Kids Who Learn Differently and Parents Who Want Answers!

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based on work of Joan A. Cotter, Ph.D.

Teaching Math

- Can be a challenge
- Especially when the child learns differently
- Often includes frustration, tears, anxiety
- Brings self-doubt for you and your child
- Learning differently is NOT an inability to learn
- Struggle is due to how math is taught; **NOT your child**

Learning Differently

- Approximately one in ten children learn differently.
- Traditional methods of teaching math are a source of frustration and failure.

"If a child can't learn the way we teach, maybe we should teach the way they learn."
– Michael J. Fox

Learning Differently

- Approximately one in ten children learn differently.
- Traditional methods of teaching math are a source of frustration and failure.
- Rote memorization is nearly impossible.

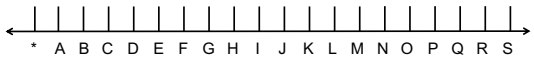
Rote Memorization

Because we're so familiar with 1, 2, 3, we'll use letters.

A = 1
B = 2
C = 3
D = 4
E = 5, and so forth

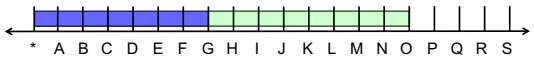
Rote Memorization

G + H = ____



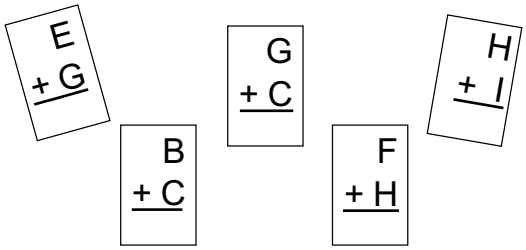
Rote Memorization

G + H = _O_



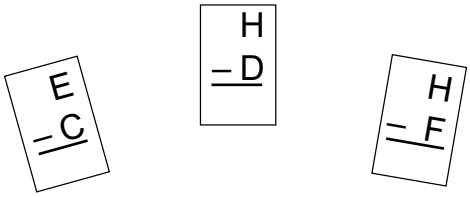
Rote Memorization

Now Memorize the Facts!!



Rote Memorization

Now try subtraction!



Learning Differently

- Approximately one in ten children learn differently.
- Traditional methods of teaching math are a source of frustration and failure.
- Rote memorization is nearly impossible.
- When memorization does occur, it doesn't stick very long.

Learning Differently

- Children learn better when they are active.
- They need to physically manipulate objects, not watch someone do it for them.
- Need to explore and learn.
- Manipulatives need to be visualizable.

Learning Differently

- Visual: seeing with our eyes.
- Visualizable: seeing in our mind's eye.
- Research shows that we learn best with visualizable images.
- For quantities to be visualizable, they must be grouped in fives and tens.

Subitizing

- Subitizing is the instant recognition of quantity.

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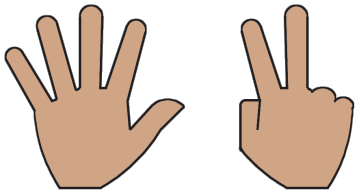
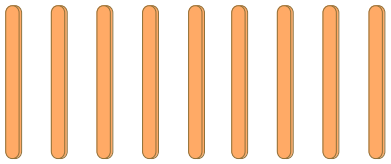


Subitizing

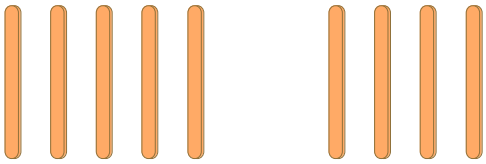
- Subitizing is the instant recognition of quantity.
- Subitizing can be extended when quantities are grouped in fives and tens.

Grouping in 5s



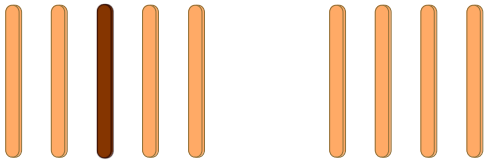
How many?

Grouping in 5s



How many?

Grouping in 5s

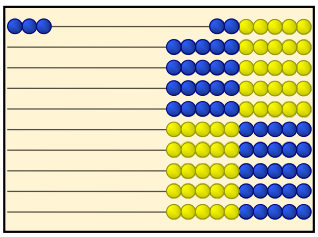


5 has a middle; 4 does not.

