Activities for Learning, Inc.

RightStart™ Mathematics

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MULTIPLICATION LESSON EXCERPTS

Transition Lessons

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Lesson T32

Working with Fours and Eights

OBJECTIVE 1. To look the 4s and 8s multiples and facts in new ways

MATERIALS Abacus
Cards and envelopes for Multiples Memory, P2 and Multiplication Memory, P10

ACTIVITIES ***Visualizing the multiples patterns.*** Memorizing the multiplication facts is approached through visualizing the multiples (also called skip counting) patterns, as shown below, and through visualizing arrays. The child should not recite multiples to find a multiplication fact.

The terms needed to discuss multiplication are: multiplicand "×" multiplier = product. That is, 8 × 4 is interpreted as 8 repeated 4 times, not as 8 groups of 4. This is consistent with the other operations of arithmetic:

8 + 4 means start with 8 and transform it by adding 4.

8 – 4 means start with 8 and transform it by removing 4.

8 ÷ 4 means start with 8 and transform it by breaking it into either 4 groups or groups of 4.

8 × 4 means start with 8 and transform it by duplicating it a total of 4 times.

2	4	6	8	10
12	14	16	18	20

The multiples of 2.

4	8	12	16	20
24	28	32	36	40

The multiples of 4.

8	16	24	32	40
48	56	64	72	80

The multiples of 8.

Multiples are learned more quickly and easily when children see patterns, see the figures at the left. In addition to these patterns being helpful for learning the multiplication facts, recognizing multiples is essential for simplifying fractions and algebra.

Arrange the 2s in two rows that shows the ones repeating. The second row is 10 plus the first row. Notice that all the multiples for the 2s are even.

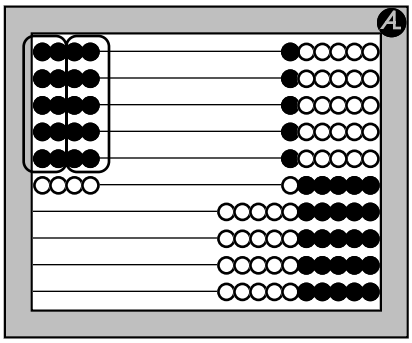
With the 4s arranged in two rows, the ones repeat. Observe that the first row of 4s is every other number of the 2s. The second row is 20 more than the first row.

The 8s, of course, are every other 4 in the first row. Also notice that in each row, the ones are the even numbers in descending order. In each row the tens increase by 1 and the second row is 40 more than the first row.

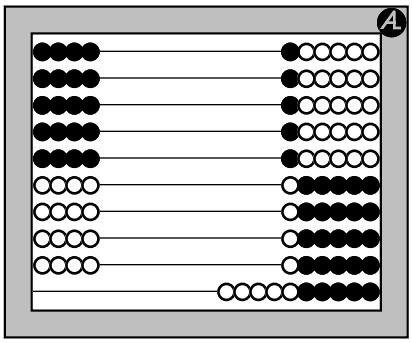
Multiples Memory game. Play the game Multiples Memory (*Math Card Games,* P2) with the child. This simple game teaches the multiples patterns visually. Play it several times, with the child using the 4s or the 8s multiples.

Abacus practice with 4s. Ask the child to enter 4 × 6 on the abacus (see figure on top left on the next page). Ask him to find its value and to explain how he did it. One way is to notice that two columns of 5 equal 10, making 20. Adding 20 to the 4 gives 24.

Next the child to enter 4 × 9 and to find the value. One way is to see 20 + 16; another way is to think if each column were 10, the total would be 10 × 4 = 40, but it's actually 9, so 40 – 4 = 36.



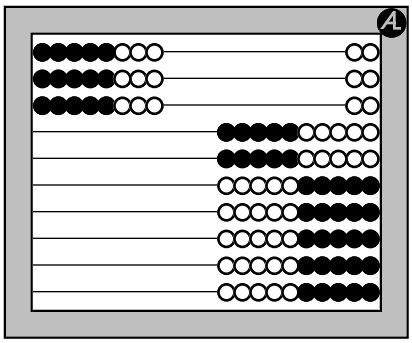
Showing 4 × 6 = 20 + 4 = 24.



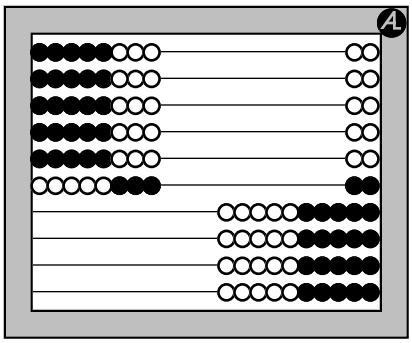
Showing 4 × 9 = 40 – 4 = 36.

