THE IMPACT OF VISUAL MATH

Based on the work of Joan A. Cotter, Ph.D.

Traditional Math Approach

Traditional math approaches often have common challenges:

- shaky reliance on rote memorization,
- lack of understanding,
- struggles to apply to new situations,
- anxiety-inducing timed tests & flashcards, and
- frequent source of frustration and failure.

Visual Math Approach

Today, much more is known about how children learn.

- Need good visual representations for mathematical concepts.
- Need hands-on exploration.
- Need to move from concrete to representations, then to abstract concepts.

Visual Math Approach

- Math more than just numbers.
- Math is about patterns and relationships.
- Therefore, we need to encourage exploration and discovery in learning.
- Children need to physically manipulate objects, not watch someone do it for them.
- They learn better when they are active.

Visual Math Approach

- Our brains prefer visual information.
- A visual image is:
  - more permanent in our memory,
  - needs less review to maintain, and
  - can be a springboard for related facts.
- A visual approach to math is important for making abstract concepts concrete.

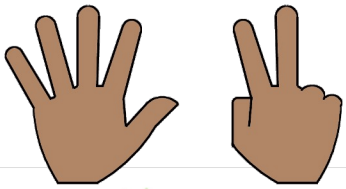
Visual Math Approach

- Subitizing
- Part-Whole Circles
- Cotter Sum Line
- Adding 4-Digits with Trading
- Multiplication Chart
- Fractions
- Area of a Circle

Quantities on Fingers

Subitizing

- Subitizing is quick recognition of quantity without counting.

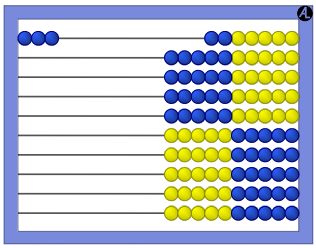


Subitizing

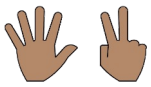
- Subitizing is quick recognition of quantity without counting.
- When grouped in **fives** and **tens**, subitizing allows quantities to be quickly recognized.

Subitizing

3



Subitizing



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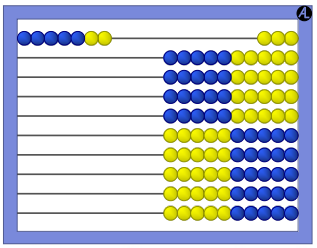
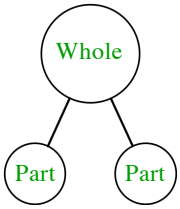


Part-Whole Circles



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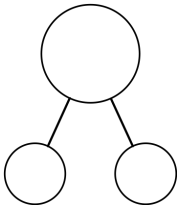
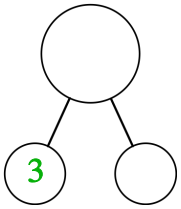


Lee received 3 goldfish as a gift. Now Lee has 5. How many did Lee have to start with?

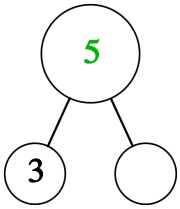
Part-Whole Circles



Is 3 a part or whole?

Lee received 3 goldfish as a gift. Now Lee has 5. How many did Lee have to start with?

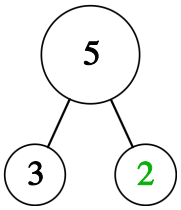
Part-Whole Circles



Is 5 a part or whole?

Lee received 3 goldfish as a gift. Now Lee has 5. How many did Lee have to start with?

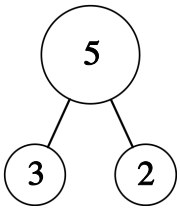
Part-Whole Circles



What is the missing part?

Lee received 3 goldfish as a gift. Now Lee has 5. How many did Lee have to start with?

Part-Whole Circles

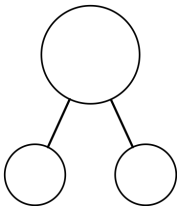


Write the equation.

3 + 2 = 5
2 + 3 = 5
5 – 3 = 2

Lee received 3 goldfish as a gift. Now Lee has 5. How many did Lee have to start with?