How the Brain Learns Best

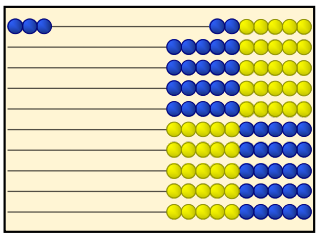
- Children need to touch quantities so they can begin to visualize the quantities.
- Children need to see math before they symbolize it with the written numerals.

Visualizable Quantities

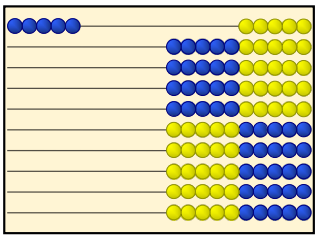


Visualizable Quantities

3

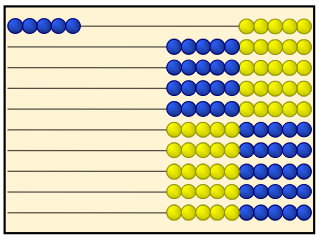


Visualizable Quantities

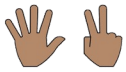


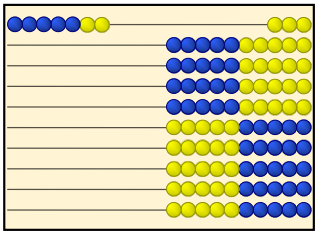
Visualizable Quantities

5



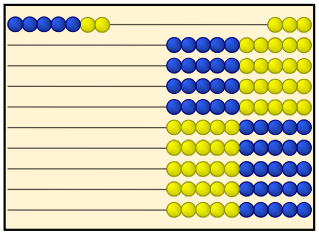
Visualizable Quantities



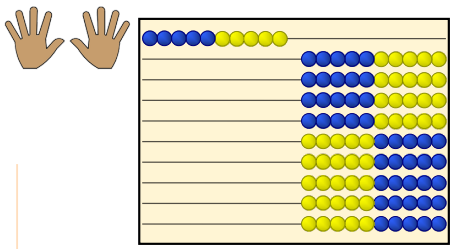


Visualizable Quantities

7

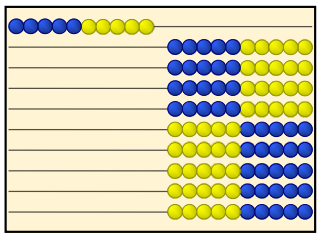


Visualizable Quantities



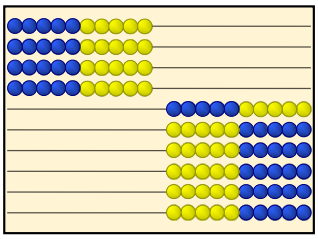
Visualizable Quantities

10



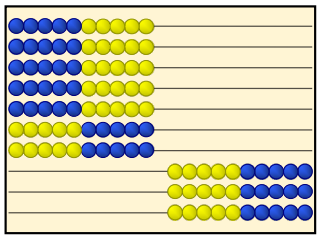
Visualizable Quantities

4-ten



Visualizable Quantities

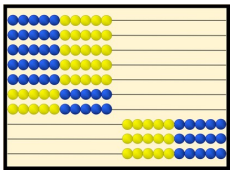
7-ten



Learning Differently

- Reduce the memory load.
  - No recital of the numbers 10 to 100.
  - Rather, use transparent number names.

7-ten



Transparent Number Names

11 = ten 1	20 = 2-ten
12 = ten 2	21 = 2-ten 1
13 = ten 3	22 = 2-ten 2
14 = ten 4	23 = 2-ten 3
. . . .	. . . .
19 = ten 9	. . . .
	99 = 9-ten 9

Transparent Number Names

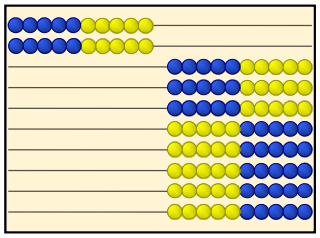
- Just as reciting the alphabet doesn't teach reading, counting doesn't teach arithmetic.
- Just as we first teach the *sound* of the letters, we first teach the *name* of the quantity (math way).

Transparent Number Names

- Only 11 words are needed to count to 100 the transparent number way; 28 in English.
- East Asian languages use the math way of number naming.
- These children understand place value in first grade; only half of U.S. children understand place value at the end of fourth grade.

