *Fraction War*

Purpose: To practice comparing ones, halves, fourths, and eighths in preparation for reading a ruler.

Goal: To capture all the cards.

Fraction War
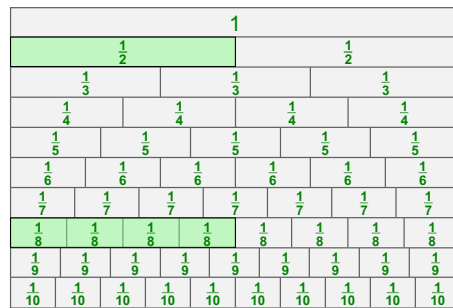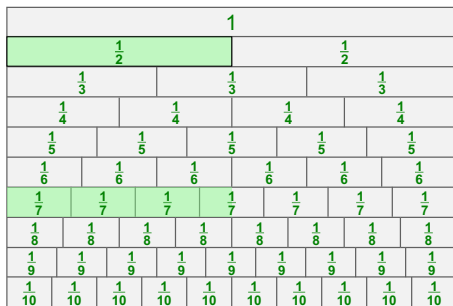

Game App
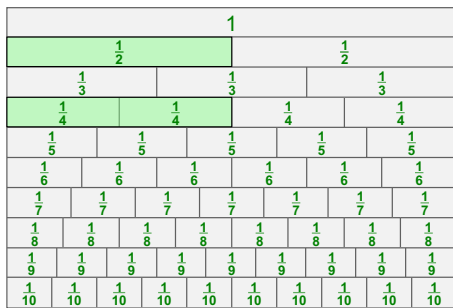
Equivalent Fractions

How many fourths equal a half?

Equivalent Fractions

How many fourths equal a half? Eighths?

Equivalent Fractions

How many fourths equal a half? Eighths? Sevenths?

Proper and Improper Fractions

$$\frac{9}{8}$$
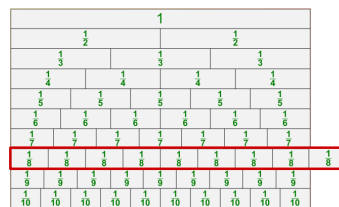
Proper and Improper Fractions
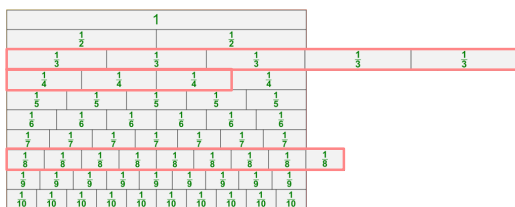
$$\frac{9}{8} \qquad \frac{5}{3} \qquad \frac{3}{4}$$

Which fraction(s) are improper?

Proper and Improper Fractions
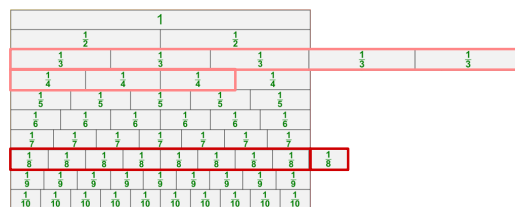
$$\frac{9}{8} = 1\frac{1}{8} \qquad \frac{5}{3} \qquad \frac{3}{4}$$

Rewrite the improper fractions using a whole number.

Proper and Improper Fractions
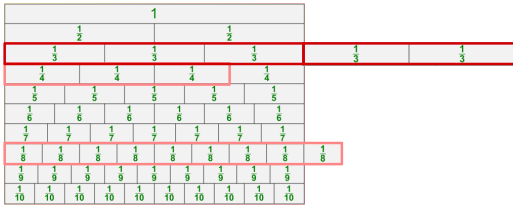
$$\frac{9}{8} = 1\frac{1}{8} \qquad \frac{5}{3} = 1\frac{2}{3} \qquad \frac{3}{4}$$

Rewrite the improper fractions using a whole number.
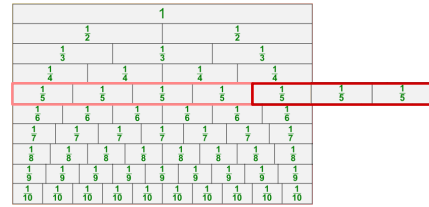
Proper and Improper Fractions
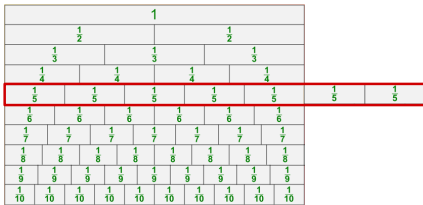
$$\frac{4}{5} + \frac{3}{5} =$$