Part-Whole Circles



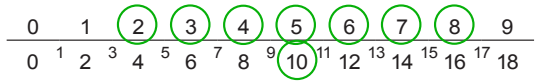
Part-whole circles help children solve problems. Writing equations does not.

Cotter Sum Line

0		1		2		3		4		5		6		7		8		9
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

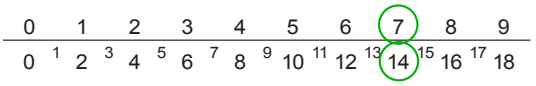
- Special number line for addition and subtraction facts.
- The sum of two numbers is found by locating the addends above the line, finding their middle, then reading the sum directly below the line.

Cotter Sum Line



$2 + 8 =$

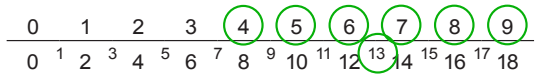
Cotter Sum Line



$2 + 8 = 10$

$7 + 7 =$

Cotter Sum Line

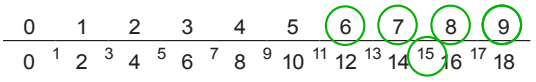


$2 + 8 = 10$

$7 + 7 = 14$

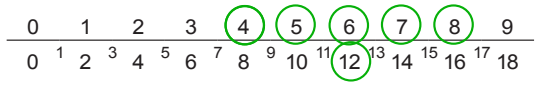
$9 + 4 =$

Cotter Sum Line



$15 - 6 =$

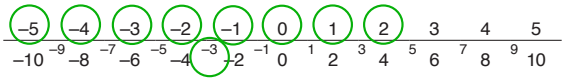
Cotter Sum Line



$15 - 6 = 9$

$12 - 8 =$

Cotter Sum Line



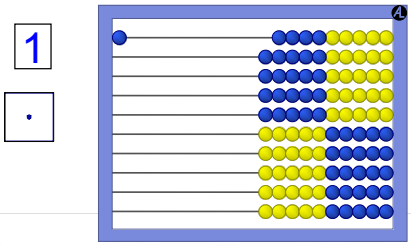
$-5 + 2 =$

Adding with Trading

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ } \\ 3658 \\ +\ 2736 \\ \hline 6394 \end{array}$$

- WHY???
- Why does this work? And what does it mean??

Visual Adding with Trading



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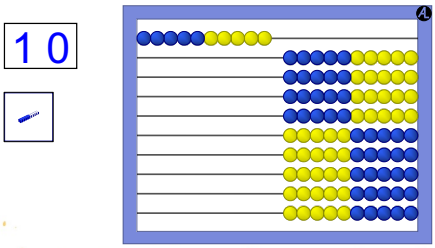

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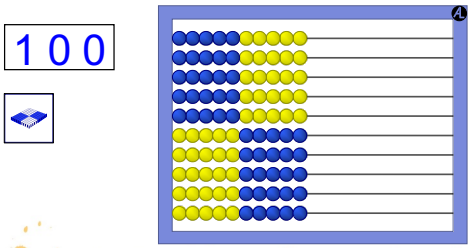