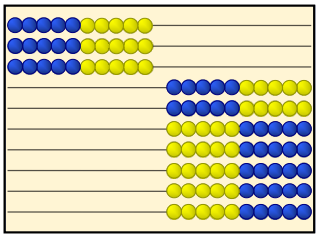
Transparent Place Value

2-ten 2 0



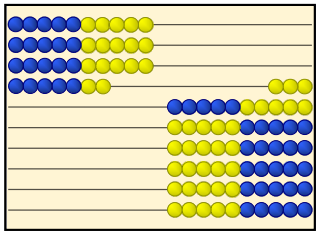
Transparent Place Value

3-ten 3 0



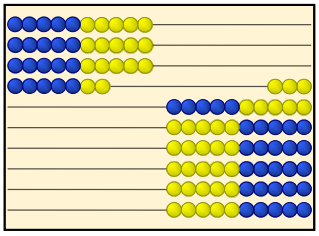
Transparent Place Value

3-ten 7 3 0 7



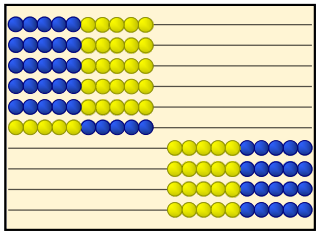
Transparent Place Value

3-ten 7 3 7



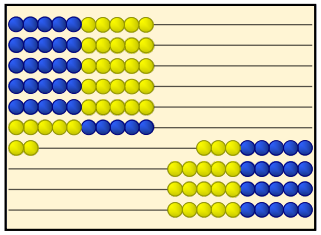
Transparent Place Value

6-ten 60



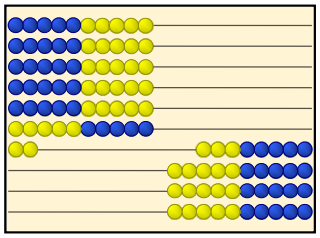
Transparent Place Value

6-ten 2 60 2



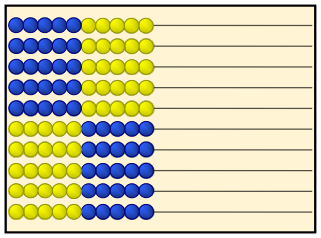
Transparent Place Value

6-ten 2 62



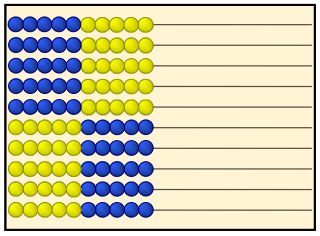
Transparent Place Value

10-ten 100



Transparent Place Value

1 hundred 100



Learning Differently

- Reduce the memory load.
  - No recital of the numbers 10 to 100.
  - Rather, use transparent number names.
    twenty-three is 2-ten 3
    seventy-four is 7-ten 4
  - Gives order and clarity to numbers.
  - Makes place value a natural part of numbers.

Learning Algorithms

- Mathematics is much more than a hodgepodge of algorithms and formulas.
- Use has changed greatly over the centuries.
- Used to add long columns of numbers and multiply large numbers together.
- Now this process is frequently handled by calculators and computers.
- Need to learn estimating skills and simple mental calculations.

Learning Algorithms

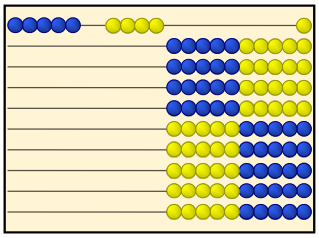
- Teach concepts before procedures.
- Research shows what is understood is retained much longer and is more likely to be applied to other situations.

Learning Algorithms

- A fact is considered to be known if it can be recalled in two or three seconds.
- With dyslexia and dyscalculia, mental processing takes longer, so three to five second is acceptable.
- Gives time to visualize, then produce the fact.

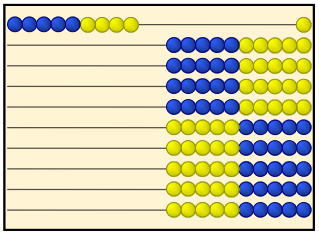
Adding

5 + 4 = ___



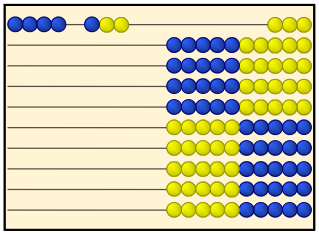
Adding

5 + 4 = 9



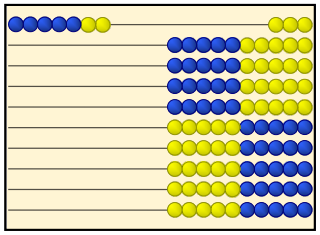
Adding

4 + 3 = ___



Adding

4 + 3 = 7



Learning Algorithms

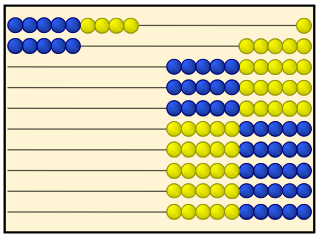
- A fact is considered to be known if it can be recalled in two or three seconds.
- With dyslexia and dyscalculia, mental processing takes longer, so three to five second is acceptable.
- Gives time to visualize, then produce the fact.
- Visual strategies help learn the facts.