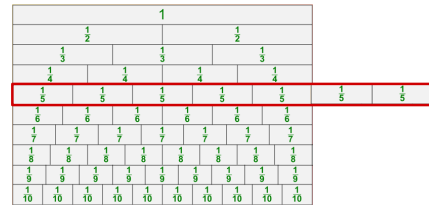## Proper and Improper Fractions
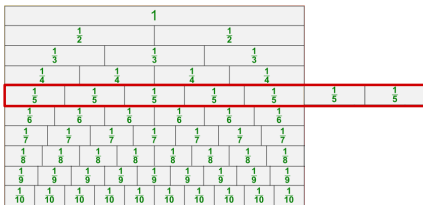
$$\frac{4}{5} + \frac{3}{5} = \frac{7}{5}$$

Proper and Improper Fractions

$$\frac{4}{5} + \frac{3}{5} = \frac{7}{5}$$

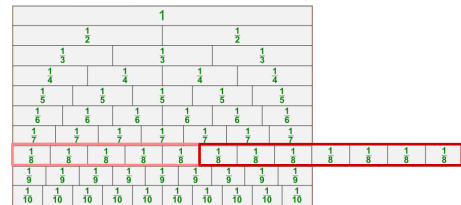Rewrite the improper fraction using a whole number.

Proper and Improper Fractions

$$\frac{4}{5} + \frac{3}{5} = \frac{7}{5} = 1\frac{2}{5}$$

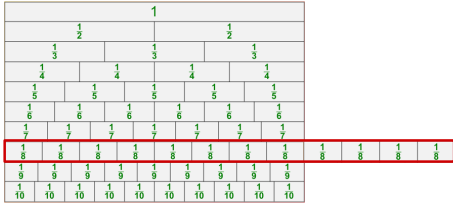Rewrite the improper fraction using a whole number.

Proper and Improper Fractions

$$\frac{5}{8} + \frac{7}{8} =$$

Proper and Improper Fractions

$$\frac{5}{8} + \frac{7}{8} = \frac{12}{8} = 1\frac{4}{8}$$

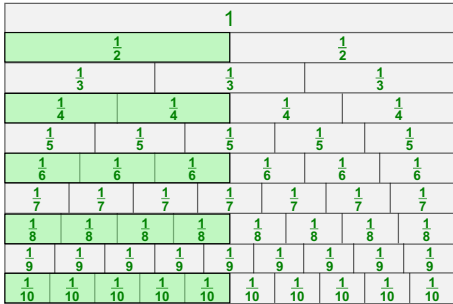Rewrite the improper fraction using a whole number.

Help your child
discover the algorithm.

"What you have been obliged to discover by yourself leaves a path in your mind which you can use again when the need arises." — *G.C. Lichtenberg*
18th century physicist

Simplifying Fractions

Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$$\frac{21}{28}$$

Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$$\frac{45}{72}$$

Simplifying Fractions

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$$\frac{45}{72}$$

Why does this work?

Fraction Chart

What is $\frac{1}{4} + \frac{1}{8}$?

Fraction Chart

What is $\frac{1}{4} + \frac{1}{8}$?

Fraction Chart

What is $\frac{1}{4} + \frac{1}{8}$?

Fraction Chart

What is $\frac{1}{4} + \frac{1}{8}$? $\frac{3}{8}$
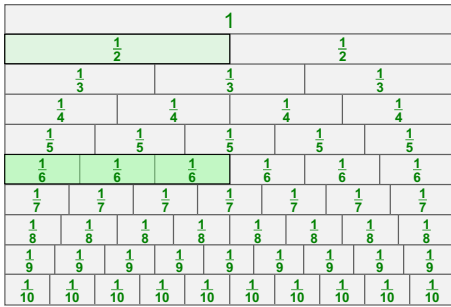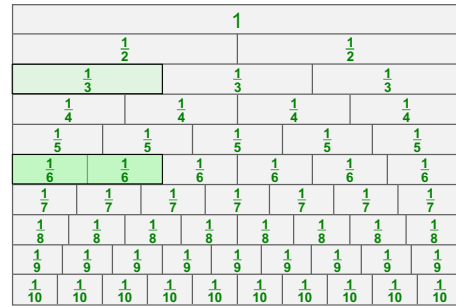
Fraction Chart

What is $\frac{1}{2} + \frac{1}{3}$?