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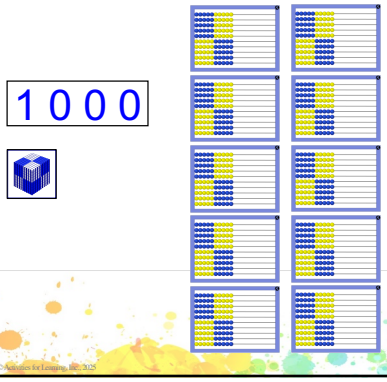
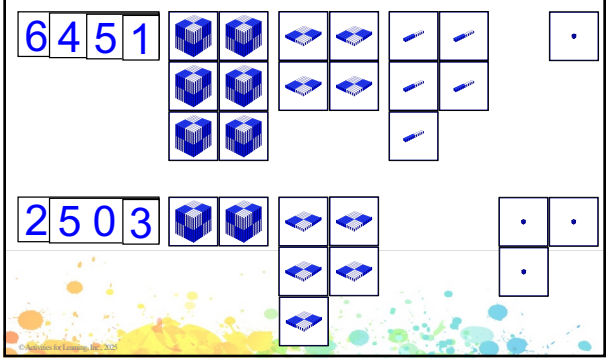
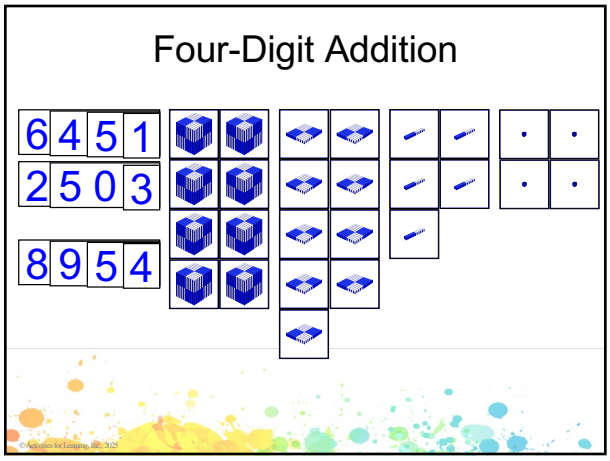
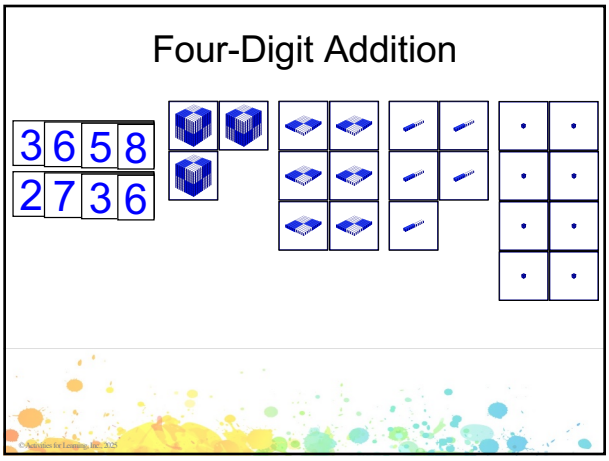
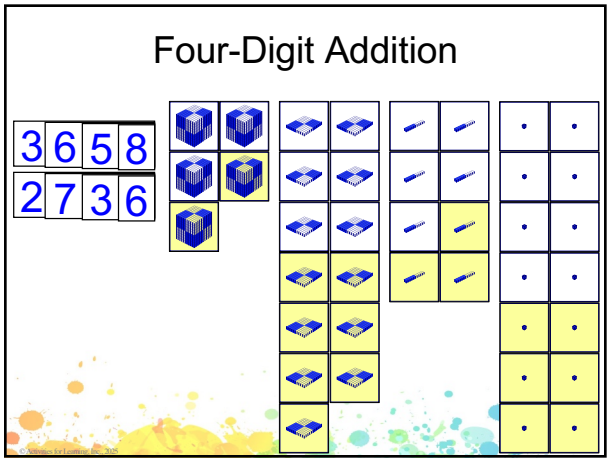