Strategies

- A strategy is a way to learn a new fact or recall a forgotten fact.
- A visual representation is a powerful strategy.

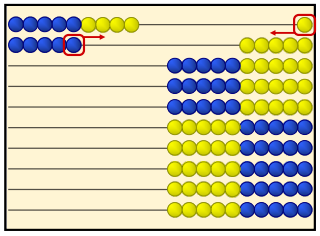
Strategy: Complete the Ten

9 + 5 = ___



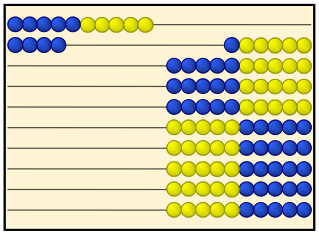
Strategy: Complete the Ten

9 + 5 = ___



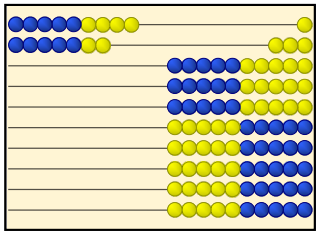
Strategy: Complete the Ten

9 + 5 = 14



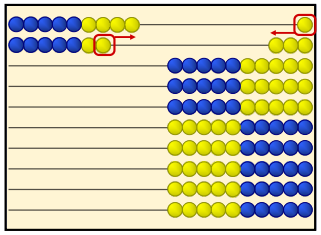
Strategy: Complete the Ten

$9 + 7 = \_\_\_$



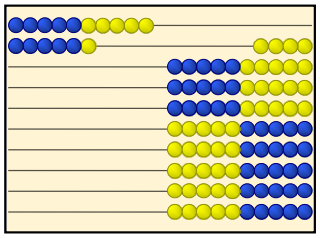
Strategy: Complete the Ten

$9 + 7 = \_\_\_$



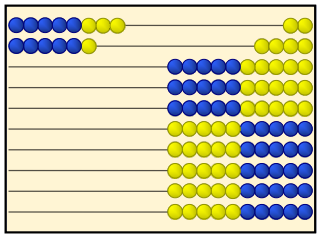
Strategy: Complete the Ten

$9 + 7 = 16$



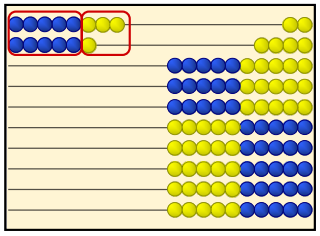
Strategy: Two Fives

$8 + 6 =$



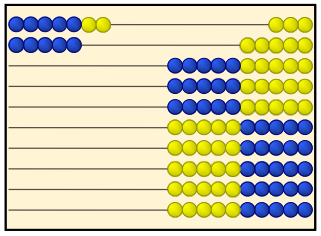
Strategy: Two Fives

$8 + 6 = 10 + 4 = 14$



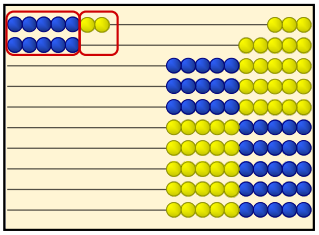
Strategy: Two Fives

$7 + 5 =$



Strategy: Two Fives

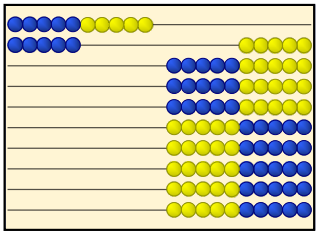
$7 + 5 = 10 + 2 = 12$



Strategy: Part from Ten

$15 - 9 = \_\_\_$

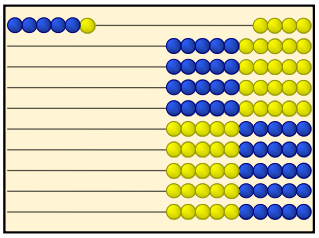
Subtract 5, then 4



Strategy: Part from Ten

$15 - 9 = \_6\_$

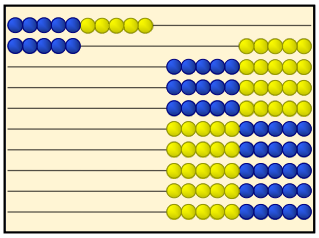
Subtract 5, then 4



Strategy: All from Ten

$15 - 9 = \_\_\_$

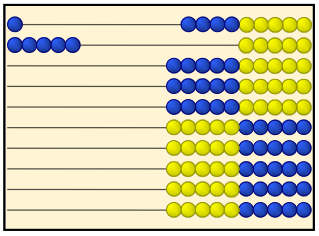
Subtract 9 from the 10



Strategy: All from Ten

$15 - 9 = \_6\_$

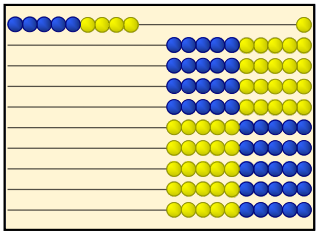
Subtract 9 from the 10



Strategy: Going Up

$15 - 9 = \_\_\_$

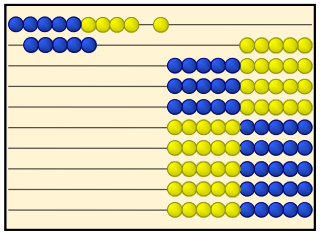
Start at 9; go up to 15



Strategy: Going Up

$15 - 9 = \underline{6}$

Start at 9; go up to 15

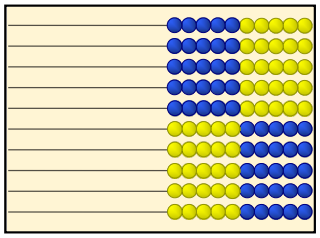


Multiplication

- Multiplication has been the mathematical downfall of many students (and adults).
- It's the problem of memorizing the 100 facts.
- Multiplication is often taught as repeated addition.
- This gives a limited view of multiplication.
- An array in rows and columns, like this abacus, makes a better model.

Multiplication

$6 \times 2 =$

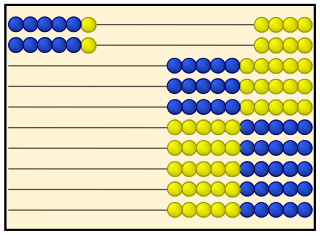


Multiplication

- There are different interpretations about the meaning of 6×2.
  - $6 + 2$: start with 6 and transform by adding 2
  - $6 - 2$: start with 6 and transform by decreasing 2
  - $6 \div 2$: start with 6 and transform it by dividing it into either 2 groups or groups of 2
- Therefore, to be consistent, 6×2 starts with 6 and transforms it by duplicating it.