Fraction Chart

What is $\frac{1}{2} + \frac{1}{3}$?
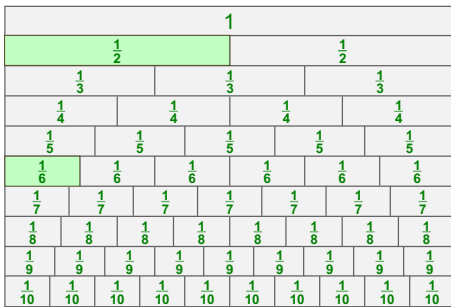
Fraction Chart

What is $\frac{1}{2} + \frac{1}{3}$? $\frac{5}{6}$

Fraction Chart

What is $\frac{1}{2}$ of $\frac{1}{2}$?

Fraction Chart

What is $\frac{1}{3}$ of $\frac{1}{2}$? That's multiplying fractions!

Multiplying Fractions

Multiplying is not exclusively repeated addition.
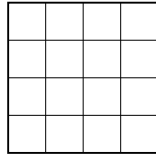
$$4 \times 4 = 4 + 4 + 4 + 4$$

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{2} + \,?$$

Multiplying Fractions

Multiplying is not exclusively
repeated addition.

Area is a better model.

4 × 4 =
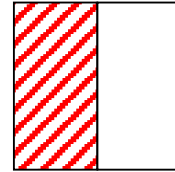
Multiplying Fractions

$\frac{1}{2} \times \frac{1}{2} =$

One half
of one half
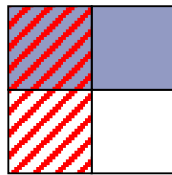
Multiplying Fractions

$\frac{1}{2} \times \frac{1}{2} =$

One half
of one half

Multiplying Fractions

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

One half
of one half
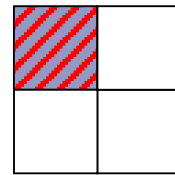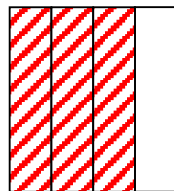
Multiplying Fractions

$\frac{2}{3} \times \frac{3}{4} =$

Three fourths
of two thirds

Multiplying Fractions
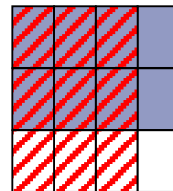
$\frac{2}{3} \times \frac{3}{4} =$

Three fourths
of two thirds

Multiplying Fractions

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

Three fourths
of two thirds

Multiplying Fractions

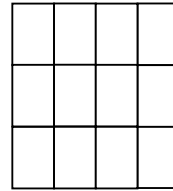$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

The total number of rectangles is 3 × 4.

Multiplying Fractions

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The total number of rectangles is 3 × 4.

The number of colored crosshatched rectangles is 2 × 3.
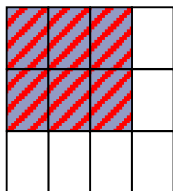
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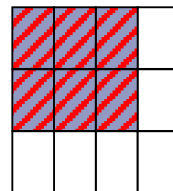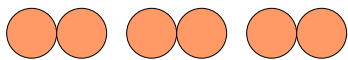
What is Division?

$$6 \div 2 = \underline{\quad}$$

How many 2s in 6?

Number of groups of 2s; 3 groups

Dividing Fractions

$$1 \div \frac{1}{2} = 2$$

How many $\frac{1}{2}$s in 1?
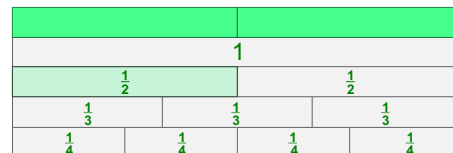
Dividing Fractions

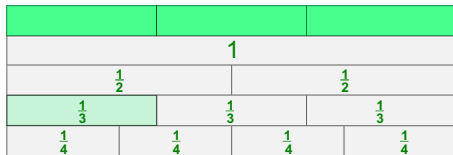$1 \div \frac{1}{2} = 2$

$1 \div \frac{1}{3} = 3$

How many $\frac{1}{3}$s in 1?