Ask the child What position are 24 and 36 on the 4s multiples patterns? [6th and 9th] Repeat for other 4s facts.

Abacus practice with 8s. Ask the child to enter 8 × 3 on the abacus (see left figure below). Ask for the total. It can be see as 15 + 9 or 30 – 6.



Showing 8 × 3 = 20 + 4.



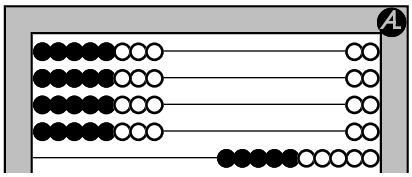
Showing 8 × 6 = 40 = 4.

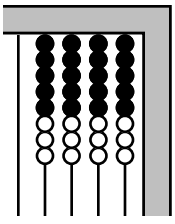
Repeat for 8 × 6 (see right figure above). One way is to notice 10 × 6 = 60 and 60 – 12 = 48. Another way is seeing the upper left dark beads as 25, then adding the 20 light beads, and finally adding the dark 3 to get 48. Of course, the child might notice 8 × 6 is double 8 × 3, so the answer would double 24, or 48.

Note: "Seeing the upper left dark beads as 25" is discussed in Lesson T18, All Five Coins.

Ask the child What position are 24 and 48 on the 8s multiples patterns? [3rd and 6th] Repeat for other 8s facts.

The commutative property. Ask the child to enter 8 × 4 on the abacus and then ask him to turn the abacus 90 degrees. See the figures below. Now what does it look like? [4 × 8] Which is more, 8 × 4 or 4 × 8? [They are the same.]





Demonstrating that 8 × 4 is equal to 4 × 8.

Multiplication Memory game. Play the game Multiplication Memory (*Math Card Games,* P10) with the child using the 4s and 8s. This game provides practice for learning those facts.

Lesson T33

Working with Threes and Sixes

OBJECTIVE 1. To look the 3s and 6s multiples and facts in new ways

MATERIALS Abacus
Cards and envelopes for Multiples Memory, P2 and Multiplication Memory, P10

WARM-UP Write the multiples patterns for the 4s and 8s.

ACTIVITIES ***The 3s multiples.*** Use the 3s envelope or write the 3s as shown on the right. Ask the child if she sees any patterns. Looking in each column, notice the ones decrease by one. Also note the 0-9 sequence starting at the lower left, moving up, and continuing at the bottom of the next column.

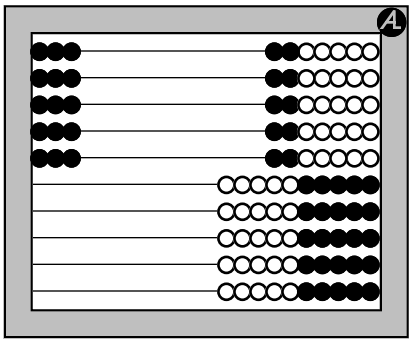
3	6	9
12	15	18
21	24	27
30		

The multiples of 3.

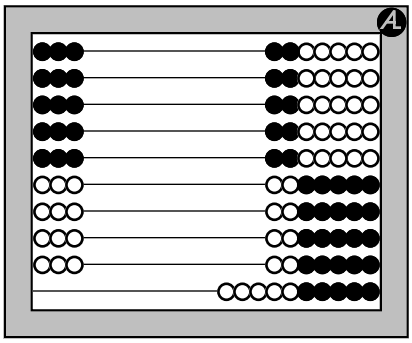
In each column the tens have the pattern, 0, 1, 2. Also add the digits: in the first column they are 3 (1 + 2 & 2 + 1); in the second column, 6; and the third, 9.

Ask the child to write the 3s pattern. Can you use the pattern and find 3 × 5? [second row, second column] Repeat for 3 × 7. [third row, first column] Repeat for other multiples.

Abacus practice with 3s. Ask the child to enter 3 × 5 on the abacus. Can you see the product (answer)? See the left figure below. Two columns of 5 make 10, and 10 + 5 = 15.



Abacus showing 3 × 5. = 10 + 5



Showing 3 × 9 = 30 – 3 = 27.

Now ask her to enter 3 × 9 to find the total. See the right figure above. One way is to add 15 (from the previous example) and 12 to get 27. Another way is think that if the columns were 10, rather than 9, the product would be 30. But that is 3 too much, so the answer is 27.