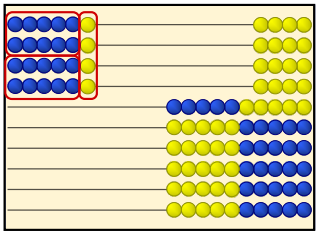
Multiplication

$6 \times 2 = 12$



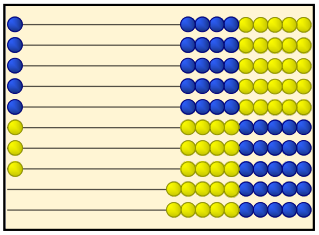
Multiplication

$6 \times 4 = 24$



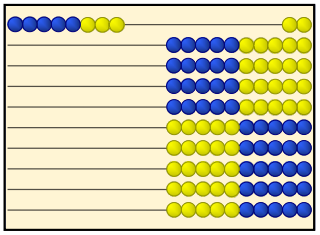
Multiplication

$1 \times 8 = 8$



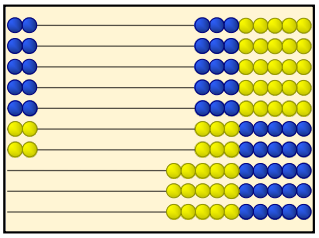
Multiplication

$8 \times 1 = 8$



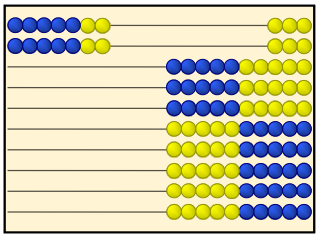
Multiplication

$2 \times 7 = 14$



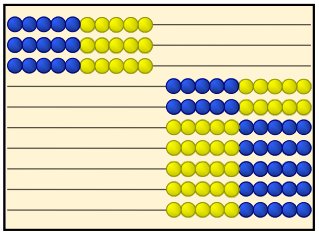
Multiplication

$7 \times 2 = 14$



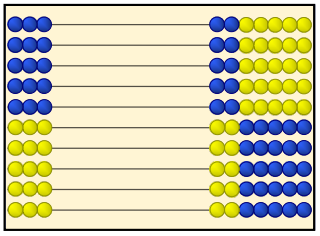
Multiplication

$10 \times 3 = 30$



Multiplication

$3 \times 10 = 30$



Multiplication

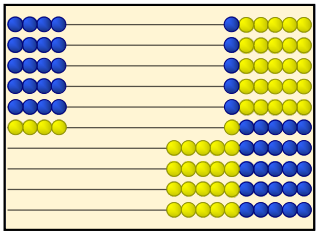
- In a 10 by 10 multiplication table, the commutative property reduces the number of facts from 100 to 55 facts.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

- Facts of 1s, 2s, and 10s are generally easy.
- Now there are only 28 facts left to learn!