Four-Digit Addition




Four-Digit Addition
6 4 5 1
2 5 0 3
8 9 5 4


Four-Digit Addition
3 6 5 8
2 7 3 6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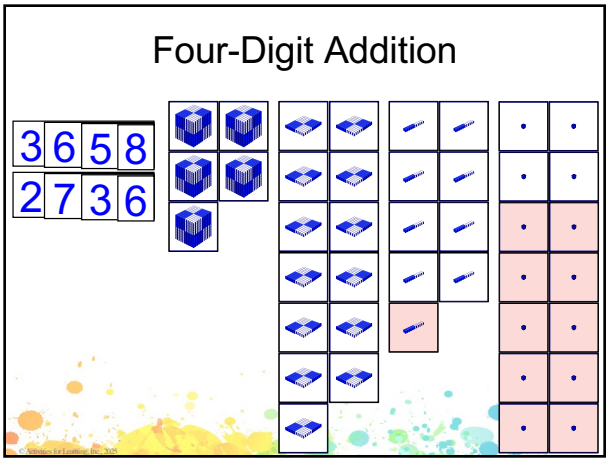
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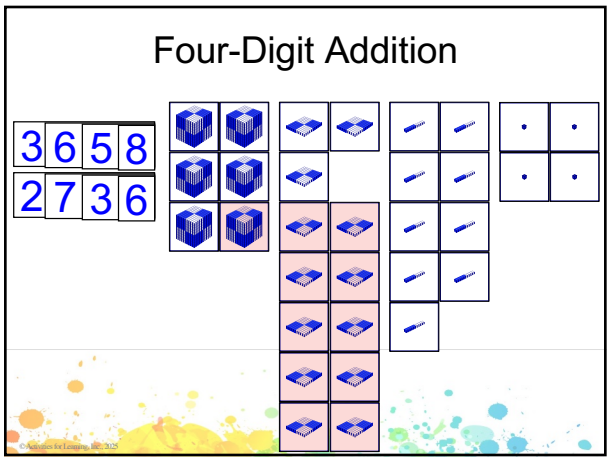
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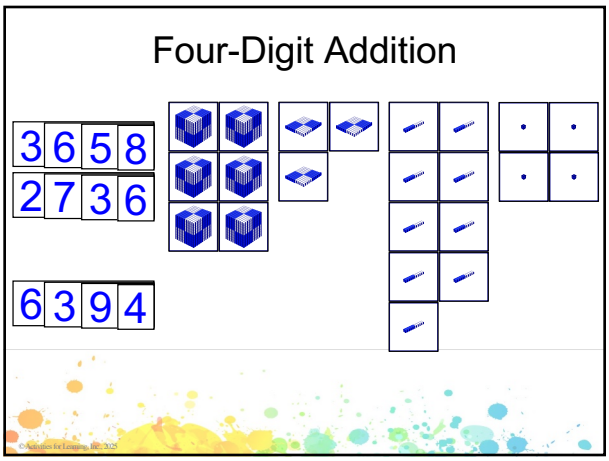
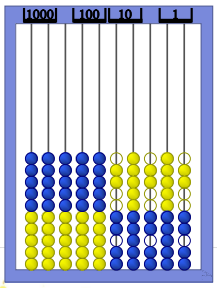
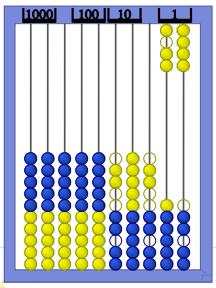

Four-Digit Addition
3 6 5 8
2 7 3 6
6 3 9 4

Addition with Trading

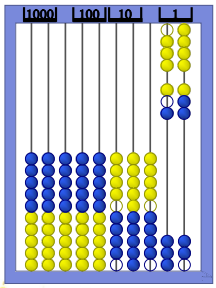


Addition with Trading



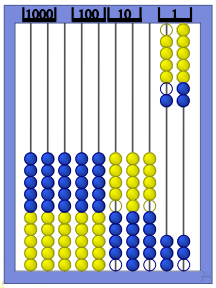
8
+ 6

Addition with Trading



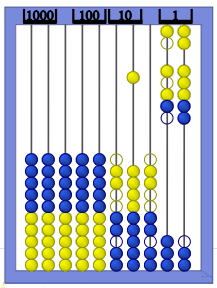
8
+ 6

Addition with Trading



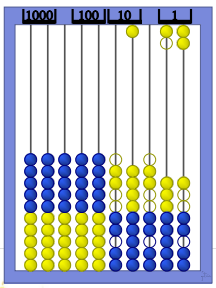
8
+ 6
14

Addition with Trading



8
+ 6
14

Addition with Trading

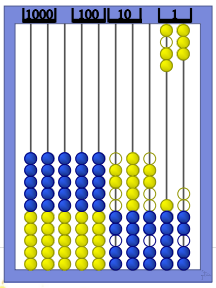


8
+ 6
14

Bead Trading Game

- Objective: To experience trading
  10 ones for 1 ten,
  10 tens for 1 hundred, and
  10 hundreds for 1 thousand.
- Goal: To reach 1000 on the abacus.