Repeat for other facts of 3, especially for a child that may have difficulty remembering. To help the child develop a mental picture of arrays, enter an array and let the child see it for only 3 seconds. Then ask her which array it was and to find the product.

The 6s multiples. Show the child the multiples of 6 in two rows as shown on the right.

6	12	18	24	30
36	42	48	54	60

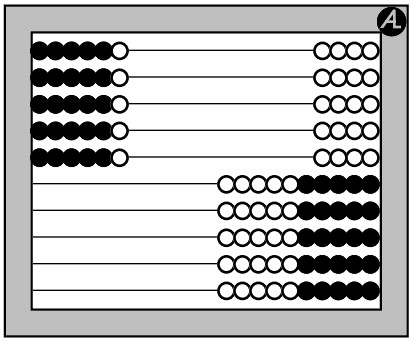
The multiples of 6.

What patterns do you see with the 6s multiples? [All the numbers are even. The ones are the same in the two rows.] What is the difference between the numbers in each row? [30]

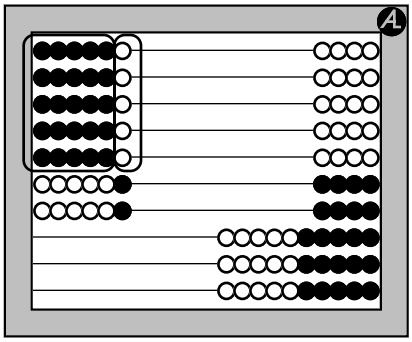
Ask the child to look at the 3s multiples. What happens if you say only the even numbers? [It's the first row of the 6s.]

While she looks at the multiples pattern, ask What is 6 × **2**? [1**2**] What is 6 × **4**? [2**4**] What is 6 × **6**? [3**6**] What is and 6 × **8?** [48**]** Ask what pattern she noticed. [The product ends in the same digit as the number being multiplied (for even numbers).] Does it works when multiplying by odd numbers, such as 6 × 3 or 6 × 5? [no]

Abacus practice with 6s. Enter 6 × 5 on an abacus and flash it for 3 seconds. See the left figure below. What is the array? [6 × 5]



Abacus showing 6 × 5.



Abacus showing 6 × 7.

Then ask her to look at the abacus and explain how she could find the total. There are six groups of 5s, making three 10s, or 30. Or, start with the 25: add the column of 5 light beads, then the 10 light beads, and finally the last 2 dark beads.

Ask the child to enter 6 × 7 and to find the product. One way is to start with the 25, and then add 5 from the adjacent column. Lastly add to the 30 a 10 and a 2, giving 42. Repeat for other facts.

Comparing 6s to 3s. Ask the child to write the multiples of 3 in a row and the multiples of 6 directly below as shown.

3	6	9	12	15	18	21	24	27	30
6	12	18	24	30	36	42	48	54	60

What pattern do you see? [The 6s are double the 3s.]

Ask the child to look at the 3s multiples pattern and say only the even threes. What pattern do you hear? [the 6s]

Games. Play Multiples Memory and Multiplication Memory (*Math Card Games,* P2 and P10) games with the child using the 3s and 6s.

Lesson T34

Working with Nines and Sevens

OBJECTIVE 1. To look the 9s and 7s multiples and facts in new ways

MATERIALS Abacus
Cards and envelopes for Multiples Memory, P2 and Multiplication Memory, P10

WARM-UP Write the multiples patterns for the 8s and 6s.

ACTIVITIES ***The 9s multiples.*** The 9s are the most interesting of all patterns; the sums of the digits in each case equal 9. The digits can also be reversed to obtain another multiple as shown in the second row, which is written backward. Also note in the rows that the ones decrease by 1 as the tens increase by 1.

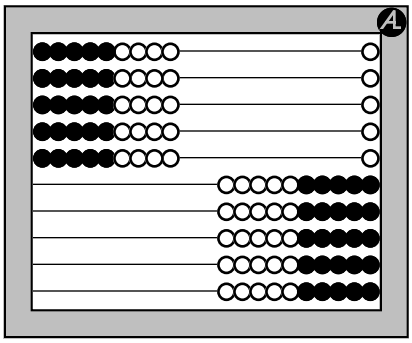
9	18	27	36	45
90	81	72	63	54

The multiples of 9.