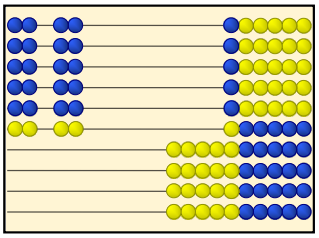
Multiplication Strategies

$4 \times 6 =$



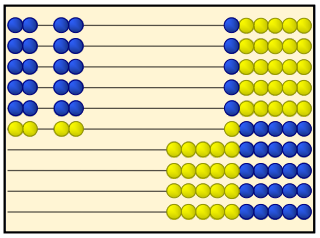
Multiplication Strategies

$4 \times 6 = 2 \times 6 \times 2$



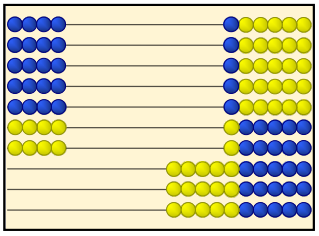
Multiplication Strategies

$4 \times 6 = 12 \times 2 = 24$



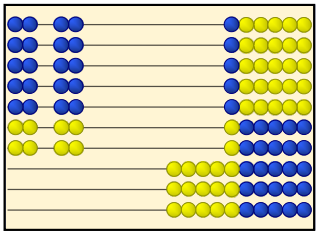
Multiplication Strategies

$4 \times 7 =$



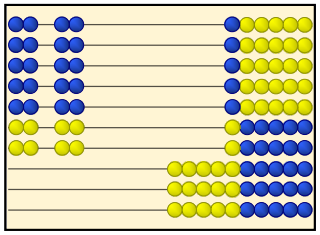
Multiplication Strategies

$4 \times 7 =$



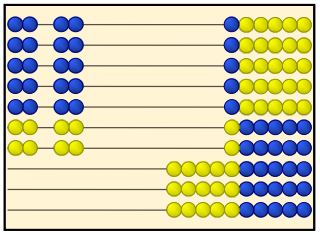
Multiplication Strategies

$4 \times 7 = 2 \times 7 \times 2$



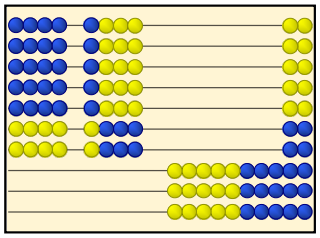
Multiplication Strategies

$4 \times 7 = 14 \times 2 = 28$



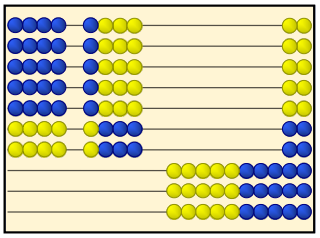
Multiplication Strategies

$8 \times 7 =$



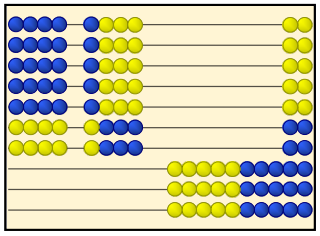
Multiplication Strategies

$8 \times 7 = 4 \times 7 \times 2$



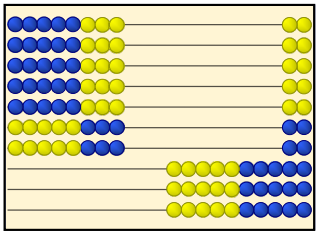
Multiplication Strategies

$8 \times 7 = 28 \times 2 = 56$



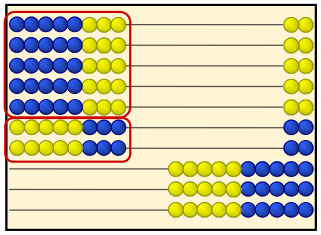
Multiplication Strategies

$8 \times 7 =$



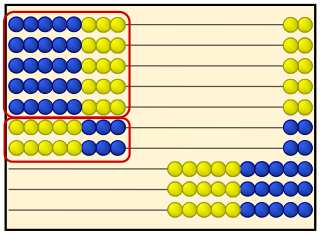
Multiplication Strategies

$8 \times 7 = 8 \times 5 + 8 \times 2$



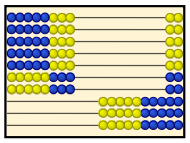
Multiplication Strategies

$8 \times 7 = 40 + 16 = 56$



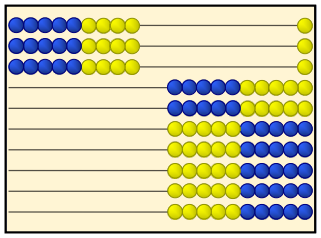