Bead Trading Game



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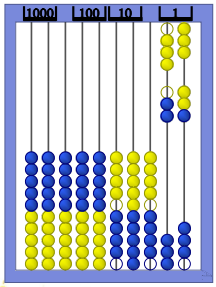
Bead Trading Game



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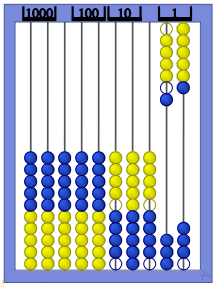
Bead Trading Game



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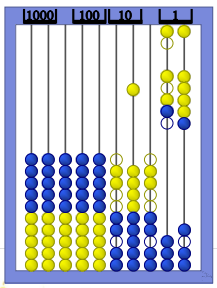

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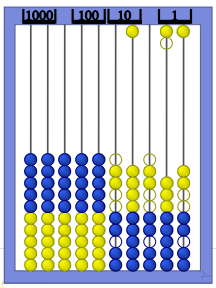

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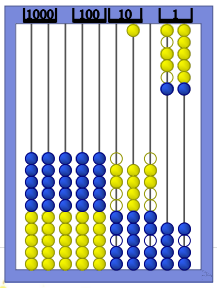

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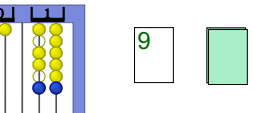

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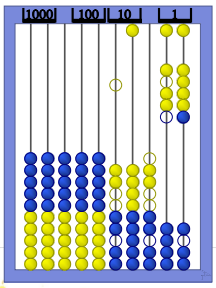

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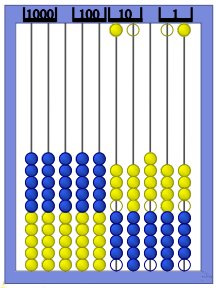

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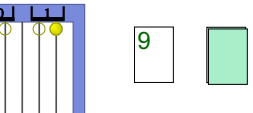

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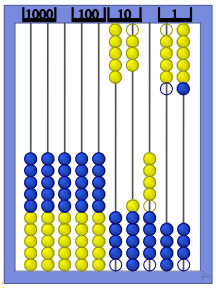

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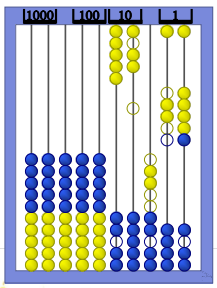
Bead Trading Game



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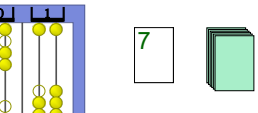

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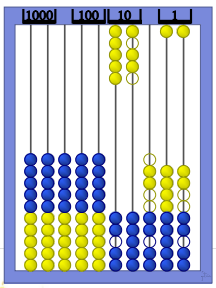

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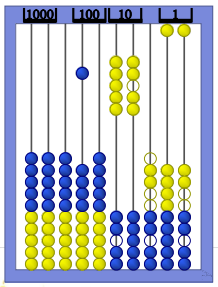

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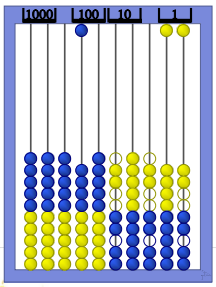

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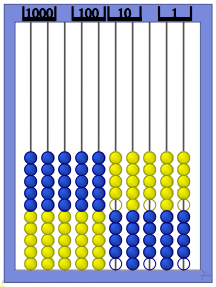
Bead Trading Game

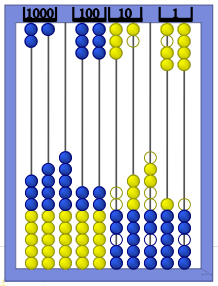
- In the Bead Trading Game, trading
  10 ones for 1 ten occurs frequently;
  10 tens for 1 hundred, less often;
  10 hundreds for 1 thousand, rarely.
- Bead trading helps the child experience the greater value of each column from left to right.
- In general, to detect a pattern, there must be at least three examples in the sequence.