Dividing Fractions

$1 \div \frac{1}{2} = 2$

$1 \div \frac{1}{3} = 3$

$1 \div \frac{1}{4} = 4$

$1 \div \frac{1}{5} = 5$

$1 \div \frac{1}{6} = 6$

Dividing Fractions

$1 \div \frac{1}{2} = 2$

$1 \div \frac{1}{3} = 3$

$1 \div \frac{1}{4} = 4$

$1 \div \frac{1}{5} = 5$

$1 \div \frac{1}{6} = 6$

$1 \div \frac{2}{3} = 1\frac{1}{2} = \frac{3}{2}$

How many $\frac{2}{3}$s in 1?

Dividing Fractions

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Dividing Fractions

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$1 \div \frac{1}{6} = 6$

$1 \div \frac{2}{3} = \frac{3}{2}$

$1 \div \frac{3}{4} = \frac{4}{3}$

$1 \div \frac{2}{5} = \frac{5}{2}$

$1 \div \frac{5}{8} = \frac{8}{5}$

$1 \div \frac{4}{7} = \frac{7}{4}$

Guide the child to making the discovery that the answers are the inverted form of the divisor.

Fraction Division War

Purpose: Practice in dividing fractions. To help the children realize the quotient is the inverted form of the divisor.

Goal: To collect all, or most, of the cards by having the greatest quotient.

Fraction Division War

Fraction Division War

1

1

Fraction Division War

1   $\frac{1}{2}$

1   $\frac{1}{4}$

Fraction Division War

1   $\frac{1}{2}$ = 2

1   $\frac{1}{4}$ = 4

Fraction Division War

1   $\frac{3}{8}$ = $\frac{8}{3}$

1   $\frac{2}{3}$ = $\frac{3}{2}$

More Dividing Fractions

$1 \div \frac{1}{4} = 4$

More Dividing Fractions

$1 \div \frac{1}{4} = 4$

$2 \div \frac{1}{4} = 2 \times (1 \div \frac{1}{4})$

$\phantom{2 \div \frac{1}{4}} = 2 \times \quad 4 \quad = 8$

More Dividing Fractions

$1 \div \frac{1}{4} = 4$

$2 \div \frac{1}{4} = 2 \times (1 \div \frac{1}{4}) = 2 \times 4 = 8$

More Dividing Fractions

$1 \div \frac{1}{4} = 4$

$2 \div \frac{1}{4} = 2 \times (1 \div \frac{1}{4}) = 2 \times 4 = 8$

$3 \div \frac{1}{4} = 3 \times (1 \div \frac{1}{4})$

$\phantom{3 \div \frac{1}{4}} = 3 \times \quad 4 \quad = 12$

More Dividing Fractions

$1 \div \frac{1}{4} = 4$

$2 \div \frac{1}{4} = 2 \times (1 \div \frac{1}{4}) = 2 \times 4 = 8$

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More Dividing Fractions

$1 \div \frac{1}{4} = 4$

$2 \div \frac{1}{4} = 2 \times (1 \div \frac{1}{4}) = 2 \times 4 = 8$

$3 \div \frac{1}{4} = 3 \times (1 \div \frac{1}{4}) = 3 \times 4 = 12$

$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times (1 \div \frac{1}{4})$

$\phantom{\frac{1}{2} \div \frac{1}{4}} = \frac{1}{2} \times \quad 4 \quad = 2$

More Dividing Fractions

$1 \div \frac{1}{4} = 4$

$2 \div \frac{1}{4} = 2 \times (1 \div \frac{1}{4}) = 2 \times 4 = 8$

$3 \div \frac{1}{4} = 3 \times (1 \div \frac{1}{4}) = 3 \times 4 = 12$

$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times (1 \div \frac{1}{4}) = \frac{1}{2} \times 4 = 2$

$\frac{1}{3} \div \frac{1}{4} = \frac{1}{3} \times (1 \div \frac{1}{4}) = \frac{1}{3} \times 4 = \frac{4}{3}$

$\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times (1 \div \frac{1}{4}) = \frac{3}{4} \times 4 = \frac{12}{4} = 3$

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Fraction Chart

Allow the child to explore the whole
picture and relationships within the whole
using the linear perspective.

In Conclusion …

Math needs to be taught so
95 percent is understood and
only 5 percent memorized.

Richard Skemp
-- major pioneer in
mathematics education

In Conclusion …

Our goal as a teacher of mathematics
is to help our children transform,
expand, and refine these beginning ideas
into deeper mathematical thinking.

– Dr. Joan A. Cotter