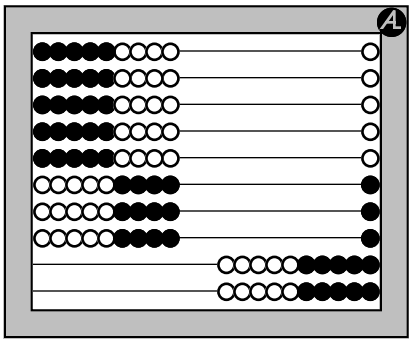
Show the child the 9s multiples. What are some special things you notice about these 9s multiples? [The second row is the same as the first row with digits reversed.] What happens when you add the digits for the these multiples? [The sum is always 9.]

Ask the child to write the 9s pattern. Can you use the pattern and find 9 × 5? [first row, last column] Repeat for 9 × 6. [second row, last column] Repeat for 9 × 9. [second row, second column]

Abacus practice with 9s. Ask the child to enter 9 × 5 on the abacus. Can you find the product? See the left figure below. The easiest way is to see it as 50 – 5.



9 × 5 = 50 – 5 = 45.



9 × 8 = 80 – 8 = 72.

Next ask for 9 × 8. See the right figure above. Again, the easiest way is to see it as 80 – 8.

The 7s multiples. The best pattern for the 7s multiples is found with only three in a row, similar to the 3s. See the pattern at the right. Each number from 1 to 9 is used once in the ones place. This means that no two 7 facts have the same number in the ones place. In each column, the ones increase by 1 and the tens increase by 2. In the rows, the tens increase by 1.

7	14	21
28	35	42
49	56	63
70		

The multiples of 7.

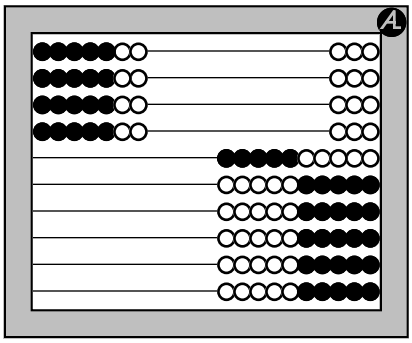
Also the ones increase by 1, starting at the right top and going down the columns going from right to left. (2**1**, 4**2**, 6**3**, 1**4**, and so

forth). And within each row, the tens increase by 1 (**0**7, **1**4, **2**1, and **2**8, **3**5, **4**2).

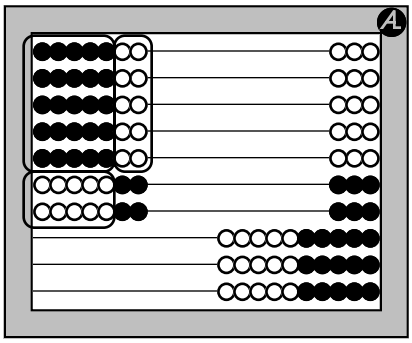
Ask, What patterns do you see with ones in the 7s multiples? Tens? Are the numbers even or odd? [both, they alternate] Ask him to look at the multiples listing. Then ask, What is 7 × 3? [21], 7 × 6? [42], 7 × 9? [63], 7 × 7? [49], and so forth.

Ask, What happens when you multiply 7 by an even number? [even number] What happens when you multiply 7 by an odd number? [odd number]

Abacus practice with 9s. Enter 7 × 4 on an abacus and flash it for 3 seconds. What is the array? [7 × 4] Then ask him to study the abacus to find the product. See the left figure below. The dark beads form two 10s; the light beads are 8, giving a total of 28.



7 × 4 = 20 + 8 = 28.



7 × 7 = 25 + 10 + 10 + 4 = 72.

Flash the abacus with 7 × 7 for 3 seconds. See the right figure above. How can you figure out how much it is? One method is to note the 5 × 5 beads equaling 25, then add the two groups of 10 light beads and the final 4.

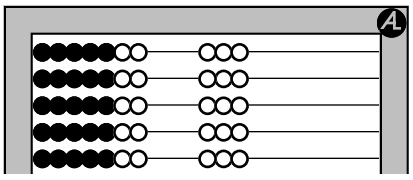
The 7s plus 3s surprise. Set the 7s multiples and 3s multiples side by side as shown on the right. Ask the child to add the corresponding multiples.

7	14	21
28	35	42
49	56	63
70		

3	6	9
12	15	18
21	24	27
30		

For example, What is 7 + 3? [10] What is 14 + 6? [20] What is 21 + 9? [30] Continue to 70 + 30. What is the pattern? [Sums are increasing multiples of 10.]

If the child is interested in why this works, ask him to enter 7 × 5. (See right figure.) Then ask him what he sees on the top 5 wires on the right side of the abacus? [3 × 5]



7 × 5 and 3 × 5 = 10 × 5.

Games. Play Multiples Memory and Multiplication Memory (*Math Card Games*, P2 and P10).