Addition with Trading



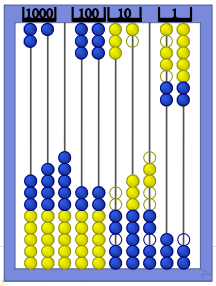
3658
+ 2736

Addition with Trading



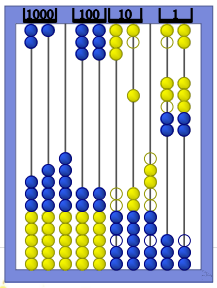
3658
+ 2736

Addition with Trading



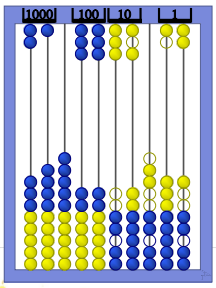
3658
+ 2736

Addition with Trading



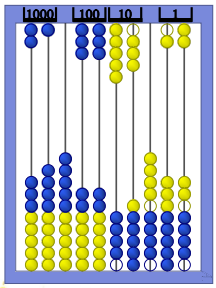
3658
+ 2736

Addition with Trading



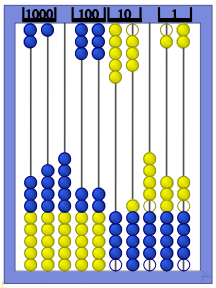
1
3658
+ 2736
4

Addition with Trading



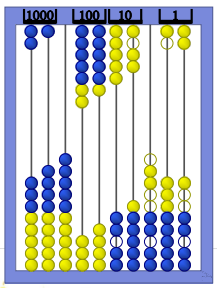
1
3658
+ 2736
4

Addition with Trading



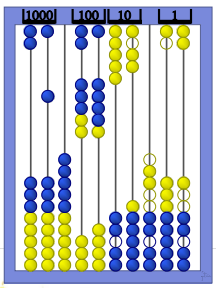
1
3658
+ 2736
94

Addition with Trading



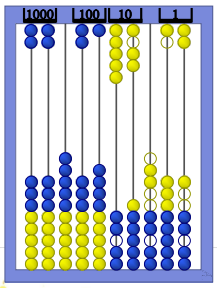
1
3658
+ 2736
94

Addition with Trading



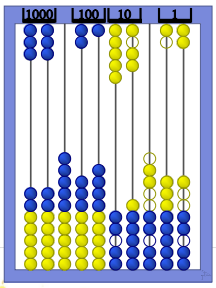
1
3658
+ 2736
94

Addition with Trading



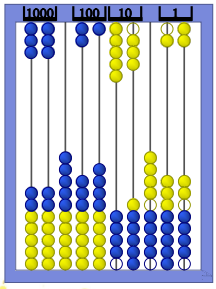
1 1
3658
+ 2736
394

Addition with Trading



1 1
3658
+ 2736
394

Addition with Trading



1 1
3658
+ 2736
6394

Advanced Bead Trading Game

- Objective: To create 3-digit and 4-digit numbers and add them all together.
- Cards: The place value cards less than 4000.
- Goal: To have a correct total sum of 10,995.

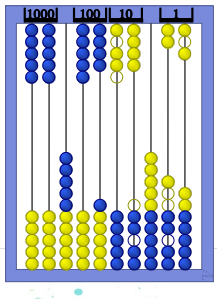
Advanced Bead Trading Game

5 | 80 | 500 | 3000 | 30 | 2 | 2000 | 90 | 20 | 60 | 7 | 400 | 800 | 3 | 700 | 1 | 9 | 600 | 1000 | 8 | 100 | 40 | 70 | 300 | 50 | 900 | 6 | 200 | 10 | 4

Advanced Bead Trading Game

2817
6394
1293
125
3968
344
571
786
452

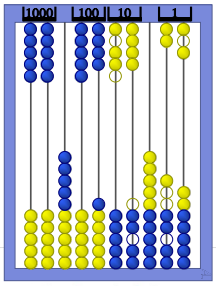
2817
+ 639
3456
+ 1293
4749
+ 125
4874
+ 3968
8842
+ 344
9186
+ 571
9757
+ 786
10543
+ 452
10995 ✓