Multiplication Strategies

- Adults generally think in pictures.
- Children definitely think in pictures.
- This approach provides solid visualizable strategies.



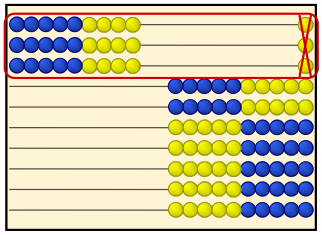
Multiplication Strategies

$9 \times 3 =$



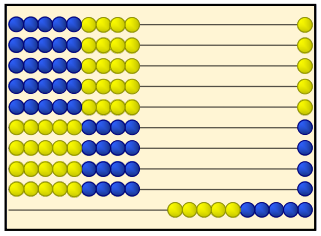
Multiplication Strategies

$9 \times 3 = 30 - 3 = 27$



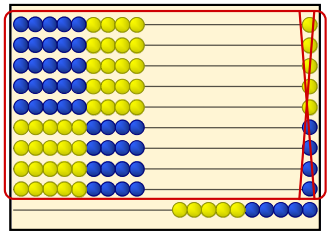
Multiplication Strategies

$9 \times 9 =$



Multiplication Strategies

$$9 \times 9 = 90 - 9 = 81$$



Learning Algorithms

- A fact is considered to be known if it can be recalled in two or three seconds.
- With dyslexia and dyscalculia, mental processing takes longer, so three to five second is acceptable.
- Gives time to think about a strategy, then produce the fact.
- Visual strategies help learn the facts.
- Use games, not worksheets, for practice.

Why Games

$$\frac{\text{Games}}{\text{Math}} = \frac{\text{Books}}{\text{Reading}}$$

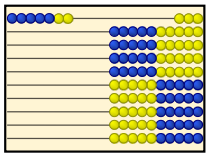
Games provide interesting repetition needed for automatic responses.

More importantly, games provide an application for the new information!

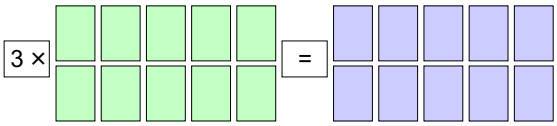
Go to the Dump Game

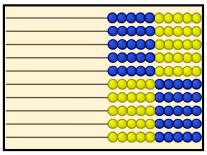
A "Go Fish" type of game where the pairs are:

1 & 9
2 & 8
3 & 7
4 & 6
5 & 5

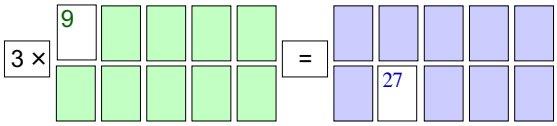


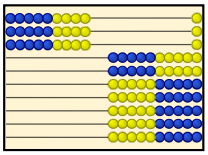
Multiplication Memory





Multiplication Memory



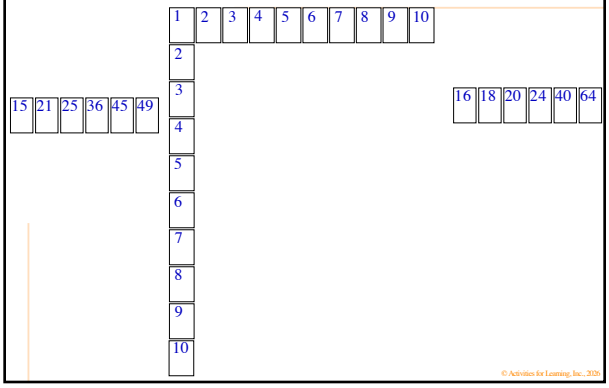


Multi-Fun

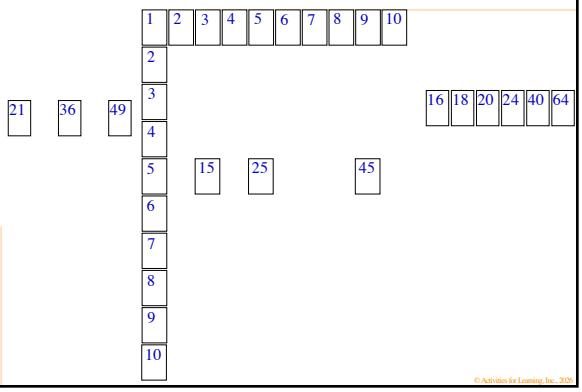
Building the multiplication table as a game.

- Each player draws six cards from the stock, then replaces the cards used, keeping six cards in hand.
- Lay as many cards as possible in the proper spaces in either a row or column.
- Only one row or column can be used during a turn.
- One point is awarded for each card played.

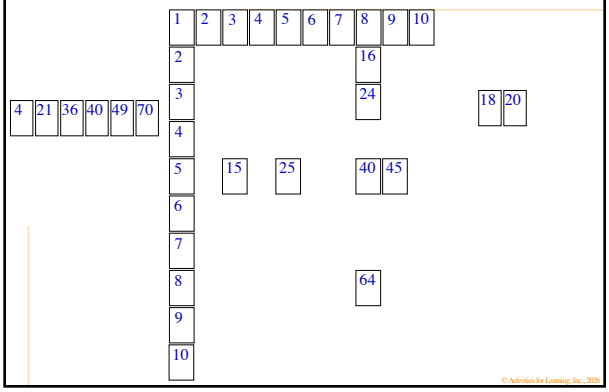
Multi-Fun



Multi-Fun



Multi-Fun



Multi-Fun

	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Division

quotient remainder

divisor) dividend

Division

- Short division before long division.

Short Division

$$3\overline{)4^{1}7^{2}1}\quad 157$$

Short Division

$$9\overline{)8\,0^{8}5^{4}3}\quad 894\text{ r}7$$

Remainder Problems

Problem 1.

Thirteen children are going on a field trip. If 4 children can ride in a car, how many cars are needed?

[4 cars]

Remainder Problems

Problem 2.

Pauline has 13 petunias to plant. She wants exactly 4 in a row. How many rows can she plant?

[3 rows]

Remainder Problems

Problem 3.

Four children have $13 to split evenly. How much does each one receive?

[$3.25]

Remainder Problems

Problem 4.

Four children divide 13 candy bars. How much does each one receive?

[$3\frac{1}{4}$ bars]

Remainder Problems

Problem 5.

Jack packages 13 cookies with 4 per bag and eats the leftovers. How many does he eat?

[1 cookie]

Remainder Problems

Five problems with same numbers, but five different answers.

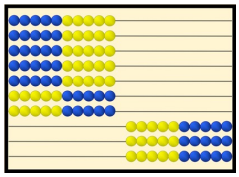
[4 cars]
[3 rows]
[$3.25]
[$3\frac{1}{4}$ bars]
[1 cookie]

Summary

- Traditionally: Insisting all children memorize the counting words to 100 before doing any meaningful math.
  - About 20% of children have difficulty with the counting words and often fall behind.
- Solution: Teach the names of quantities to 10, use transparent number names for 11 to 99, then connect to the traditional names.

Summary

- Traditionally: Ignoring children's ability to visualize.
- Solution: Use visualizable manipulatives that group quantities in fives as well as tens.



Summary

- Traditionally: Using flash cards and timed tests.
- Solution: Teach strategies for learning facts.
- Solution: Use games the children enjoy for practice.

Summary

- Traditionally: Teaching math like it's a bunch of rules without rhyme or reason.
  - This makes advanced math much more difficult and applications mystifying.
- Solution: Teach for understanding.
  - Ask questions that require the child to think, not just parrot an answer.

In Conclusion …

Math needs to be taught so
95 percent is understood and
only 5 percent memorized.

– Richard Skemp
major pioneer in mathematics education

In Conclusion …

Our goal as a teacher of mathematics
is to help our children transform,
expand, and refine these beginning ideas
into deeper mathematical thinking.

– Dr. Joan A. Cotter