Advanced Bead Trading Game

2	5	9
6	9	4
9	1	7
2	3	8	3
1	7	6	8
1	2	1
4	4	5
3	5	7	2
8	3	6

259
+ 694
953
+ 917
1870
+ 2383
4253
+ 1768
6021
+ 121
6142
+ 445
6587
+ 3572
10159
+ 836
10995 ✓



Multiplication Chart

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$4 \times 3 =$

Multiplication Chart

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$4 \times 3 = 12$

Multiplication Chart

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

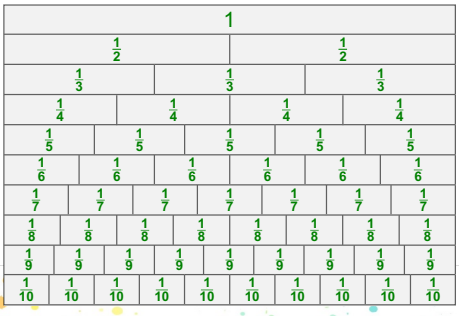
$5 \times 4 = 20$

Multiplication Chart

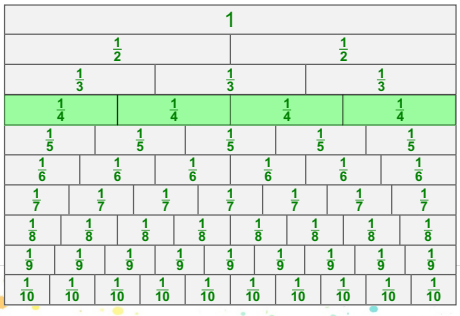
1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$8 \times 7 = 56$

Fraction Chart

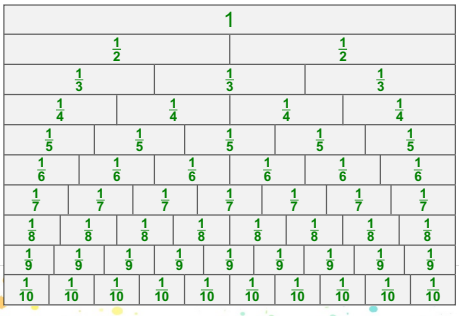


Fraction Chart



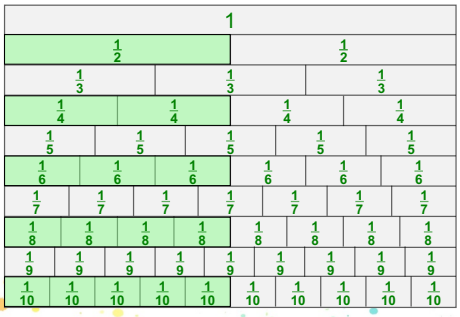
How many fourths in a whole?

Fraction Chart

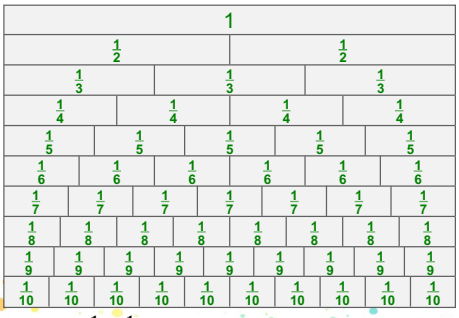


How many fourths in a whole? Fifths? Eighths?

Simplifying Fractions

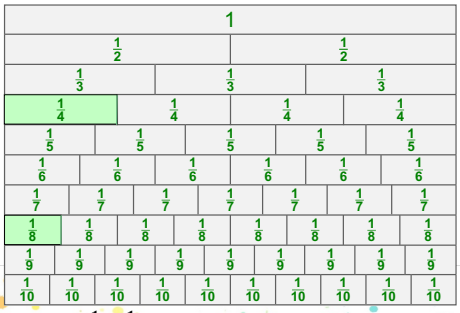


Fraction Chart



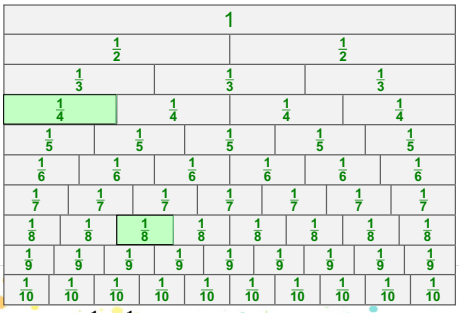
What is $\frac{1}{4} + \frac{1}{8}$?

Fraction Chart



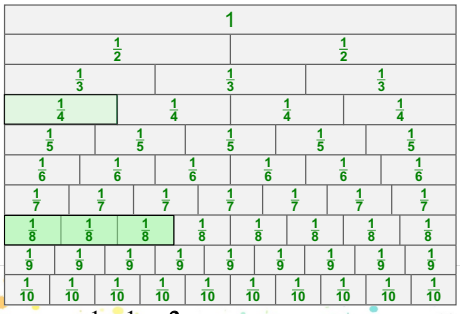
What is $\frac{1}{4} + \frac{1}{8}$?

Fraction Chart



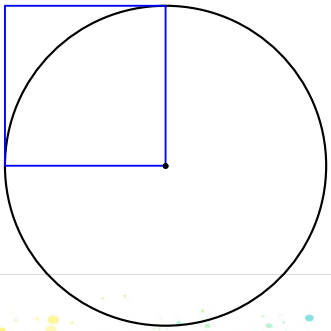
What is $\frac{1}{4} + \frac{1}{8}$?

Fraction Chart

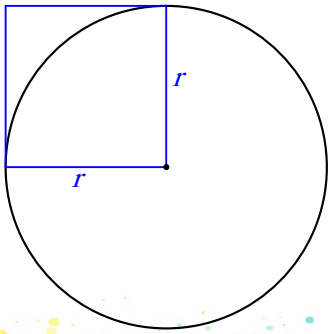


What is $\frac{1}{4} + \frac{1}{8}$? $\frac{3}{8}$

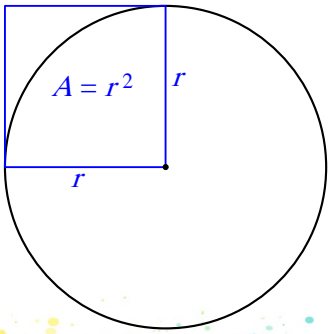
Area of a Circle



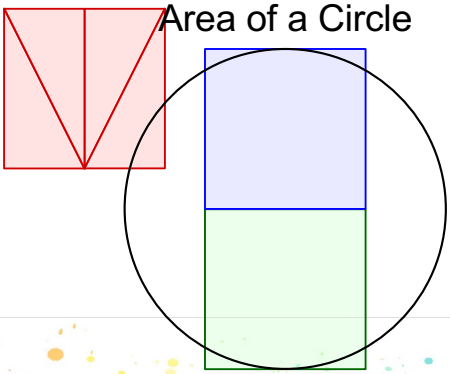
Area of a Circle



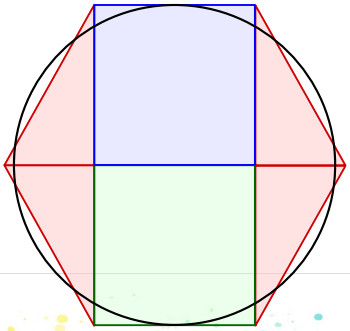
Area of a Circle



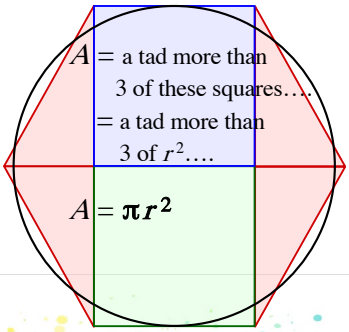
Area of a Circle



Area of a Circle



Area of a Circle



Visual Math Approach

- Math more than just numbers.
- Math is about patterns and relationships.
- Therefore, we need to encourage exploration and discovery in learning.
- Children need to see, touch, and physically manipulate objects, not watch someone do it for them.
- Children learn better when they are active.

Visual Math Approach

- Need good visual representations for mathematical concepts.
- Need hands-on exploration.
- **A visual approach is important for making abstract concepts concrete.**